AST3310 • Lecture #2

assumptions

i) a star is spherically symmetrical

ii) star is static → hydrostatic equilibrium
- gravitational force on a mass dm is cancelled by the outward force due to pressure gradient. From NLL:

$$dm\frac{\partial^2 r}{dt^2} = F_g + F_{P,t} + F_{P,b}$$

F_g: negative; $F_{P,t}$: negative

gravitational force

$$F_g = -\frac{GM_r dm}{r^2}$$

Pressure : $\frac{\text{force}}{\text{area}}$

$$P \equiv F/A$$

differential force

$$dF_P = A\,dP$$

density ρ

$$\Rightarrow dm = \rho A\,dr$$

$$F_{P,t} = -\left(F_{P,b} + dF_P\right)$$

$$\Rightarrow dm\frac{\partial^2 r}{\partial t^2} = F_g - dF_P$$

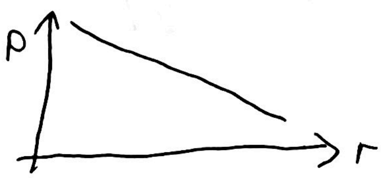
$$\Rightarrow dr\,\rho A\frac{\partial^2 r}{\partial t^2} = -G\frac{M_r \rho A\,dr}{r^2} - A\,dP$$

$$\Rightarrow \rho\frac{d^2 r}{dt^2} = -G\frac{M_r \rho}{r^2} - \frac{dP}{dr} = 0$$

static

$$\Rightarrow \frac{dP}{dr} = -\underbrace{G\frac{M_r}{r^2}}_{g}\rho = -g\rho \quad \text{hydrostatic equilibrium}$$

g: local gravitational acceleration at r



- hydrogen fusion: PP chain $\rightarrow \varepsilon_{pp} \propto T_6^4$ $\quad T = 10^6 T_6$

CNO cycle $\rightarrow \varepsilon_{CNO} \propto (T_6)^{20}$

- most of these stars have similar temperatures

$\rightarrow T_{(core)} \simeq \underbrace{1.5 \cdot 10^7}_{T_0} \left(\frac{M}{M_\odot}\right)^{1/3} K$

triple alpha

3α: $He \rightarrow C$ $\quad \varepsilon_{3\alpha} \propto T_8^{41}$ $\quad T = 10^8 T_8$

radius: Virial theorem

- gravitational potential energy is 2x kinetic energy

$$2K + V = 0$$

- grav. potential energy

$$E_{pot} = -\iint \frac{G}{r} dm_1 dm_2 \approx -\frac{GM^2}{R}$$

- kinetic energy

$$E_{kin} = \frac{3}{2} N k_B T \approx \frac{M}{m_p} kT$$

$$\Rightarrow 2 \frac{M}{m_p} kT - \frac{GM^2}{R} = 0$$

$$\frac{2 k_B T_0}{m_p} \left(\frac{M}{M_\odot}\right)^{1/3} - \frac{GM}{R} = 0$$

$$\Rightarrow R = \frac{m_p G \cancel{M}}{2 k_B T_0} \left(\frac{M}{M_*}\right)^{2/3}$$

$R \sim 10^9 \left(\frac{M}{M_\odot}\right)^{2/3} m$

$R_\odot = 7 \cdot 10^8 m$

Luminosity L : $\frac{E_{ph}}{\Delta t}$

. resistance of gas against radiation : opacity κ $[\underset{\text{cross-section}}{cm^2}/g]$

diffusion process $f = \underset{\text{constant}}{c} \frac{dU}{dr}$

flux: energy per area per sec :

$$\frac{L}{4\pi R^2} = \left(\frac{c}{\kappa\rho}\right)\frac{dU_{ph}}{dr}$$

radiative energy

$U_{ph} = \frac{a}{3}T^4$ (Stefan - Boltzmann)

a: radiation constant

$$\rho = \frac{M}{V} \approx \frac{M}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \frac{L}{4\pi R^2} = \frac{ac}{3\kappa\rho}\frac{dT^4}{dr}$$

$$\sim \frac{T^4}{R} \approx \frac{4\pi}{3} R \frac{ac}{\kappa\rho} T^4$$

$\Rightarrow L \propto M^3 \sim M^4$ ~ actually closer to 4

Lifetime

$E = pMc^2$

p: efficiency and fraction $\sim 7 \cdot 10^{-4}$

$$L = \frac{pMc^2}{t_{life}} \Rightarrow t_{life} \propto \frac{1}{M^{\cancel{3}2}}\left(\sim\frac{1}{M^3}\right)$$

Central pressure

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho$$

$dP : \overset{\approx 0}{P_{surface}} - P_{core}$

$dR : R$

$$\Rightarrow \; -\frac{\cancel{\Delta}P_{core}}{R} = -\frac{GM}{R^2}\frac{M}{\frac{4}{3}\pi R^3}$$

$$\cancel{\Delta}P_{core} = \frac{3}{4}\frac{G}{\pi}\frac{M^2}{R^4} \Rightarrow P_{core} \propto \frac{M^2}{R^4}$$

$$P_{core}^{sun} = 3.45 \cdot 10^{16}\ Pa \quad (1 \text{ atmosphere} \sim 10^5\ Pa)$$