Modelling energy production

Term project 1

This project involves calculating the energy production at the center of a star given a temperature and density. The goal is to write a class/function that accepts a temperature and density, and then calculates the amount of energy produced by the fusion chains discussed in Chapter 3. The class/function is going to be used for your later model of a star in Project 2. Read through the entire project description before you start with the coding.

Assumptions

- You can assume that the mass fraction of each atomic species is independent of radius, and given by

$$\begin{aligned} X &= 0.7 \\ Y_{^3_2\mathrm{He}} &= 10^{-10} \\ Y &= 0.29 \\ Z_{^7_3\mathrm{Li}} &= 10^{-7} \\ Z_{^7_4\mathrm{Be}} &= 10^{-7} \\ Z_{^{14}_7\mathrm{N}} &= 10^{-11} \end{aligned}$$

- When calculating the electron density you can assume that all elements are fully ionized.

- You need to include the processes making up the PP fusion chains and the CNO[1] cycle. Assuming that produced $^2_1$D is immediately consumed

[1]The dominant CNO cycle, given in Eqs. 3.13-3.18 in the lecture notes.

by the next step to produce $^3_2\mathrm{He}$, you may combine the first two steps of the PP chain as a single step with the same reaction rate.

- You do not need to consider changes over time, as you are looking at a snapshot of a star at a particular moment in time.

Tasks

- Before you begin coding, identify the SI units of the following variables:

$$T_9 \qquad \lambda_{ik} \qquad r_{ik} \qquad Q_{ik} \qquad \varepsilon$$

  For some of these, you may want to use "number of reactions" (or just "reactions") as one of the units involved, making it clearer what the variables represent. In Table 3.1, Avogadro's constant is considered to be unitless. If you want to express N_A in moles, you may consider the reaction rates in the table to be given in units of $\mathrm{cm}^3\ \mathrm{s}^{-1}\ \mathrm{mol}^{-1}$.

- Write a class/function for calculating the energy production (in SI units), including the PPI, PPII and PPIII branches, as well as the CNO cycle. You should include the upper limit of the $^7_4\mathrm{Be}$ electron capture.

- Once this is done, adjust your class/function so that no step consumes more of an element than the previous steps are able to produce.

Sanity check

In order to verify your code, you need to implement a sanity check. Using the temperature and density of the solar core (see Appendix B), you should get the following results:

$$\begin{aligned}
r_{^1_1\mathrm{H},^1_1\mathrm{H}}(Q_{^1_1\mathrm{H},^1_1\mathrm{H}} + Q_{^1_1\mathrm{H},^2_1\mathrm{D}})\rho &= 4.04 \cdot 10^{2}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^3_2\mathrm{He},^3_2\mathrm{He}}Q_{^3_2\mathrm{He},^3_2\mathrm{He}}\rho &= 8.68 \cdot 10^{-9}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^3_2\mathrm{He},^4_2\mathrm{He}}Q_{^3_2\mathrm{He},^4_2\mathrm{He}}\rho &= 4.86 \cdot 10^{-5}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^7_4\mathrm{Be},\mathrm{e}^-}Q_{^7_4\mathrm{Be},\mathrm{e}^-}\rho &= 1.49 \cdot 10^{-6}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^7_3\mathrm{Li},^1_1\mathrm{H}}Q_{^7_3\mathrm{Li},^1_1\mathrm{H}}\rho &= 5.29 \cdot 10^{-4}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^7_4\mathrm{Be},^1_1\mathrm{H}}Q_{^7_4\mathrm{Be},^1_1\mathrm{H}}\rho &= 1.63 \cdot 10^{-6}\ \mathrm{J\ m^{-3}\ s^{-1}} \\
r_{^{14}_7\mathrm{N},^1_1\mathrm{H}}Q_{^{14}_7\mathrm{N},^1_1\mathrm{H}} &= 9.18 \cdot 10^{-8}\ \mathrm{J\ m^{-3}\ s^{-1}}
\end{aligned}$$

The sanity check needs to be implemented in your code, and it must be possible to turn on and off so that the instructors can check if the sanity check is correctly implemented. 10 points

Code

The code has to be written using the `python 3` programming language. The way your code is written impacts the number of points you get for this project. It should be easy to read, well commented and logically structured. The instructors should be able to run your code. 10 points

Report

You are required to write a report in this project. The report should include an introduction, method, results, discussion and conclusion. The maximum number of pages is 10 (excluding front page and references). How your report is written impacts the amount of points you get on this project. It is recommended that you write the report using LaTeX. Use a spellchecker to avoid spelling mistakes and typos. Your figures should have a clear layout with proper axis labels and units, with a caption explaining what it shows. All figures should have a reference in the main text.

You should include answers and explanations to as many as possible of the following bullet points. Start from the top and work your way down.

- Explain the different equations that goes into solving this project (you do not need to include the expression from Table 3.1). 5 points
- Calculate the energy output from the reactions in the PP chain and CNO cycle. For each of the branches (PPI, PPII, PPIII and CNO), how large a percentage of the total energy is lost due to neutrinos? 15 points
- Why do we only need to use the reaction rate from

  $$^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + 7.30\ \mathrm{MeV}$$

  for the entire CNO cycle? And where does the $^{12}_{6}\mathrm{C}$ in the first CNO reaction (Eq. 3.13) come from? 10 points

- Explain how you make sure that no step consumes more of an element than the previous steps are able to produce. 10 points
- Calculate the energy production from each of the PP branches and the CNO cycle, and explain how you do it. 10 points
- Plot the relative energy production from each of the PP branches and the CNO cycle as functions of temperature, where $T \in [10^4, 10^9]$. Explain. 15 points
- Your report should be well structured and easy to read. Include a summary in your conclusion where you explain what you have learned from this exercise and if you had any trouble on the way. 15 points

The maximum score of this project is 100 points. The report and code is delivered at `https://devilry.ifi.uio.no/`. There will be no extension to the deadline, except in the case of documented medical circumstances. If you cannot solve the project, write the report anyway, explaining your problems and what you have tried in order to solve them.