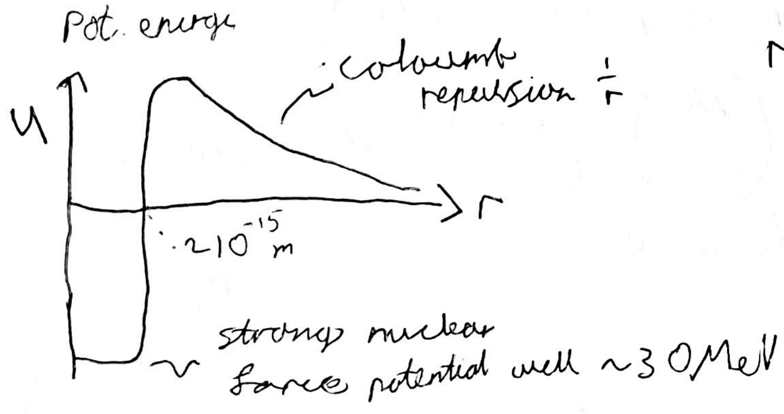
AST3310: lecture 4

- interaction of two particles ($2p^+$)



reduced mass:

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

- overcome Coulomb barrier by thermal energy:
  - initial kinetic energy = pot. energy of Coulomb barrier:

$$\frac{1}{2} m v^2 = \frac{3}{2} kT = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{r}$$

ε_0: permativity of free space

$$T_{classical} = \frac{Z_1 Z_2 e^2}{6\pi\varepsilon_0 k r}$$

temperature needed to overcome barrier

$$T^{p-p}_{class}(r \approx 10^{-15}) \approx \sim 10^{10} K$$

- $T_{\odot,0} \approx 15 \cdot 10^6 K$

- Quantum mechanical tunneling - explains the lacking temp
  - from Heisenberg uncertainty $\Delta x \Delta p \geq \frac{h}{2}$
  - de Broglie wavelength of particle $\lambda = \frac{h}{p}$

$\frac{1}{2} m v^2 = \frac{p^2}{2m}$

$$\Rightarrow \frac{p^2}{2m} = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{\lambda} = \frac{(h/\lambda)^2}{2m} = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{\lambda^2}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0} \frac{2m}{h^2} \Rightarrow T_{quantum} = \frac{Z_1 Z_2 e^2}{6\pi\varepsilon_0 k} \frac{1}{\lambda} = \frac{Z_1^2 Z_2^2 e^4 m}{12\pi^2 \varepsilon_0^2 h^2 k}$$

$$T^{p^+}_{quantum} \approx 10^7 K$$

assume: gas in ⊙ in thermal equilibrium

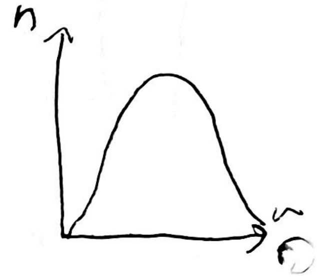
→ many collisions

→ Maxwell-Boltzmann distribution

number density ? $n_v dv = n \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{1}{2}\frac{mv^2}{kT}} 4\pi v^2 dv$

peak at most probable speed

$v_{mp} = \sqrt{\frac{2kT}{m}}$

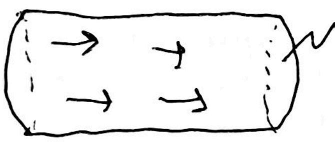


Energy:

$$n_E \, dE = \frac{2\pi}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \sqrt{E}\, e^{-E/kT} \, dE$$

n: total no. of particles per unit volume

consider particles that will interact with k



$\sigma(E)$ - cross-section

$dS = V(E) dT$

• k - target k will be hit by particles with energy E in a volume $\sigma(E) v(E) dt$ in time dt

$n_i dE$

number of reactions: ~~dN~~

n_i: total number density of incoming particles

n_{iE}: — " — with correct energy

$$dN_E = n_{iE} \, dE \cdot \sigma(E) v(E) dt$$

$$n_{iE} dE = \frac{n_i}{n} n_E dE$$

reaction rote:

$$\frac{\text{reactions per nucleus}}{\text{time interval}} = \frac{dN_E}{dt} = \sigma(E)\, v(E) \frac{n_i}{n} n_E\, dE$$

reaction rate (per unit volume per unit time)

$$r_{in} = \int_0^\infty n_n n_i \sigma(E) v(E) \frac{n_E}{n} dE$$

$$\text{cross-section } \sigma(E) = \underbrace{S(E)}_{\text{weak dependence on energy}} \cdot \underbrace{S^*(E)}_{\text{strong dependence on energy}} =$$

1 area → $\sigma(E) \propto \pi\lambda^2$ ~ de Broglie $\lambda = \frac{h}{p}$, $p = mv$, $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$$\propto \pi \frac{h^2}{p^2} \propto \frac{1}{E}$$

2. probability → exponential

Coulomb barrier

$$\sigma(E) \propto e^{-2\pi^2 U_c/E}$$

$$\frac{U_c}{E} = \frac{Z_1 Z_2 e^2 / 4\pi\varepsilon_0 r}{\frac{1}{2}mv^2} = \frac{Z_1 Z_2 e^2}{2\pi\varepsilon_0 h v}$$

$$r \cdot mv \cdot v = \frac{h}{p} pv$$

$$-2\pi^2 \frac{Z_1 Z_2 e^2}{2\pi\varepsilon_0 h v} = -\pi \frac{Z_1 Z_2 e^2}{\varepsilon_0 h} \frac{1}{v} = -\pi \frac{Z_1 Z_2 e^2}{\varepsilon_0 h} \sqrt{\frac{m}{2}} \sqrt{\frac{1}{E}}$$

$$\Rightarrow \sigma(E) \propto e^{-G/\sqrt{E}}, \quad G \equiv \frac{\pi Z_1 Z_2 e^2}{\varepsilon_0 h} \sqrt{\frac{m}{2}}$$

slowly varying function

$$\Rightarrow \sigma(E) = S(E) \cdot S^*(E) = \frac{1}{E} e^{-G/\sqrt{E}} S(E)$$

Maxwell-Boltzmann distribution: $n_E dE = \frac{2n}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \sqrt{E}\, e^{-E/kT} dE$

reaction rate integral: $r_{ik} = \int_0^\infty n_k n_i \,\sigma(E)\, v(E) \frac{n_E}{n} dE$

"cross-section": $\sigma(E) = \frac{1}{E} e^{-G/\sqrt{E}} \cdot S(E)$

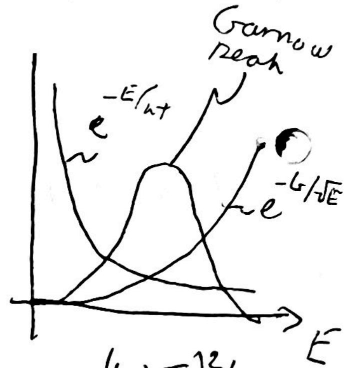
velocity:

$$v(E) = \sqrt{\frac{2E}{m}}$$

$$\Rightarrow \quad r_{ik} = \int_0^\infty n_k n_i \frac{1}{E} e^{-G/\sqrt{E}} S(E) \sqrt{\frac{2E}{m}} \frac{1}{n} \frac{2n}{\sqrt{\pi}} \frac{1}{(kT)^{2/3}} \sqrt{E}\, e^{-E/kT} dE$$

$$= 2\sqrt{2} \frac{1}{(kT)^{3/2}} \frac{n_k n_i}{\sqrt{m\pi}} \int_0^\infty S(E)\, e^{-G/\sqrt{E}} e^{-E/kT} dE$$

=per unit volume per second



$$E_0 = \left(\frac{GkT}{2}\right)^{2/3}$$

$$r_{ik}^{\text{lecture notes}} = \frac{r_{ik}}{\rho} = \frac{n_k n_i}{\rho} \lambda_{ik} \quad [m^3/s]$$