FORMULA-UNIT:1

Types of roots	Complementary function
Roots are real & distinct say $m_{1,} m_{2,} ......m_n$	$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + ...c_n e^{m_n x}$
Roots are real & repeated say, $m_1 = m_{2,}$ & $m_3, m_4, ...m_n$ **are distinct.**	$y_c = (c_1 x + c_2) e^{m_1 x} + c_3 e^{m_3 x} + ... + c_n e^{m_1 x}$
Roots are complex and distinct , $m_1 = \alpha + i\beta$, $m_2 = \alpha - i\beta$	$y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$
Roots are complex and repeated , $m_1 = \alpha + i\beta, m_2 = \alpha - i\beta, m_3 = \alpha + i\beta,$ $m_4 = \alpha - i\beta$	$y_c = e^{\alpha x}((c_1 x + c_2)\cos \beta x + (c_3 x + c_4)\sin \beta x)$
Particular integral Shortcut Mehotd	
$y_p =$ Ge	$\frac{1}{\phi(D)} f(x)$
$f(x) = e^{ax}$	$y_p = \frac{1}{\phi(D)} e^{ax} = \frac{1}{\phi(a)} e^{ax}$ provided $\phi(a) \neq 0$ (D=a)
$f(x) = a^x$	$y_p = \frac{1}{\phi(D)} a^x = \frac{1}{\phi(\log a)} a^x$ provided $\phi(\log a) \neq 0$ (D=loga)
$f(x) = k$, where k is constant	$y_p = \frac{1}{\phi(D)} k = \frac{1}{\phi(0)} k$ provided $\phi(0) \neq 0$ (D=0)
f(x)= sin(ax+b)or cos(ax+b)/sinax or cosax	$y_p = \frac{1}{\phi(D^2)} \sin(ax+b) = \frac{1}{\phi(-a^2)} \sin(ax+b)$ Provided $\phi(-a^2) \neq 0$
Case of Failurer first time	If $\phi(a) = 0$ or $\phi(-a^2) = 0$ then $y_p = \frac{1}{\phi(D)} e^{ax} = \frac{x}{\phi'(D)} f(x)$
Case of Failurer second time	If $\phi^1(a) = 0$ or $\phi^1(-a^2) = 0$ then $y_p = \frac{x}{\phi^1(D)} f(x) = \frac{x^2}{\phi^{''}(D)} f(x)$
$f(x) = x^m$	$y_p = \frac{1}{\phi(D)} x^m = [\phi(D)]^{-1} x^m$ where $\phi(D)$ is any one of the form (1+z) or (1-z) $(1+z)^{-1} = 1 - z + z^2 - z^3 + -....$ $(1-z)^{-1} = 1 + z + z^2 + z^3 + -....$
$f(x) = e^{ax} V$ where V is a function of x	$y_p = \frac{1}{\phi(D)} e^{ax} V,$ then substitute D=D+a

	$y_p = e^{ax}\frac{1}{\phi(D+a)}V$
$f(x) = xV$,where V is a function of x	$y_p = \frac{1}{\phi(D)}xV = \left[x - \frac{\phi'(D)}{\phi(D)}\right]\frac{1}{\phi(D)}V$

General Method

$y_p = \frac{1}{D+a}f(x) =$	$e^{-ax}\int e^{ax}f(x)dx$

Method of variation of parameters

Step 1:Find Complimentary function of the given LDE form

$$C.F = c_1 y_1 + c_2 y_2$$

Step 2:Find Wronkian,W= $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

Step 3: Find u&v, $u = \int \frac{-y_2 f(x)}{w}dx$ $v = \int \frac{y_1 f(x)}{w}dx$

Step 4:Find $P.I = uy_1 + vy_2$

Step 5:Write complete solution y=C.F+P.I

Legendre's linear differential equation

Step: I Reduce the given differential equation into linear differential equation by

substituting (ax+b) = e^z, D $\equiv \frac{d}{dz}$, and log(ax+b)=z……………….R.H.S

$(ax+b)^2\frac{d^2y}{dx^2} = a^2D(D-1)y\,,(ax+b)\frac{dy}{dx} = aDy$ ………………………….L.H.S

Step II: Find C.F and P.I in terms of z.

Step III: Find C.F and P.I in terms of x and y by substituting back z = log (ax+b).

Cauchy's linear differential equation

Same as above put a=1