COUPLED PENDULUMS

CASPAR LANT

The Objective of this week's experiment is to explore the interactions of two pendulums which share a common axis, as a way to frame our studies of waves and harmonic motion.

Theoretical Background/Abstract:
Way back in the mid 17th century, the brilliant Huygens noticed that the two grandfather clocks on his mantlepiece (who keeps two grandfather clocks on their mantlepiece?) had fallen into synchrony! The pendulums that drove the two clocks were not oscillating in phase, as one might expect, but 180 out of phase. We will learn later that this is one of 2 normal modes that exist for a two-pendulum oscillating system. From here, Huygens formulated an equation that could generalize the motion of two pendulums. Before this, let's assume the following conditions, which give rise to force equations for each pendulum.

Let's assume that l and m are the same for each pendulum, that each pendulum oscillates in the same plane, angles are small; $sin\theta \approx \theta$, and that $F_a = -Kl(\theta_a - \theta_b)$, $F_b = -Kl(\theta_a - \theta_b)$, $\Sigma F = 0$.

$$ml\frac{d^2\theta_a}{dt^2} = -mg\theta_a - Kl_0(\theta_a - \theta_b) \tag{1}$$

$$\frac{d^2\theta_a}{dt^2} + \frac{g}{l}\theta_a + K\frac{l_0}{ml}(\theta_a - \theta_b) = 0? \tag{2}$$

$$\frac{d^2\theta_a}{dt^2} + \omega^2\theta_a + C(\theta_a - \theta_b) = 0\Big(\omega_0 = \sqrt{\frac{g}{l}}\Big) \tag{3}$$

Where l_0 is the position of the spring coupling between the pendulums, with $l_0 = 0$ at the axis of oscillation.

Normal modes denote starting conditions under which the motion of the pendulum is stable, and constant in phase-space. This is to say that, if you start a two pendulum in one of its two normal modes, it will continue to oscillate in this mode, with no energy trade-off between the pendulums. These modes are eigenvectors of the equation to follow. The so-called "slow mode" is given by the initial condition $\theta_a = \theta_b$, and is shown in vector-form by $[1, 1]$. The second, "fast mode" is given by $\theta_a = -\theta_b$, or $[1, -1]$.

For arrangements of pendulums that cannot be classified as either of the the system's two normal modes, we find the following equation:

$$\vec{X}(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}(a_1cos\omega_1t + b_1sin\omega_1t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix}(a_2cos\omega_2t + b_2sin\omega_2t) \tag{4}$$

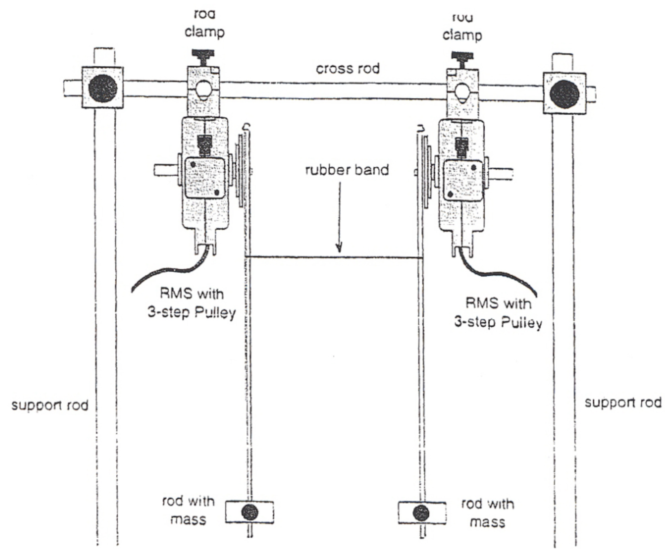

FIGURE 1. Experimental Setup

Experimental Procedure?

- Set up the two pendulum system in the manner detailed by the above diagram.
- Connect the two rotary distance sensors to Data Studio via the provided interface.
- Create a display in Data Studio that graphs the position of each distance sensor over time.
- Match the frequency of one pendulum to that of its partner by adjusting the position of its mass. It's of note here that we disregard the mass of the rod in our calculations.
- Plot the oscillation of each pendulum in Data Studio and compute its frequency with a best-fit sine function.
- Connect the rubber band to each pendulum at a distance of 5cm.
- Start the pendulums off in the "slow mode" and record their oscillations in Data Studio.
- Do the same for the "fast mode".
- Now for something more interesting: start the pendulum off in a "non-normal" mode by displacing one pendulum and keeping the other at zero position. Ensure the the angle of displacement is sufficiently small, in accordance with the paraxial approximation on page one.
- Repeat each experiment for rubber band length distances 10cm, 15cm, 20cm, and so on. How does the position of the rubber band affect the rate of the pendulums' oscillation?

hello

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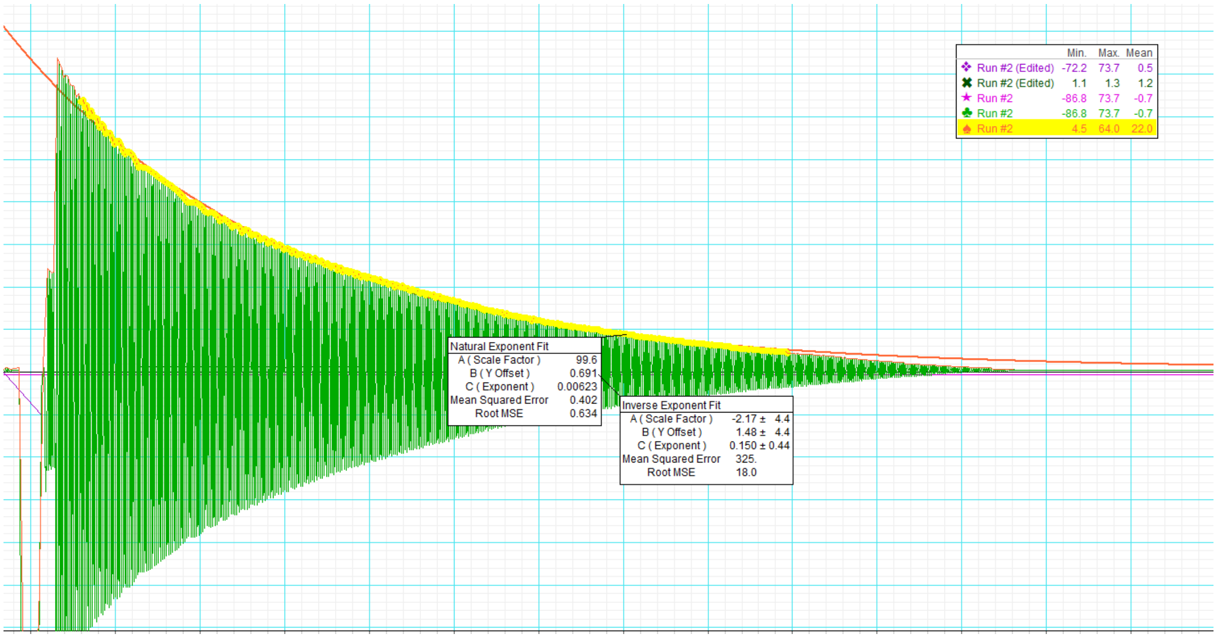

FIGURE 2. Extra Credit: Calculating the Damping of a Single Pendulum

$$DampingCurve : 97e^{-0.00594t}, \gamma = 0.00594$$

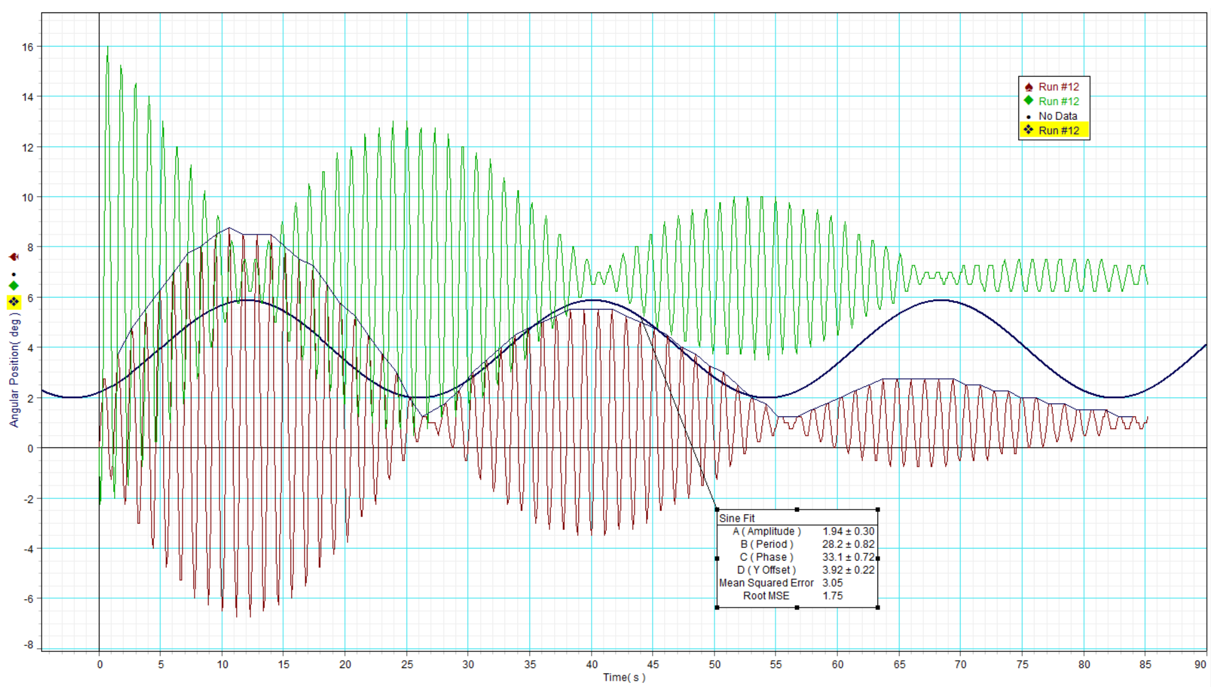

FIGURE 3. Harmonic Resonance of two Pendulums

Table 1. Experimental Data

Pend Mass (g)	Rubber Band Height (cm)	ω_b	ω_{av}	ω_1	ω_2
74.4	5	0.018	0.901	0.855	0.893
74.4	10	0.060	0.990	0.862	1.000
74.4	20	0.380	0.862	0.862	1.403

Table 2. Theoretical Calculations

Pend Mass (g)	Rubber Band Height (cm)	ω_b	ω_{av}	ω_1	ω_2
74.4	5	0.019	0.874	0.855	0.893
74.4	10	0.069	0.931	0.862	1.000
74.4	20	0.270	1.132	0.862	1.403