Modelling energy production

Term project 1 - how to get a working code

As you await feedback on Project 1, you should go through this document and check that your code is working properly. In order to solve Project 2, the code from Project 1 has to be correct. Some of you may have problems with the sanity check, others may have gotten errors in the calculations of energy production. Read through the entire document and check that your code is producing the correct output.

Element consumption

No step should consume more of an element than the previous steps are able to produce. Many of you have the correct idea of how to do this, but there are some errors in the execution. In order to account for element consumption, you need to start at the first reaction which is the common step for all PP branches. This step must occur at least once to produce enough ${}^3_2\text{He}$ for PPI and the first reaction of PPII/PPIII. An `if`-statement can then check if the reaction rate of the common step, r_{pp}, is smaller than the reaction rates of $2r_{33} + r_{34}$ ($2r_{33}$ since PPI requires the common step to happen twice to produce enough ${}^3_2\text{He}$). If this is the case, r_{33} and r_{34} should be recalculated so that $2r_{33} + r_{34}$ does not exceed r_{pp}. This can be done as

$$
\begin{aligned}
r_{\text{ratio}} &= \frac{r_{\text{pp}}}{2r_{33} + r_{34}} \\
r_{33,\text{new}} &= r_{33} \cdot r_{\text{ratio}} \\
r_{34,\text{new}} &= r_{34} \cdot r_{\text{ratio}}
\end{aligned}
$$

where r_{ratio} is a dimensionless quantity. Similar corrections should be made for $r_{\text{e}7}$ and r_{17} that depend on r_{34}, and r'_{17} that depends on $r_{\text{e}7}$.

An additional sanity check

In order to check that you have done the element consumption correctly, you should perform an additional sanity check. This is a check where all the `if`-statements should be entered, making sure that you account for element consumption correctly. The sanity check to perform is the following:

$$
\begin{aligned}
r_{{}^1_1\mathrm{H},{}^1_1\mathrm{H}}(Q_{{}^1_1\mathrm{H},{}^1_1\mathrm{H}} + Q_{{}^1_1\mathrm{H},{}^2_1\mathrm{D}})\rho &= 7.34 \cdot 10^4 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^3_2\mathrm{He},{}^3_2\mathrm{He}} Q_{{}^3_2\mathrm{He},{}^3_2\mathrm{He}} \rho &= 1.09 \cdot 10^1 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^3_2\mathrm{He},{}^4_2\mathrm{He}} Q_{{}^3_2\mathrm{He},{}^4_2\mathrm{He}} \rho &= 1.74 \cdot 10^4 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^7_4\mathrm{Be},\mathrm{e}^-} Q_{{}^7_4\mathrm{Be},\mathrm{e}^-} \rho &= 1.22 \cdot 10^{-3} \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^7_3\mathrm{Li},{}^1_1\mathrm{H}} Q_{{}^7_3\mathrm{Li},{}^1_1\mathrm{H}} \rho &= 4.35 \cdot 10^{-1} \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^7_4\mathrm{Be},{}^1_1\mathrm{H}}(Q_{{}^7_4\mathrm{Be},{}^1_1\mathrm{H}} + Q_{\mathrm{decay}})\rho &= 1.26 \cdot 10^5 \text{ J m}^{-3} \text{ s}^{-1} \\
r_{{}^{14}_7\mathrm{N},{}^1_1\mathrm{H}} Q_{\mathrm{CNO}} \rho &= 3.45 \cdot 10^4 \text{ J m}^{-3} \text{ s}^{-1}
\end{aligned}
$$

If you do not get these results, please contact Helle or write ask on Piazza. Getting help from other students is a great way to learn!

Energy production

When calculating the energy production from each branch, many of you have included the common step in your calculation. It can be confusing considering the equation

$$\varepsilon = \sum r_{ik} Q'_{ik}$$

We will go through how to calculate the energy production of the PPI branch, then you should try to calculate the energy production from PPII and PPIII yourself.

When calculating the energy production from PPI, the common step is required in terms of element consumption, but not as an actual reaction. That means that the energy output Q' from the common step should be included in the calculation, but not its reaction rate. You know that the common step must occur twice in order to produce enough ${}^3_2\mathrm{He}$, and the energy production can then be calculated as

$$\varepsilon_{\mathrm{PPI}} = r_{33}(Q'_{33} + 2[Q'_{\mathrm{pp}} + Q'_{\mathrm{pd}}])$$

Make sure that the calculation is correct by checking that you get the following values of ε when using a temperature $T = 1.57 \cdot 10^7$ K and density $1.62 \cdot 10^5$ kg m^{-3}:

$$\varepsilon_{\text{PPI}} = 1.09 \cdot 10^{-13} \text{ J kg}^{-1} \text{ s}^{-1}$$
$$\varepsilon_{\text{PPII}} = 4.84 \cdot 10^{-9} \text{ J kg}^{-1} \text{ s}^{-1}$$
$$\varepsilon_{\text{PPIII}} = 1.57 \cdot 10^{-9} \text{ J kg}^{-1} \text{ s}^{-1}$$
$$\varepsilon_{\text{CNO}} = 5.67 \cdot 10^{-13} \text{ J kg}^{-1} \text{ s}^{-1}$$

If you do not get these values, contact Helle.

Temperature plot

When you have implemented everything properly, you should get something similar to Figure 1 (do not worry if you do not get the line reversal at $T = 10^6$ K).

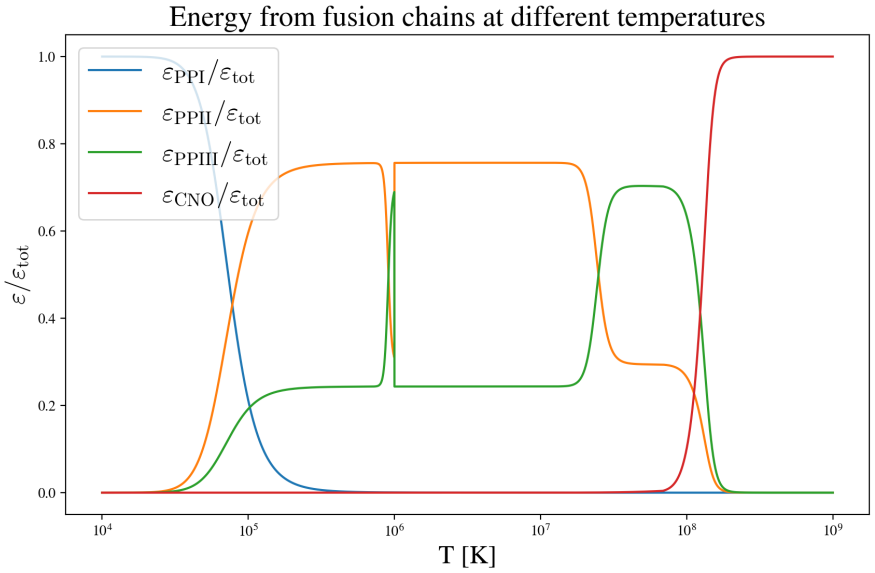

Figure 1: Relative energy production from the PP chain and CNO cycle as functions of temperature.

Many of you have plotted both axes logarithmically. Keep in mind that the task asks for the relative energy production, meaning that you should

divide each branch by the total energy production. Hence a logarithmic y-axis should not be used. Again, if you do not get this result after implementing the above corrections, contact Helle.