Lab 2:

Mechanical Equivalent of Heat

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Intermediate Experimental Physics
Section 002

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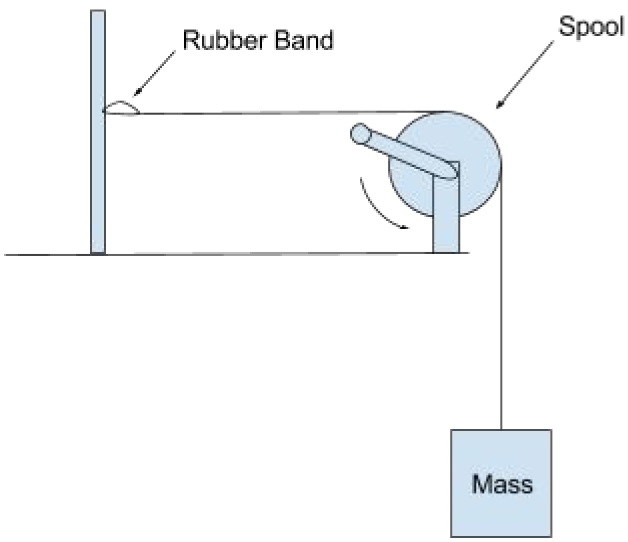
The Objective of this week's experiment is to verify the widely-accepted idea that heat can be said to have mechanical equivalence. That is to say that, for a given amount of **work**, there exists some corresponding **heat** that is produced by a change in **temperature**.

Theoretical Background/ Abstract:

The work done on an object is often defined as the integral of force on the object with respect to the distance over which the force is applied. This can be stated mathematically as:

$$W = \int_a^b F \cdot dx$$

Where W is the work done on the object and F is the force applied.



In this experiment, we will use a tension system in which a mass is connected to an aluminum spool via a non-elastic cord. The cord will be wrapped around the spool a few times before meeting a rubber band connected to an anchored post. The rubber band is put in place to ensure that the full weight of the mass is taken up by the spool, by keeping the tension in the upper length of cord to a minimum. The cord is wrapped around the spool several times for the same reason.

The mass is suspended just above the floor and should not move when the spool is turned. This is to say that the cord should slip completely. It is this slippage, and the resultant frictional forces, that heat up the spool in this experiment.

Because the hanging mass does not move, no work is done on it by the turning of the crank. Instead, it is the lower portion of *cord* that does work on the *spool* when the crank is turned. "How so?" you may ask...

How so?

Glad you asked! As we know, the work done on an object (in this case, the spool) is equal to the integral of Force with respect to distance, or in the case of a constant force, force times the distance traveled by the object. But in this case, you may ask, *where is the distance component?* The spool doesn't appear to traverse over any distance. It turns out that the spool *does* travel over some distance, relative to the stationary cord: this is given by the circumference of the spool times the number of times it was turned: $N2\pi R$. The work done on the spool by the cord is equal to the tension force in the string, F_T, which is equal to the weight of the hanging mass, mg. The product of these quantities gives us the work done on the spool by the cord:

$$W = N2\pi R \cdot mg$$

Where N is the number of turns of the spool, and R is the radius of the spool. This makes $2\pi R$ the circumference of the spool, and $N2\pi R$ the “distance” that the string traveled along the spool. mg is the mass of the pale, times the acceleration due to gravity; the “weight” of the object.

Procedure:

1. Procure a spool, mass, and a length of cord; as well as a multimeter and a pair of leads.
2. Set up the spool, mass, and cord to mirror the diagram on the previous page. Connect the leads to the multimeter and the terminals found on the non-crank side half of the spool.
3. Bathe the spool in a bucket of ice water for several minutes
4. When finished, dry off the spool and re-attach it to the base. Wrap the cord around it several times, and record the resistance of the thermistor.
5. Turn the spool until the thermistor until it has reached a resistance that corresponds to a temperature as far away from room temperature as the spool was when initially attached to the base. Assuming that the spool was 0ºC when it came out of the ice water, this temperature should be in the neighborhood of 40ºC– quite hot. This is all to say that the graph of the spool’s temperature should be symmetric about the ambient temperature of the lab environment.
6. Record the final resistance of the semiconductor.
7. Cool the spool in the ice water bath again and, with the multimeter, record the length of time that it takes to reach thermal equilibrium with the air in the room .
8. Heat the spool on a hot plate and, with the multimeter, record the length of time that it takes to reach thermal equilibrium with air in the room.

Experimental Data:

1. **Measurements of the Cylinder and Paint Can**

Spool Mass (kg)	Spool Diameter (cm)	Mass of Can (kg)
0.2033	5.36	9.368

2. **Measuring Temperature vs. Number of Turns**

Turns	Resistance (kΩ)	Uncertainty (kΩ)	Temperature (ºC)	Temperature (K)	Terror (K)
0	173	1	13.5	286.65	0.2
338	55.2	1	38	311.15	0.2

3. **Heating The Spool**

Resistance (kΩ)	Uncertainty (kΩ)	Temperature (ºC)	Temperature (K)	Terror (K)	time (s)
59	1	36.5	309.65	0.2	0
64	1	34.7	307.85	0.2	261

4. **Cooling the Spool**

Resistance (kΩ)	Uncertainty (kΩ)	Temperature (ºC)	Temperature (K)	Terror (K)	time (s)
37.5	1	46.7	319.85	0.2	84
42.5	1	44.5	317.65	0.2	296

Analysis:

$$W = N2\pi R \cdot mg = (338\, turns)(2\pi)(0.0268m) \cdot (9.368kg)(9.8m/s^2)$$ = 5225.2J

$$Q = m_{spool} c(T_f - T_i) = (203.0g)(0.220\frac{cal}{g \cdot {}^\circ C})(311K - 287K)$$ = 1095.8 calories

$$c_{aluminum} = 0.220\frac{cal}{g \cdot {}^\circ C} \qquad \frac{W}{Q} = 4.77\frac{Joules}{calorie}$$

Answers to Questions:

1. When you turn the crank, what would be the problem with cranking too slowly?

   Turning the crank too slowly would result in a dissipation of heat into the surrounding environment. Ideally, all heat produced from the work done by the experimenter on the wire (through turning the spool) goes directly into thermal energy involved in heating the spool and thermistor.

2. What role does the heat capacity and heat conductivity of the cord play in the accuracy of this experiment?

   The heat conductivity must be as low as possible to ensure that cord is heated as little as possible. Again, all thermal energy should "stay" in the spool for a proper measure of work-energy equivalence.

3. Can you think of advantages and disadvantages of making the temperature interval larger or smaller?

   Making the temperature interval larger would require more turns of the spool. This results in a longer experimental time, which, as stated in the answer to question #1, is dangerous.

   Making the temperature interval smaller would make it difficult to make accurate generalizations about the mechanical equivalent of heat— the table provided with the thermistor is experimentally derived, and fine changes in temperature/resistance are not useful.

4. Why are the lower and higher temperatures chosen to be symmetric about room temperature?

   The temperature extrema of the spool are chosen to be symmetric about the spool to "balance" the flow of heat into the cylinder (after it has been cooled) with the flow of heat out of the cylinder.

5. Any heat flow into or out of the cylinder will contribute to error. How is this error minimized?

   Heat flow out of the cylinder is a source of major concern in this experiment. It can be minimized through the use of non-conductive materials, especially those with come into direct contact with the spool. This is to say that it is of little consequence if the hanging mass is conductive, as long as the interface between it and the spool (the cord) is not. The cord is chosen to be made of a non-conductive material as to prevent heat flow out of the cylinder. Additionally, the crank must be turned at a considerable clip to prevent the flow of heat from the cylinder into the surrounding environment, as stated in the answer to question one.

   Heat flow into the cylinder is unlikely in the first stage of our experiment, when the cylinder is heated from room temperature, by virtue of the fact that its temperature never falls below the ambient temperature of the room. The same goes for part two, when we heat the cylinder and let its temperature reach equilibrium with the room. After having cooled the cylinder in an ice water bath, there is little that we can do to mitigate the flow of heat.

6. What will be the effect on your results if there is moisture on the cylinder when you start turning the crank for data.

   Moisture on the cylinder will reduce the amount of friction between the cord and the cylinder, potentially causing the cylinder to make contact with the floor, or producing a tension in the portion of cord between the spool and the supporting post. We have assumed that the spool supports the full weight of the hanging mass, so either of these occurrences would result in an error in calculation

Results and Error Analysis:

After much satisfying work at the ol' lab bench, my partner and I were able to make some important conclusions about the mechanical equivalence of heat. Namely, that heat has some (expected) mechanical equivalence.

As stated in Taylor's book, for calculating the compound error in quantities that are sums of independent measurements, we should take the root of the sum of the squares of the errors associated with each quantity:

$$\delta q = \sqrt{(\delta q_1)^2 + (\delta q_2)^2 + \ldots + (\delta q_n)^2}$$

When multiplying quantities with independent random errors, we take the root of the sum of the squares of the errors associated with each quantity divided by the magnitude of said quantity:

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta q_1}{q_1}\right)^2 + \left(\frac{\delta q_2}{q_2}\right)^2 + \ldots + \left(\frac{\delta q_n}{q_n}\right)^2}$$

Take the equation:

$$Q = m_{spool} c_{Al} (T_f - T_i)$$

Because the difference in temperature requires summing two values of the same quantity, we must add their respective errors. When multiplying this quantity with the other two–specific heat and the mass of the spool–we must use Taylor's second equation. This gives us an error in *Q* of:

$$|c_{Al}| \frac{\delta Q}{|Q|} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta \Delta T}{\Delta T}\right)^2} \quad \Rightarrow \delta \mathrm{Q} = 45.05\mathrm{K}$$

You'll notice that the quantity c_{Al} does not come into play in calculating the error of Q in the same way as our other quantities. This is because it's a constant rather than a quantity derived through (our) experimental procedure.

Calculating the error in W requires similar calculations, where the mass and radius of spool are given some tolerance for error, and pi, g, and N fall into the same category as c:

$$\frac{\delta W}{|W|} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta R}{R}\right)^2} \quad \Rightarrow \delta \mathrm{W} = 55.78\mathrm{J}$$

This makes the error bars on W/Q equal to:

$$\sqrt{\left(\frac{\delta W}{W}\right)^2 + \left(\frac{\delta Q}{Q}\right)^2} \qquad = 0.042 Joules / Calorie$$

Which is slightly smaller than we'd like, given that our measurement for W/Q is 4.77Joules/calorie.

We gave a whopping 1 Kelvin margin for error to our temperature measurements, because the precision of the table that we used to equate electrical resistance to temperature was 1K (that is to say that the table had increments of one Kelvin). Our margin for error of mass was 0.01kg, as was implicit from the sig-figs on the label of the paint can.

Our main sources of error were likely the fact that we could not account for, or control, the dissipation of heat into the surrounding environment. It's likely that we would have obtained a better measurement if we had turned the crank a fewer number of times, and instead adhered to the notion that we should make the temperature change of the cylinder symmetric about room temperature.