Lab 3:

Heat Engine

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Intermediate Experimental Physics
Section 002

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The Objective of this week's experiment is to create and analyze a **heat engine**, which will behave in a manner similar to that of the idealized heat engines we have learned about in our physics lectures.

Theoretical Background/ Abstract:

A heat engine is defined by its ability to produce useful energy from a flow of heat due to a temperature differential. A heat engine typically operates in some kind of cycle, where heat is added and released at opposite ends. This is all dependent on the First Law of Thermodynamics, which states that the energy put into a body is equal to sum of the the body's internal energy and the work that it does:

$$Q = \Delta U + W$$

The efficiency of a heat engine is given by the ratio of its useful output (work) and its input heat, Q_H:

$$\epsilon = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H}$$

As we know from last week's experiment, this work has some mechanical equivalence, and can be move objects. A train car, for instance. We can calculate the work output of a heat engine by evaluating the following integral, where *W* is work, *p* is pressure, and *V* is volume:

$$W = \int_a^b p dV = p\Delta V|_{p=const.}$$

We can derive the pressure experimentally, and calculate the change in the volume of the cylinder with the equation:

$$\Delta V = \pi r_{cylinder}^2 (h_i - h_f)$$

We can further calculate the energy put into the engine, Q_H, with the following well-known formula:

$$Q_H = nC_v \Delta T = nC_{v_{air}}(T_H - T_C)$$

This procedure also works for calculating the waste heat, Q_C. With these three values, we can finally calculate the efficiency of our engine.

Experimental Procedure:

1. Procure a piston, mass, and a length of cord; as well as two sections of piping, and a small cylinder.
2. Set up these components as detailed in the above diagram. Connect one length of piping to the cylinder and the other to a pressure sensor. Both leads should end up at the mounted piston. The piston should be attached to a cord and a distance gauge, as a means of determining the

piston's position. Uncork the cylinder to ensure that it is at pressure equilibrium with the surrounding environment. This will be our reference pressure.

3. Submerge the cylinder in a bucket of hot water, whose temperature has been recorded. The volume of the gas contained in the cylinder-piston-tubing system should increase, as evidenced by an increase in the height of the piston.
4. A mass should then be gently placed on the top of the piston. It is important to do this slowly, as a fast change in the system would screw things up…
5. Record the changes to the height of the piston. It is important to do this quickly, to mitigate any heat-loss; we are pretending that this step in the heat cycle is isothermal.
6. Bathe the cylinder in a volume of ice water, and record its final temperature, as well as the new height of the piston.
7. Remove the mass from the piston, taking care to—again—do it slowly.
8. Record the final height of the piston. You're done!
9. Repeat until satisfied.

Experimental Data:

1. **Preliminary Measurements of Experimental Setup**

Piston Diameter (mm)	Piston Mass (g)	Tubing Diameter (mm)	Pressure Gauge Tube (mm)	Air Chamber Tube (mm)
32.5	35.0	3.175 ±0	610.0 ±0.5	470.0 ±0.5
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2. **Measurements Over One (Reversible) Heat Engine Cycle without**

Height (mm)	Uncertainty in Height	Volume in Cylinder (mm^3)	Uncertainty	Total Volume (mm^3)	Temperature (ºC)
45.5	0.5	37745.74	142.94	46296.43	57.0 ±1
25.5	0.5	21154.21	80.11	29704.89	5.0 ±1
41	0.5	34012.65	128.81	42563.34	55.0 ±1
19.5	0.5	16176.75	61.26	24727.44	4.0 ±1

Graph:

dx_a (m)	dx_b (m)	dx_c (m)	dx_d (m)	p_bc (Pa)	p_da (Pa)
-0.0287	-0.0320	-0.0035	-0.0002	285000	40000

From this table we can extrapolate the values for work in segments bc and da.

dV_ab	dV_bc	dV_cd	dV_da	W_bc (J)	W_da (J)
-23.643	-2.738	23.643	2.738	780217	-109504

Answers to Questions:

1. From looking at a p-V diagram that you have taken, how would you know if any working substance had been lost during a cycle?

    If air molecules had been lost over the course of a cycle, our value for PV would decrease, because PV = nRT. This makes sense, because the volume of anything decreases if you remove some of its constituents. If this were to happen, our plot would steadily shrink over each cycle.

2. How much work is done by the working substance and how much heat is added to the working substance for processes bc and da? What is the change in internal energy of the working substance between b and c, and between d and a?

    The work done by the gas through one cycle of our heat engine is given by the equation:

    $$\Sigma W = W_{bc} + W_{da} = \int_b^c pdV = \int_d^a pdV$$

    Using the values in the above table, we can show that the net work is 670713 Joules. Further, we show that the heat added to the gas in processes bc and da over one cycle is equal to

3. How much heat is added to the working substance for processes ab and cd? What is the change in internal energy of the working substance between a and b, and between c and d?

    Using the equation $Q_H = Q_{ab} + Q_{cd} = nC_v \Delta T = nC_{v_{air}}(T_H - T_C)$,
    We can should that the heat added to the gas in processes ab and cd over one cycle is equal to

4. For one complete cycle, what is the change in internal energy of, the net heat added to, and the the net work done by, the working substance.

    Over one complete cycle, the change in internal energy of the gas is given by the equation:
    $\Delta U = Q - W$.

5. Why do the PV curves just taken look a little different from each other?

Heat flow out of the cylinder is a source of major concern in this experiment. It can be minimized through the use of non-conductive materials, especially those with come into direct contact with the spool. This is to say that it is of little consequence if the hanging mass is conductive, as long as the interface between it and the spool (the cord) is not. The cord is chosen to be made of a non-conductive material as to prevent heat flow out of the cylinder. Additionally, the crank must be turned at a considerable clip to prevent the flow of heat from the cylinder into the surrounding environment, as stated in the answer to question one.

Heat flow into the cylinder is unlikely in the first stage of our experiment, when the cylinder is heated from room temperature, by virtue of the fact that its temperature never falls below the ambient temperature of the room. The same goes for part two, when we heat the cylinder and let its temperature reach equilibrium with the room. After having cooled the cylinder in an ice water bath, there is little that we can do to mitigate the flow of heat.

6. Why will leakage be less of a problem with the 100 g mass? Why are the p-V curves different

Moisture on the cylinder will reduce the amount of friction between the cord and the cylinder, potentially causing the cylinder to make contact with the floor, or producing a tension in the portion of cord between the spool and the supporting post. We have assumed that the spool supports the full weight of the hanging mass, so either of these occurrences would result in an error in calculation

Results and Error Analysis:

After much satisfying work at the ol' lab bench, my partner and I were able to make some important conclusions about the work output and efficiency of our heat engine.

As stated in Taylor's book, for calculating the compound error in quantities that are sums of independent measurements, we should take the root of the sum of the squares of the errors associated with each quantity:

$$\delta q = \sqrt{(\delta q_1)^2 + (\delta q_2)^2 + \ldots + (\delta q_n)^2}$$

When multiplying quantities with independent random errors, we take the root of the sum of the squares of the errors associated with each quantity divided by the magnitude of said quantity:

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta q_1}{q_1}\right)^2 + \left(\frac{\delta q_2}{q_2}\right)^2 + \ldots + \left(\frac{\delta q_n}{q_n}\right)^2}$$

Take the equation:

$$Q = nC_{v_{air}}(T_f - T_i)$$

Because the difference in temperature requires summing two values of the same quantity, we must add their respective errors. When multiplying this quantity with the other two–specific heat and the mass of the spool–we must use Taylor's second equation. This gives us an error in *Q* of:

$$|C_{air}|\frac{\delta Q}{|Q|} = \sqrt{\left(\frac{\delta n}{n}\right)^2 + \left(\frac{\delta \Delta T}{\Delta T}\right)}$$

You'll notice that the quantity c_{Al} does not come into play in calculating the error of Q in the same way as our other quantities. This is because it's a constant rather than a quantity derived through (our) experimental procedure. Calculating the error in W requires similar calculations:

$$\frac{\delta W_{engine}}{|W_{engine}|} = \sqrt{\left(\frac{\delta p}{p}\right)^2 + \left(\frac{\delta \Delta V}{\Delta V}\right)}$$

We gave a whopping 1 Kelvin margin for error to our temperature measurements, because. Our margin for error of mass was 0.01kg, as was implicit from the sig-figs on the label of the paint can.

Our main sources of error were likely the fact that we could not account for, or control, the dissipation of heat into the surrounding environment. It's likely that we would have obtained a better measurement if we had turned the crank a fewer number of times, and instead adhered to the notion that we should make the temperature change of the cylinder symmetric about room temperature.