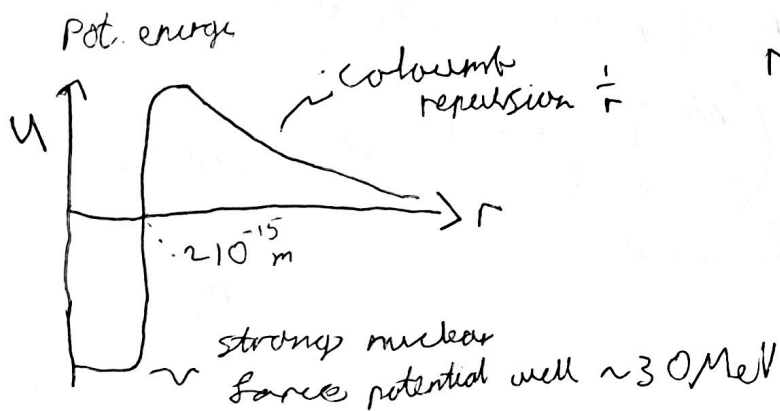


AST3310: lecture 4

interaction of two particles ($2p^+$)



reduced mass =

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

- overcome Coulomb barrier by thermal energy:
initial kinetic energy = pot. energy of Coulomb barrier:

$$\frac{1}{2} m v^2 = \frac{3}{2} kT = \frac{1}{4\pi\epsilon_0} \frac{z_1 z_2 e^2}{r}$$

ϵ_0 : permittivity of free space

$$T_{\text{classical}} = \frac{z_1 z_2 e^2}{6\pi\epsilon_0 k r}$$

temperature needed to overcome barrier

$$T_{\text{class}}^{p-d} (r \approx 10^{-15}) \approx 10^{10} \text{ K}$$

$$T_{e,0} \approx 15 \cdot 10^6 \text{ K}$$

- Quantum mechanical tunneling - explains the lowering temp

• from Heisenberg uncertainty $\Delta x \Delta p \geq \frac{\hbar}{2}$

- de Broglie wavelength of particle $\lambda = \frac{h}{p}$

$$\Rightarrow \frac{p^2}{2m} = \frac{1}{4\pi\epsilon_0} \frac{z_1 z_2 e^2}{\lambda} = \frac{(\hbar/\lambda)^2}{2m} = \frac{1}{4\pi\epsilon_0} \frac{z_1 z_2 e^2}{\lambda^2}$$

$$\frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{z_1 z_2 e^2}{4\pi\epsilon_0} \frac{2m}{\hbar^2} \Rightarrow T_{\text{quantum}} = \frac{z_1 z_2 e^2}{6\pi\epsilon_0 \hbar} \frac{1}{\lambda} = \frac{z_1^2 z_2^2 e^4 m}{12\pi^2 \epsilon_0^2 \hbar^2 k}$$

$$T_{\text{quantum}}^{p^+} \approx 10^7 \text{ K}$$

assume: gas in \odot in thermal equilibrium

→ many collisions

- Maxwell-Boltzmann distribution

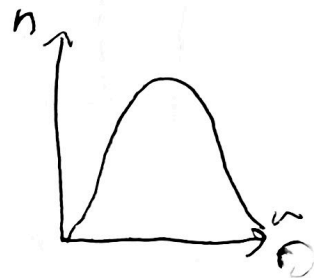
$$\text{number density} \quad n(v) dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{1}{2} \frac{mv^2}{kT}} 4\pi v^2 dv$$

reach at most probable speed

$$U_{mp} = \sqrt{\frac{2ut}{m}}$$


Energy:

$$n_E dE = \frac{2\pi}{\sqrt{\pi}} \frac{1}{(hT)^{3/2}} \sqrt{E} e^{-E/hT} dE$$



A hand-drawn diagram of a cell. The cell is an irregular oval shape. Inside, there are several organelles labeled with letters: 'a' is a small circle at the top; 'b' is a small circle at the bottom; 'c' is a small circle on the left; 'd' is a small circle on the right; 'e' is a small circle at the top right; 'f' is a small circle at the bottom right; 'g' is a small circle at the bottom left; 'h' is a small circle at the top left; 'i' is a small circle at the top center; 'j' is a small circle at the bottom center; 'k' is a small circle at the top center; 'l' is a small circle at the bottom center; 'm' is a small circle at the top center; 'n' is a small circle at the bottom center; 'o' is a small circle at the top center; 'p' is a small circle at the bottom center; 'q' is a small circle at the top center; 'r' is a small circle at the bottom center; 's' is a small circle at the top center; 't' is a small circle at the bottom center; 'u' is a small circle at the top center; 'v' is a small circle at the bottom center; 'w' is a small circle at the top center; 'x' is a small circle at the bottom center; 'y' is a small circle at the top center; 'z' is a small circle at the bottom center.

n : total no. of particles per unit volume
consider particles that will interact with h


 $\sigma(E)$ - cross-section
 h - target h particles

$$dS = V(E)dT$$

- h - target h will be hit by particles with energy E in a volume $\sigma(E) v(E) dt$ in time dt

hide

number of reactions: ~~2~~

$$dN_E = n_{iE} dE \cdot \sigma(E) v(E) dt$$

$$n_i dE = \frac{n_i}{n} n_E dE$$

n_i : total number density of incoming particles

n_{LE} : — 11 —
with correct
energy

reaction rate:

$$\frac{\text{reactions per nucleus}}{\text{time interval}} = \frac{dN_E}{dt} = \sigma(E) v(E) \frac{n_i}{n} n_E dE$$

reaction rate
(per unit volume
per unit time)

$$r_{in} = \int_0^{\infty} n_n n_i \sigma(E) v(E) \frac{n_E}{n} dE$$

cross-section $\sigma(E) = S(E) \cdot S^*(E) =$

weak dependence on energy strong dependence on energy

1 area $\rightarrow \sigma(E) \propto \pi \lambda^2 \sim$ de Broglie $\lambda = \frac{h}{p}$, $p = mv$, $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$$\propto \pi \frac{h^2}{p^2} \propto \frac{1}{E}$$

2 probability \rightarrow exponential
Coulomb barrier

$$\sigma(E) \propto e^{-2\pi^2 U_C/E}$$

$$\frac{U_C}{E} = \frac{Z_1 Z_2 e^2 / 4\pi\epsilon_0 r}{\frac{1}{2}mv^2} = \frac{Z_1 Z_2 e^2}{2\pi\epsilon_0 h v}$$

$$\downarrow$$

$$r \cdot mv \cdot v = \frac{h}{p} p v$$

$$\frac{-2\pi^2 Z_1 Z_2 e^2}{2\pi\epsilon_0 h v} = -\pi \frac{Z_1 Z_2 e^2}{\epsilon_0 h} \frac{1}{v} = -\pi \frac{Z_1 Z_2 e^2}{\epsilon_0 h} \sqrt{\frac{m}{2}} \sqrt{\frac{1}{E}}$$

$$\Rightarrow \sigma(E) \propto e^{-G/\sqrt{E}}, \quad G = \frac{\pi Z_1 Z_2 e^2}{\epsilon_0 h} \sqrt{\frac{m}{2}}$$

slowly varying
 \sim function

$$\Rightarrow \sigma(E) = S(E) \cdot S^*(E) = \frac{1}{E} e^{-G/\sqrt{E}} S(E)$$

Maxwell-Boltzmann distribution: $n_E dE = \frac{2n}{\sqrt{\pi}} \left(\frac{1}{hT}\right)^{3/2} \sqrt{E} e^{-E/hT} dE$

reaction rate integral: $r_{ih} = \int_0^{\infty} n_h n_i \sigma(E) v(E) \frac{n_E}{n} dE$

"cross-section": $\sigma(E) = \frac{1}{E} e^{-G/\sqrt{E}} S(E)$

velocity:

$$v(E) = \sqrt{\frac{2E}{m}}$$

$$\Rightarrow r_{ih} = \int_0^{\infty} n_h n_i \frac{1}{E} e^{-G/\sqrt{E}} S(E) \sqrt{\frac{2E}{m}} \frac{2n}{\sqrt{\pi}} \left(\frac{1}{hT}\right)^{3/2} \sqrt{E} e^{-E/hT} dE$$

$$= 2\sqrt{2} \frac{1}{(hT)^{3/2}} \frac{n_h n_i}{\sqrt{m}} \int_0^{\infty} S(E) e^{-G/\sqrt{E}} e^{-E/hT} dE$$

= per unit volume
per second

$$r_{ih}^{\text{lecture notes}} = \frac{r_{ih}}{E} = \frac{n_h n_i}{E} \lambda_{ih} \quad [m^3/s]$$

