Lab 6:
Oscillations on a String

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Intermediate Experimental Physics
Section 002

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The Objective of this week's experiment is to become familiar with the ways in which strings respond to tension forces, specifically with regard to the frequencies of their harmonic series. We will further go on to analyze the *damping factor* associated with our string.

Theoretical Background/ Abstract:

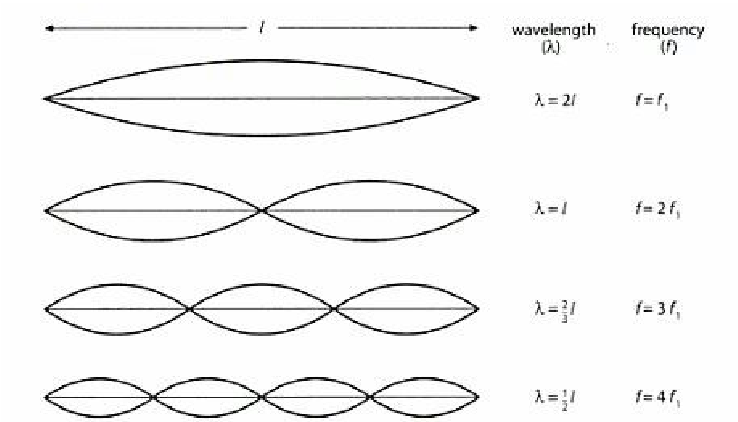
Strings, when met with some force, tend to oscillate at a characteristic set of frequencies, called their **harmonics**. This is similar to the harmonics we observed in the resonant tube lab. Strings, when met with an oscillating force with a frequency matching that of one of their harmonics, will produce **standing waves**. These waves, crossing the length of the string to-and-fro, will produce patterns as depicted in the diagram below. The frequency of these waves is given by integer multiples of the string's **natural resonant frequency**:

$$f_n = n f_1 \Rightarrow \lambda = H_n l \qquad H_n = \sum_{k=1}^{n} \frac{1}{k}$$

The natural resonant frequency of the string, or the first fundamental, as it's called, can be found using the following expression:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\rho}} \Rightarrow f_n = \frac{n}{2L}\sqrt{\frac{T}{\rho}}$$

Where T is the tension on the string and rho is the linear mass density of the string



The **amplitude** of a string is given by the equation:

$$\frac{1}{\sqrt{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}}$$

*where omega is the frequency and gamma is the **damping factor***

Experimental Procedure:

1. Connect a length of cord a secure post at the end of your workbench. Wrap the cord around the post several times to ensure that it is secure.
2. Lay the length of cord on a pulley attached to the other end of your workbench. Use a weight at this end of the cord to take up the slack.

3. Place a “mechanical vibrator” under the post-end of the cord, about 5cm from its end. Make sure that the cord touches the vibrator, and will not jiggle off when the device is in motion.
4. Starting from 5 Hz, ramp the frequency of the vibrator up until the first harmonic is clear on the string. Record this frequency
5. Continue to increase the frequency of vibration until the cord reaches the second, third, and fourth harmonics.
6. Repeat this procedure with varying mass: 200g and 400g. Record the stretched length of string for each mass.
7. Abruptly stop the oscillation of the string. Record the amplitude of the string at regular time intervals until it ceases motion. This will help to determine the *damping factor*.

Experimental Data:

1. Preliminary Measurements

Vibrator to Post (cm)	Total String Length (cm)
15	138.3

2. Values for Frequency of Each Node for a Given Mass

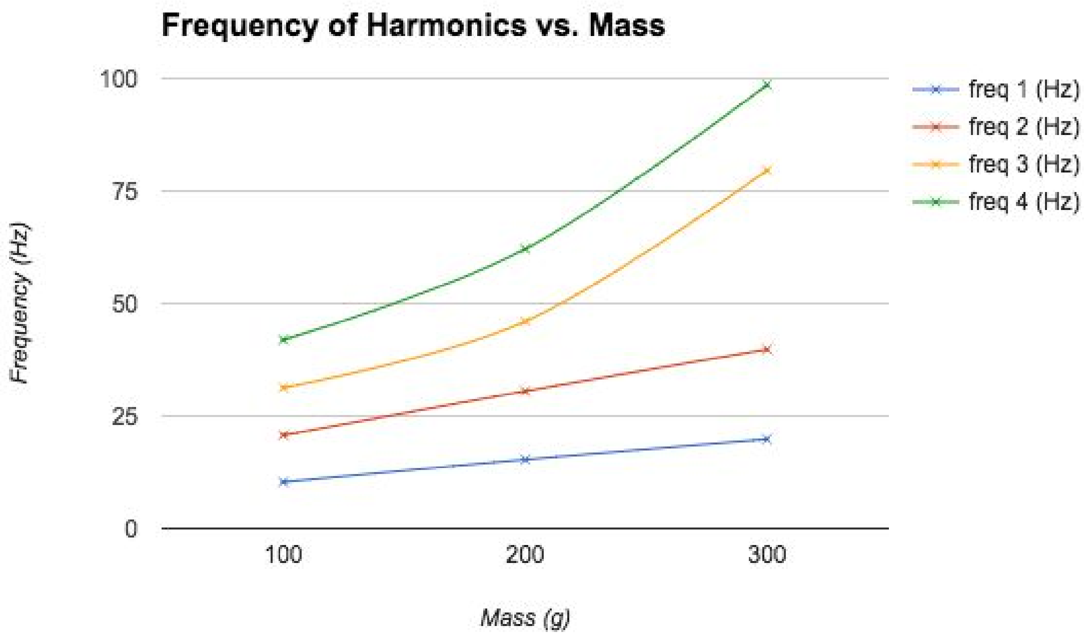
Mass (g)	freq 1 (Hz)	freq 2 (Hz)	freq 3 (Hz)	freq 4 (Hz)
100	10.3	20.7	31.3	41.9
200	15.3	30.5	46	62.1
300	19.8	39.8	79.6	98.6

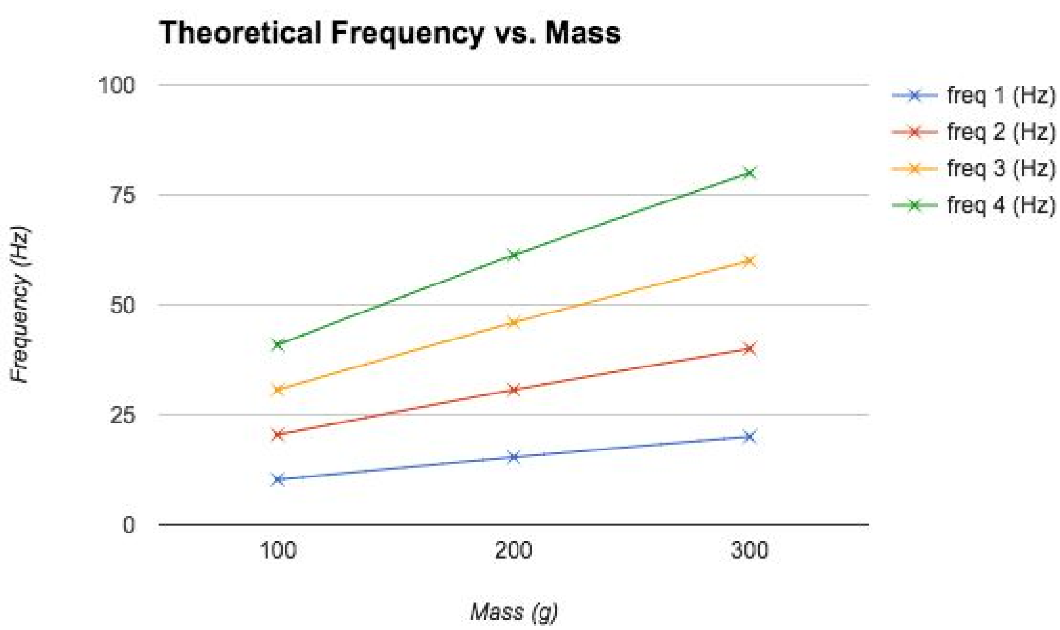
3. Cord Density Calculations

String Mass (kg)	Total String Length (m)	Mass (kg)	Cord Density (kg/m)
0.009	1.25	0.0	0.0072
0.009	1.38	0.1	0.0065
0.009	1.55	0.2	0.0058
0.009	1.76	0.3	0.0051

4. Theoretical Frequency Calculations

Cord Density (kg/m)	theo freq 1 (Hz)	theo freq 2 (Hz)	theo freq 3 (Hz)	theo freq 4 (Hz)
0.00652	10.22	10.22	30.66	40.88
0.00581	15.32	30.64	45.96	61.27
0.00511	19.99	39.98	59.97	79.97





Notice that the observed frequencies differ substantially from theoretical ones at high harmonics and mass.

Damping Factor:

$$A(t) = A_o e^{-\frac{\gamma t}{2}} \qquad \lim_{t \to \infty} A_o e^{-\frac{\gamma t}{2}} = 0$$

The time required for the amplitude to (approximately) reach 0 is 7.6, with an error of ±0.7.
The initial amplitude is was roughly 2cm.
Because 7.6/2 is not very large, we know that gamma, our damping factor must be very large indeed.

Results and Error Analysis:

You'll see that most of our theoretical calculations for frequency are fairly close to the observed values, and well within the error bounds:

$$\frac{\delta f}{|f|} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta T}{T}\right)^2 + \left(\frac{\delta \rho}{\rho}\right)^2} \Rightarrow \delta f = \pm 6.8 Hz$$

The two salient exceptions are of course, the frequencies for the third and fourth harmonics at greatest mass, coming in at close to 20 Hz. The differences between our experimental data and theoretical results never exceed our error bounds in for all other values frequencies. This provides a fairly convincing argument towards the claim that the frequency of the n^{th} overtone of a string is dependent on the string's tension, length, and linear mass density in the way expressed on the first page of this lab report.

Our main sources of error were likely due to the presence of non-conservative forces, like air-resistance and mechanical friction. The interface between the vibrator and the string can't have been perfect. I imagine that, each time the mechanism moved up or down, the string bounced ever so slightly, losing miniscule amounts of energy to the surrounding environment. The "error bars" surrounding our measurements of frequency are related to the **quality factor** of the string, equivalent to the width of the resonance spike.

To answer the question posed on the final page of the lab manual:
A mechanical wristwatch is an excellent example of a system that employs a (damped) oscillation. Of course, the watchmaker strives to minimize the damping factor of his masterpiece, so as to create a most-precise timepiece that requires little from its bearer. But of course he fails, as he foists life into a contraption born of an imperfect world, in which the presence of air molecules barrs the possibility of continuous and perpetual motion. The watch-bearer winds the watch, lest it slither off into uselessness.