

## Mechanical Equivalent of Heat

Equipment Mechanical Equivalent of heat apparatus, bucket of sand, Fluke Multimeter, calipers for measuring diameter of cylinder, scale to measure mass of cylinder, S.S. insulated bucket with ice, No. 18 rubber band, lint free towel for drying cylinder

### 1 Remark

WHEN YOU ENTER THE LAB, PLEASE DO NOT TURN THE CRANK. The apparatus will be assembled with a cord wound around a cylinder. One of your first experimental activities will be to measure room temperature which will be done by measuring the temperature of the cylinder. If you turn the crank the temperature of the cylinder will rise and no longer be the desired value.

### 2 Heat and Internal Energy

Consider two objects in vacuum at different temperatures. Assume that the objects are cool enough so that their radiation is negligible. If the two objects are placed in contact, the hotter object gets colder and the cooler one gets hotter. Energy has been transferred by conduction from the hotter object to the cooler object. The energy in transit from the hotter object to the cooler object is called heat flow. The energy that has been transferred is called heat. In physics, heat is rather narrowly defined as energy that has been transferred due to a temperature difference. (One of the reasons thermodynamics is usually a difficult subject is that colloquially heat has many other meanings.) After a time the two objects will have the same temperature. We cannot say that the initially hotter object has less heat and the initially cooler object has more heat, for there is more than one way to change the temperature of an object. For example, rubbing and compressing, which involve doing work, will also increase the temperature of an object. What we *can* say is that the initially hotter object has less internal energy than it did and that the initially cooler object has more internal energy than it did. Internal energy  $U$  is the total kinetic and potential energy of the particles that make up the object. Internal energy does not include center of mass motion and center of mass potential energy.

Both heat and work are energy but historically have been measured in different units. Joules are the units of work in SI units. Traditionally heat has been measured in the units of calories (cal). At around 15 deg C (degrees centigrade), an energy of 1 cal raises the temperature of 1 g of water 1 deg C. In this experiment the equivalence between these two units will be measured.

The Calorie (Cal) used for food (note the capital C) is 1000 cal, or a kcal.

### 3 Overview of the Experiment

See Fig. 1. An aluminum cylinder is rotated by a crank. A cord or rope under known tension rubs against the cylinder and heats it by friction. The mechanical work in joules done on the system is determined. The temperature rise of the cylinder is measured and the amount

of heat in calories that would be necessary to produce that temperature rise is calculated. The amount of work done in joules is equated to the calculated heat in calories to obtain what is known as the mechanical equivalent of heat.

It is worthwhile emphasizing that the temperature rise of the cylinder is produced directly by work done on it. The heat is a calculated quantity. Ideally, there would be no heat flow in this experiment. There is a small amount of heat flow which leads to some error.

## 4 Theory

It is necessary to determine the amount of work done on the cylinder, measure the temperature rise of the cylinder, and calculate the amount of heat that would produce the same temperature rise.

### 4.1 Work and Energy

Letting  $W$  be the work done on an object,  $\vec{F}$  be the net force on the object, and  $\vec{r}$  be the position of the object, a differential element of work is defined by  $dW = \vec{F} \cdot d\vec{r}$ . If  $\vec{F}$  is the net force on a point object, the work is equal to the change in kinetic energy of the object. If the net force is zero, the kinetic energy of the object does not change. With a frictional force present it is possible for the net force to be zero and for the frictional force to raise the temperature of the object, increasing the internal energy, with the energy supplied by the mechanical work done. A simple example in one dimensional motion would be a block pulled at a constant speed by a horizontal string along a horizontal frictional plane. The force of friction would be canceled by the tension in the string. Assume that the block can conduct heat but that the plane cannot. The person pulling the string is doing work, and the temperature and internal energy of the block rises. The frictional rubbing sets the molecules at the plane-block intersection vibrating and increases the local temperature. Heat flows into the block but not the plane because the plane was assumed to be a thermal insulator.

This experiment uses a rotational analog of the above situation. The cylinder is rotated at a fairly constant angular velocity about a horizontal axis by a crank. The crank applies a torque  $\tau$  to the cylinder and does positive work. An opposite torque is applied to the cylinder by cord with a weight of mass  $M$  hanging on it. The cord is wrapped a number of times around the cylinder and held so lightly at the other end that the torque applied by this end is negligible. The torque applied by the crank and cord are equal in magnitude but opposite in direction, and the rotating cylinder is in what we might call “rotational equilibrium.” The magnitude of the torque  $\tau$  applied by the crank is then equal to the torque of the cord which is  $MgR$ , where  $g$  is the acceleration of gravity and  $R$  is the radius of the cylinder. The positive work done by turning the crank is  $W = \int \tau d\theta$ , where  $\theta$  is the angular rotation of the drum in radians. If the cylinder is rotated  $N$  turns, this gives

$$W = 2\pi MgRN. \quad (1)$$

### 4.2 Calculation of Heat

The temperature rise of the cylinder is produced by mechanical work, and we now calculate the heat that would produce the same temperature rise. The specific heat  $c$  of a substance

is defined as the heat  $Q$  added to unit mass of the substance that will raise the temperature one degree. In SI units the units of  $c$  are  $J/kg \cdot K$ , where  $J$  is joules,  $kg$  is kilograms, and  $K$  is degrees Kelvin. The units of  $c$  used in this experiment are  $cal/g \cdot deg\ C$ . Let  $T_i$  and  $T_f$  be the initial and final temperatures of the cylinder and let  $m$  be the mass of the cylinder in grams. The heat  $Q$  that will give the temperature rise is

$$Q = mc(T_f - T_i). \quad (2)$$

The cylinder used is aluminum which has a specific heat of 0.220  $cal/g \cdot deg\ C$ , about 1/5 that of water.

## 5 First Law of Thermodynamics

For completeness we mention how heat  $Q$  and work  $W$  fit into the first law of thermodynamics. The temperature  $T$  and internal energy  $U$  of a system can be raised by heat flow or by doing work on the system. A system cannot be said to have a certain amount of heat or a certain amount of work. This translates into the statement that neither  $dQ$  or  $dW$  is a perfect differential of the system, and that when you bring a system from one state to a different state the amount of heat added or the amount of work done depends on how you change the state of the system. What *is* a perfect differential is the internal energy  $U$ , and the first law, which is a conservation of energy statement, is

$$dU = \widetilde{dQ} - \widetilde{dW}, \quad (3)$$

where  $dQ$  is positive if heat is added to the system and  $dW$  is positive if work is done by the system. Note the minus sign. The tildes above the  $dQ$  and  $dW$  emphasize that neither is a perfect differential. (Textbooks have various other ways of indicating this.)  $Q$  is the energy transferred by a temperature difference.  $W$  is all other kinds of energy, be it mechanical, electrical, magnetic, or gravitational.

## 6 Apparatus

See Fig. 1. An aluminum cylinder is mounted with its axis horizontal. The cylinder can be rotated by a crank, and can removed by unscrewing a knob. One end of a nylon cord with a flattened cross section is attached to a bucket of sand. The cord is then wrapped 4 to 6 times around the cylinder. The other end of the cord is attached to a rubber band, and the rubber band is attached to a rod that can be moved with respect to the cylinder. See Fig. 2. The crank has a projection on it that advances a counter with every revolution. The counter can be zeroed by a knob. To raise the temperature by doing work on the cylinder the crank is turned. The number of times the cord is wrapped around the cylinder and the tension of the rubber band are adjusted so that the bucket moves a few cm off the floor and the rubber band has very little tension when the crank is turned.

The temperature of the cylinder is measured by a solid state device called a thermistor. This is a contraction of “thermal resistor.” A resistor opposes the flow of electric current, and the amount it opposes the flow is given by its resistance, which is measured in units ohms ( $\Omega$ ). The resistance of most resistors does not depend very strongly on temperature, increasing slowly as the temperature rises. But the resistance of a thermistor depends strongly on

temperature, decreasing as the temperature increases. This property makes a thermistor a good thermometer for many applications. There is a thermistor buried in the cylinder. The two leads connected to the thermistor are attached to slip rings, and two stationary brushes slide against the slip rings. See Fig. 3. Two leads attached to the brushes are connected to a Fluke multimeter. You can measure the resistance of the thermistor by turning the dial on the Fluke to  $\Omega$ . The display will probably read in kilo-ohms, or  $k\Omega$ . A table at the back of this write-up, and also on the apparatus, allows the resistance measured to be converted to temperature. You should use a linear extrapolation between table entries.

## 7 Procedures

### 7.1 Safety

The bucket that supplies the cord tension has a mass of about 10  $kg$  (weighs about 22 lbs). Don't let it drop on your foot. Do not put your feet under the bucket. Check that the apparatus is securely clamped to the table, that the knob that holds the drum in place is moderately tight (please do not strip the threads), and that the knot holding the bucket is secure (knots in nylon can slip). As discussed below, adjust the number of turns of the cord and the position of the rubber band so that the bucket is never more than a few cm above the floor.

The cord has some powdered graphite on it. When handling the cord you will pick up some graphite on your hands. Try not to touch your clothing, or better yet, don't wear your Sunday best.

### 7.2 Temperature

While the cylinder is at room temperature, measure this temperature using the resistance of the thermistor as determined by the Fluke multimeter. Check that the Fluke leads are plugged into the sockets on the apparatus and turn the Fluke dial to  $\Omega$ . The resistance can be read on the display and the temperature interpolated from the thermistor table.

By doing mechanical work on the cylinder the temperature of the cylinder will be raised from below room temperature to above room temperature. It is desirable to make the change symmetric with respect to room temperature. A suitable temperature change is  $\pm 8$  deg  $C$  from room temperature, although you can use a somewhat different number if you want. Add and subtract 8 deg  $C$  from room temperature and determine the resistance of the thermistor at those 2 temperatures. You will need these values later.

### 7.3 Mass of Bucket

Determine the mass  $M$  of the bucket if an appropriate scale is available. If a scale is not available, record the mass given on the bucket.

### 7.4 Familiarization

Get a feel for the proper adjustment of the cord and rubber band. Put the bucket on the floor under the apparatus. Take the cord, pass it through the notch in the base of the apparatus, and wind it perhaps 5 (actually  $5\frac{1}{4}$ ) turns around the cylinder. Don't let the cord

twist, that is, keep one flat surface of the cord on the cylinder. Take the other end of the cord with the rubber band and loop the rubber band around the rod supported by a table clamp. Adjust the position of this table clamp so that the rubber band has a modest amount of tension in it. Turn the crank. If the adjustment is good, the bucket will rise a few cm above the floor and stay there while the crank is being turned and the rubber band will have little or no tension in it. If the cord does not slip on the drum there are too many turns of the cord. If the cord slips too easily on the drum there are not enough turns of the cord. If the bucket rises too high the initial tension of the rubber band was too high. If the bucket does not get off the floor either the initial number of cord turns was too few or the initial tension of the rubber band was too low. Experiment until you can get the right conditions in a reasonable amount of time. When you do the experiment the adjustments may not be exactly the same and you do not want to spend too much time getting them right. Remember, your two adjustments are the number of turns of the cord and the position of the rod that holds the rubber band.

### 7.5 Removing the Cylinder

Remove the rubber band from the rod and the cord from the cylinder. Unscrew the knob holding the cylinder to the apparatus and note that the exposed end of the cylinder is aluminum, smooth, and not segmented. There are also slots in the black plastic insert in the middle of the cylinder end which *are not used*. Pull the cylinder off the apparatus and inspect the other or inner end. Note the two slip rings, the brass color, and the two holes for the thermistor wires. Observe the transverse pin on the axle which must be aligned with the slots in the black plastic insert when the cylinder is put back on the apparatus. Measure the mass  $m$  of the cylinder and its diameter.

### 7.6 Cooling the Cylinder

Place the cylinder gently on top of the ice provided so that the axis of the cylinder is horizontal and so that water does not get into the hole in the middle of the cylinder. In 1 minute the cylinder should be below the lower temperature that you have already determined. Remove the cylinder from the ice and hold it with the toweling provided to minimize heat transfer from your hands. Use the towel to dry any moisture on the cylinder. Put the cylinder back on the apparatus, *being sure that the slip ring end goes on first and that the orientation allows the pin to lock the angular position of the cylinder*. Put the knob back on and tighten it securely.

### 7.7 Doing Work

Monitor the resistance of the thermistor. Adjust the cord and rubber band for proper operation and turn the crank a bit to be sure that these adjustments are satisfactory. If the air is moist and there is more condensation on the cylinder, dry the cylinder again. When the resistance of the thermistor approaches the value corresponding to your lower chosen temperature, set the counter to zero and do not crank until the desired lower temperature is reached. When it is reached, start cranking briskly. When the resistance approaches the value corresponding to the higher selected temperature, stop cranking. Keep monitoring the resistance and record the lowest resistance value reached. Do not worry if the resistance

reached is not exactly what you were aiming for. Calculate the temperature and record the temperature and number of turns of the crank.

### 7.8 Analysis

Use Eq.1 work and Eq.2 heat to calculate the work done and the heat necessary to produce the same temperature rise. Equate these two quantities to obtain the mechanical equivalent of heat and compare your result to the accepted value of 4.186 J/cal.

## 8 Questions

1. When you turn the crank, what would be the problem with cranking too slowly?
2. What role does the heat capacity and heat conductivity of the cord play in the accuracy of this experiment?
3. Can you think of advantages and disadvantages of making the temperature interval larger or smaller?
4. Why are the lower and higher temperatures chosen to be symmetric about room temperature?
5. Any heat flow into or out of the cylinder will contribute to error. How is this error minimized?
6. What will be the effect on your results if there is moisture on the cylinder when you start turning the crank for data?

## 9 Historical Comments

### 9.1 Count Rumford

In the 18th century the relationship between heat, work and energy was very poorly understood. The rise or fall of the temperature of a body was supposed to be due to a flow of a substance called caloric. Perhaps the first person to shed light on the nature of heat was Count Rumford, who has to be one of the most interesting and controversial people who ever lived. At the end of this write-up is a brief comment on this man supplied by PASCO. If you ever find yourself going through Concord, NH, you might enjoy visiting the museum which has some material on the Count.

### 9.2 James Prescott Joule

In the middle of the 19th century Joule made the first credible measurements of the mechanical equivalent of heat. His best value was within 0.6 % of the modern value. He used several methods: electrical work, frictional work, and mechanical work (compression of gases). His best known apparatus consisted of paddle wheels rotating in water and powered by falling weights, somewhat akin to the experiment you are doing.

## **10 Finishing Up**

Please leave the bench as you found it. Thank you.

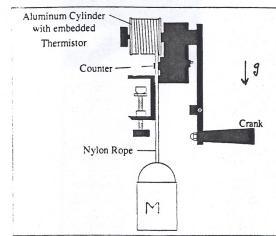


Figure 1: Mechanical Equivalent of Heat Apparatus

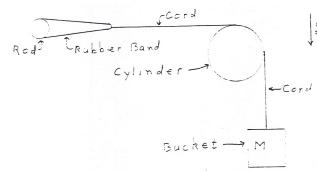


Figure 2: Setup cross section

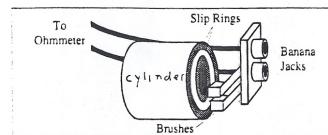


Figure 3: Measuring the Cylinder Temperature

## Thermistor Specifications:

**Temperature Versus Resistance**

Res. ( $\Omega$ )	Temp. ( $^{\circ}\text{C}$ )	Res. ( $\Omega$ )	Temp. ( $^{\circ}\text{C}$ )	Res. ( $\Omega$ )	Temp. ( $^{\circ}\text{C}$ )
351,020	0	66,356	34	16,689	68
332,640	1	63,480	35	16,083	69
315,320	2	60,743	36	15,502	70
298,990	3	58,138	37	14,945	71
283,600	4	55,658	38	14,410	72
269,080	5	53,297	39	13,897	73
255,380	6	51,048	40	13,405	74
242,460	7	48,905	41	12,932	75
230,260	8	46,863	42	12,479	76
218,730	9	44,917	43	12,043	77
207,850	10	43,062	44	11,625	78
197,560	11	41,292	45	11,223	79
187,840	12	39,605	46	10,837	80
178,650	13	37,995	47	10,467	81
169,950	14	36,458	48	10,110	82
161,730	15	34,991	49	9,767.2	83
153,950	16	33,591	50	9,437.7	84
146,580	17	32,253	51	9,120.8	85
139,610	18	30,976	52	8,816.0	86
133,000	19	29,756	53	8,522.7	87
126,740	20	28,590	54	8,240.6	88
120,810	21	27,475	55	7,969.1	89
115,190	22	26,409	56	7,707.7	90
109,850	23	25,390	57	7,456.2	91
104,800	24	24,415	58	7,214.0	92
100,000	25	23,483	59	6,980.6	93
95,447	26	22,590	60	6,755.9	94
91,126	27	21,736	61	6,539.4	95
87,022	28	20,919	62	6,330.8	96
83,124	29	20,136	63	6,129.8	97
79,422	30	19,386	64	5,936.1	98
75,903	31	18,668	65	5,749.3	99
72,560	32	17,980	66	5,569.3	100
69,380	33	17,321	67		

## The Incredible Career of Count Rumford

One of the most incredible men associated with science was Benjamin Thompson, later titled Count Rumford. Aside from making as many enemies as friends, this man amassed a large list of honorary titles and contributed significantly to scientific knowledge. He never let an opportunity for advancement escape him and many claimed he had "no real love or regard for his fellow men." Nevertheless he was one of the first American scientists and his career was probably the strangest of all American success stories.

Thompson was born into a Massachusetts farming family in 1763. He was a strange boy who fancied he could build a perpetual motion machine and took great interest in eclipses. He became an itinerant teacher and was hired by a wealthy family in Rumford, Massachusetts. After endearing himself to nearly everyone, Benjamin married the daughter of the household and was accepted into high society. So favorably did he impress the local military officers that he was made a major at age 19. This undeserved honor made him quite unpopular with the local citizenry. In fact as the political climate ripened for revolution, Thompson was arrested "upon suspicion of being inimical to the liberties of this Country." Perhaps he was a spy, but most likely he was indifferent to the revolutionary cause. When released he left his wife and fled to England.

His charming manner and good looks won the friendship of the War Minister and soon he was elected to the Royal Society and named Under Secretary in the War Department. He returned to America to command the Queen's Horse Dragoons against the colonists. During this time he strangely enough began systematic lunar observations and extensive experiments with gunpowder and shell velocity.

At age 30 he returned to England and traveled to Bavaria. He won the friendship of the duke of Bavaria and in due time was made a Count of the Holy Roman Empire—Count Rumford. Thompson was bright enough and had enough power to apply his cherished ideas of enlightened despotism; he established a successful welfare system in Munich.

This was the time he made his greatest contribution to science. While watching a cannon being bored he noted the extreme amount of heat produced. After careful experiments he was able to deduce that heat was molecular motion, not a fluid. This was a breakthrough.



Benjamin Thompson

1763-1814

Count Rumford was a careful observer. He installed a glass door in his fireplace, watched the flame carefully, and soon designed better stoves and better chimneys. He built up quite a reputation as a nutritionist; he wrote several essays on the benefits of coffee over tea. Many credit him with inventing the folding bed and he made many improvements in the design of lamps. His main scientific accomplishment in later life was his large role in founding the Royal Institution in 1800. It was Count Rumford who hired Humphrey Davy as lecturer at the Institution and it was Count Rumford's money that kept the Institution going in the beginning. Soon, however, the Institution became too theoretical for Thompson and he severed connection with it to move to France. He died in 1814 of a fever. He left his gold watch to Sir Humphrey Davy and much of his money to Harvard University.

Although much of what Benjamin Thompson did in his lifetime was simply not cricket, he was an "enlightened philanthropist" and did more for society and science than most men.

Reference: *Count Rumford of Massachusetts*  
Thompson, James Alden  
Farrar & Rinehart, New York 1935

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