MCQ's.

Fourier Integral Representation, Fourier transform and Inverse Fourier transform

A. Fourier transform on $(-\infty, \infty)$

Definition: $\left\{ F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du \right\}$

1. Fourier transform of a function $f(x)$ defined in the interval $-\infty < x < \infty$ is.

A). $\int_{-\infty}^{\infty} f(u)\, e^{iu}\, du$ B). $\int_{-\infty}^{\infty} f(u)\, e^{-\lambda u}\, du$

c) $\int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du$ D). $\int_{0}^{\infty} f(u)\, e^{-i\lambda u}\, du$.

Sol: C

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2. The fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$ is

A). $i\lambda$ B). $\frac{1}{i\lambda}$ c) $\frac{1}{\lambda}$ D). λ.

Explaination

since $F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du$

$$\therefore\ F(\lambda) = \int_{-\infty}^{0} f(u)\, e^{-i\lambda u}\, du + \int_{0}^{\infty} f(u)\, e^{-i\lambda u}\, du$$

$$= 0 + \int_{0}^{\infty} e^{-i\lambda u} du = \left[\frac{e^{-i\lambda u}}{-i\lambda}\right]_0^{\infty} = \frac{1}{i\lambda}$$

Ans: (B)

3. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} e^{-x}, & x>0 \\ 0, & x<0 \end{cases}$ is

A). $\frac{1-\lambda}{1+\lambda^2}$ B) $\frac{1-i\lambda}{1+\lambda^2}$

C) $\frac{1-i\lambda}{1-\lambda^2}$ D) $\frac{1}{1+\lambda^2}$

Explaination

$$F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u} du$$

$$= \int_{-\infty}^{0} f(u)\, e^{-i\lambda u} du + \int_{0}^{\infty} f(u)\, e^{-i\lambda u} du$$

$$= 0 + \int_{0}^{\infty} e^{-u}\, e^{-i\lambda u} du$$

$$= \int_{0}^{\infty} e^{-(1+i\lambda)u}\, du$$

$$= \left[\frac{e^{-(1+i\lambda)u}}{-(1+i\lambda)} \right]_0^{\infty}$$

$$= \frac{1}{1+i\lambda} \times \frac{1-i\lambda}{1+i\lambda}$$

$$= \frac{1-i\lambda}{1+\lambda^2}$$

$\boxed{\text{Ans :(B)}}$

4. If $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x < 0 \end{cases}$ then fourier transform $F(\lambda)$ of $f(x)$ is

A). $\frac{e^{i\lambda\pi}+1}{1+\lambda^2}$ B). $\frac{e^{i\lambda\pi}+1}{1-\lambda^2}$

C). $\frac{e^{-i\lambda\pi}+1}{1-\lambda^2}$ D). $\frac{e^{-i\lambda\pi}+1}{1+\lambda^2}$

Explaination

$$F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du$$

$$= \int_{-\infty}^{0} f(u)\, e^{-i\lambda u} du + \int_{0}^{\pi} f(u)\, e^{-i\lambda u} du + \int_{0}^{\infty} f(u) \cdot e^{-i\lambda u} du$$

$$= 0 + \int_{0}^{\pi} \sin x \; e^{-i\lambda u} \cdot du + 0.$$

$$= \int_{0}^{\pi} \sin x \, e^{-i\lambda u} du.$$

$$= \left\{ \frac{e^{-i\lambda u}}{(i\lambda)^2+1} \left[-i\lambda \sin u - (1)\cos u \right] \right\}_0^{\pi}$$

$$= \frac{e^{-i\lambda\pi}}{1-\lambda^2} \left[-i\lambda \sin\pi - \cos\pi \right] - \frac{e^0}{1-\lambda^2} [0-1]$$

$$= \frac{e^{-i\lambda\pi}}{1-\lambda^2} + \frac{1}{1-\lambda^2}$$

$$= \frac{1+e^{-i\lambda\pi}}{1-\lambda^2}$$

$\boxed{\text{Ans: C}}$

5. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} \cos x, & x > \\ 0, & x < \end{cases}$ is

A). $\frac{i\lambda}{1-\lambda^2}$ B). $\frac{-i\lambda}{1-\lambda^2}$

C) $\frac{-i\lambda}{1+\lambda^2}$ D). $\frac{i\lambda}{1+\lambda^2}$

Explanation

$$F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du$$

$$= \int_{-\infty}^{0} f(u) \cdot e^{-i\lambda u}\, du + \int_{0}^{\infty} f(u)\, e^{-i\lambda u}\, du$$

$$= 0 + \int_0^{\infty} \cos x\, e^{-i\lambda u}\, du$$

$$= \left\{ \frac{e^{-i\lambda u}}{(-i\lambda)^2+1} \left[-i\lambda \cos u + \sin u \right] \right\}_0^{\infty}$$

$$= \frac{i\lambda}{1-\lambda^2}$$

$\boxed{\text{Sol: A}}$

6. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} \sin x, & x > 0 \\ 0, & x < 0 \end{cases}$ is.

A). $\dfrac{1}{1-\lambda^2}$ B). $\dfrac{1}{1+\lambda^2}$

C). $\dfrac{i\lambda}{1-\lambda^2}$ D). $\dfrac{i\lambda}{1+\lambda^2}$

Explaination

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du.$$

$$= \int_{-\infty}^{0} f(u) \cdot e^{-i\lambda u} du + \int_{0}^{\infty} f(u) e^{-i\lambda u} du$$

$$= 0 + \int_{0}^{\infty} \sin u \, e^{-i\lambda u} du$$

$$= \left\{ \frac{e^{-i\lambda u}}{(-i\lambda)^2+1} \left[(-i\lambda) \sin u - \cos u \right] \right\}_0^{\infty}$$

$$= \frac{1}{1-\lambda^2}$$

$\boxed{\text{Ans: A}}$

7. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} x, & x>0 \\ 0, & x<0 \end{cases}$ is

A). 0 B). $\frac{1}{\lambda^2}$

C). λ^2 D). $\frac{-1}{\lambda^2}$.

Sol: D.

Explaination

$$F(\lambda) = \int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u} du$$

$$= \int_{-\infty}^{0} f(u)\, e^{-i\lambda u} du + \int_{0}^{\infty} f(u)\, e^{-i\lambda u} du$$

$$= 0 + \int_{0}^{\infty} u\, e^{-i\lambda u} du.$$

$$= \left\{ u \frac{e^{-i\lambda u}}{-i\lambda} - \frac{e^{-i\lambda u}}{(-i\lambda)^2} \right]_0^{\infty}$$

$$= \left\{ \frac{u e^{-i\lambda u}}{-i\lambda} + \frac{e^{-i\lambda u}}{\lambda^2} \right\}_0^{\infty}$$

$$= \frac{-1}{\lambda^2}$$

8. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} x^2, & x>0 \\ 0, & x<0 \end{cases}$ is

A). $\frac{-2i}{\lambda^3}$ B). $\frac{1}{i\lambda^3}$ C). $\frac{2i}{\lambda^3}$, D). $\frac{-1}{i\lambda^3}$

Sol: C

Explaination

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$$

$$= \int_{-\infty}^{0} f(u) e^{-i\lambda u} du + \int_{0}^{\infty} f(u) e^{-i\lambda u} du$$

$$= 0 + \int_{0}^{\infty} u^2 e^{-i\lambda u} du$$

$$= \left[u^2 \frac{e^{-i\lambda u}}{-i\lambda} - 2u \frac{e^{-i\lambda u}}{(-i\lambda)^2} + 2 \frac{e^{-i\lambda u}}{(-i\lambda)^3} \right]_0^{\infty}$$

$$= \frac{2i}{\lambda^3}$$

9. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} x - x^2 & x > 0 \\ 0, & x < 0 \end{cases}$ is

A). $\frac{2}{\lambda^2} + i\frac{1}{\lambda^3}$

B). $\frac{1}{\lambda^2} - i\frac{2}{\lambda^3}$

C). $\frac{1}{\lambda^2} + i\frac{2}{\lambda^3}$

D). $-\frac{1}{\lambda^2} + i\frac{2}{\lambda^3}$.

Sol: D.

Explaination

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$$

$$= \int_{-\infty}^{0} f(u) e^{-i\lambda u} du + \int_{0}^{\infty} f(u) e^{-i\lambda u} du$$

$$= 0 + \int_{0}^{\infty} (u - u^2) e^{-i\lambda u} du$$

$$= \int_{0}^{\infty} u e^{-\lambda u} du + \int_{0}^{\infty} + u^2 e^{-i\lambda u} du.$$

$$= \left[u \frac{e^{-i\lambda u}}{-i\lambda} - (1) \frac{e^{-i\lambda u}}{(-i\lambda)^2} \right]_0^\infty + \left[u^2 \frac{e^{-i\lambda u}}{-i\lambda} - 2u \frac{e^{-i\lambda u}}{(-i\lambda)^2} + 2 \frac{e^{-i\lambda u}}{(-i\lambda)^3} \right.$$

$$= \frac{-1}{\lambda^2} + \frac{2i}{\lambda^3}$$

$$= \underline{\underline{\frac{2i}{\lambda^3} - \frac{1}{\lambda^2}}}$$

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10. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 2+x, & x>0 \\ 0, & x<0 \end{cases}$

A). $\frac{-1}{\lambda^2} - \frac{i2}{\lambda}$ B). $\frac{1}{\lambda^2} - \frac{i2}{\lambda}$

C). $\frac{1}{\lambda^2} + \frac{i2}{\lambda}$ D). $\frac{-1}{\lambda^2} + \frac{i2}{\lambda}$.

Sol: D.

<u>Explaination</u>

$$F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du.$$

$$= \int_{-\infty}^{0} f(u) e^{-i\lambda u} du + \int_{0}^{\infty} f(u) e^{-i\lambda u} du$$

$$= 0 + \int_0^\infty (2+u) e^{-i\lambda u} du$$

$$= 2\int_0^\infty e^{-i\lambda u} du + \int_0^\infty u e^{-i\lambda u} du$$

$$= \left[2 \frac{e^{-i\lambda u}}{-i\lambda} + u \frac{e^{-i\lambda u}}{-i\lambda} - \frac{e^{-i\lambda u}}{(-i\lambda)^2} \right]_0^\infty$$

$$= \frac{+2}{i\lambda} + \frac{1}{-\lambda^2}. = \underline{\underline{\frac{2}{i\lambda} - \frac{1}{\lambda^2}}}$$

∴ Function is even in $(-\infty, \infty)$.

$f(\lambda)$ reduces to Fourier Cosine transform

is $\left\{ F_c(\lambda) = \int_0^\infty f(u) \cos \lambda u \cdot du \right.$ or

or $\left. 2 \int_0^\infty f(u) \cos \lambda u \, du. \right\}$

11. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ is

A). $\frac{2 \sin \lambda a}{\lambda}$ B). $\frac{e^{-i\lambda a}}{\lambda}$

C). $\frac{e^{i\lambda a}}{\lambda}$ D). $\frac{2 \cos \lambda a}{\lambda}$.

Sol: A

Explaination

Since $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ is even

$F(\lambda) \Rightarrow F_c(\lambda) = 2 \int_0^\infty f(u) \cos \lambda u \cdot du$

$= 2 \int_0^a f(u) \cos \lambda u \cdot du + 2 \int_a^\infty f(u) \cos \lambda u \cdot du$

$= 2 \cdot \int_0^a \cos \lambda u \cdot du + 0 .$

$= 2 \cdot \left[\frac{\sin \lambda u}{\lambda} \right]_0^a$

$= \frac{2 \sin \lambda a}{\lambda}$ //

12. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ is

A). $\frac{-4}{\lambda^3}(\sin\lambda - \lambda\cos\lambda)$ B). $\frac{4}{\lambda^3}(\sin\lambda - \lambda\cos\lambda)$

C). $\frac{4}{\lambda^2}(\sin\lambda - \lambda\cos\lambda)$ D). $\frac{4}{\lambda^3}(\sin\lambda + \lambda\cos\lambda)$

Sol : B.

Explaination

Since $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ is even

$$F(\lambda) = F_c(\lambda) = 2\int_0^\infty f(u)\cos\lambda u\,du$$

$$= 2\int_0^1 f(u)\cos\lambda u\,du + 2\int_1^\infty f(u)\cos\lambda u\,du$$

$$= 2\int_0^1 (1-u^2)\cos\lambda u\,du + 0$$

$$= 2\int_0^1 \cos\lambda u\,du - 2\int_0^1 u^2\cos\lambda u\,du$$

$$= 2\left[\frac{\sin\lambda u}{\lambda}\right]_0^1 - \left[2\; u^2\frac{\sin\lambda u}{\lambda} + 2u\frac{-\cos\lambda u}{\lambda^2} - 2\frac{-\sin\lambda u}{\lambda^3}\right]$$

$$= \frac{2\sin\lambda}{\lambda} - \left[\frac{2\sin\lambda}{\lambda} - \frac{2\cos\lambda}{\lambda^2} + \frac{2\sin\lambda}{\lambda^3}\right]$$

$$= \frac{-2\cos\lambda}{\lambda^2} + \frac{2\sin\lambda}{\lambda^3} = \frac{4}{\lambda^3}[\sin\lambda - \lambda\cos\lambda]$$

13. The fourier transform $F(\lambda)$ of $f(x) = e^{-|x|}$ is given by.

A). $\frac{1}{1+\lambda^2}$ B). $\frac{1}{1-\lambda^2}$

C) $\frac{\lambda}{1-\lambda^2}$ D). $\frac{2}{1+\lambda^2}$

Sol: D.

Explaination

Since $f(x) = e^{-|x|}$ is even

$$F(\lambda) = F_c(\lambda) = 2\int_0^\infty \cancel{e^{x}}\, f(u)\, e^{-i\lambda u}\, du.$$

$$= 2.\int_0^\infty f(u) \cos\lambda u . du$$

$$= 2\int_0^\infty e^{-u} \cos\lambda u . du.$$

$a = -1$
$b = \lambda$.

$$= 2\left\{\frac{e^{-u}}{1+\lambda^2}\left[-\cos\lambda u + \lambda \sin\lambda u\right]\right\}_0^\infty$$

$$= \frac{2}{1+\lambda^2}[+1] = \frac{+2}{1+\lambda^2}$$

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B. Fourier transform in the interval $(0,\infty)$.

15 The Fourier cosine transform of

$$f(x) = \begin{cases} \frac{\pi}{2}, & 0<x<\pi \\ 0, & x>\pi \end{cases}$$

A). $\frac{\pi}{2}\left[\frac{1-\sin\lambda\pi}{\lambda}\right]$

B). $\frac{1-\sin\lambda\pi}{\lambda}$

C). $\frac{\pi\sin\lambda\pi}{2\lambda}$

D). $\left(\frac{\sin\lambda\pi}{\lambda}\right)$

Sol: C.

Explaination

$$F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\,du$$

$$= \int_0^\pi f(u)\cos\lambda u\,du + \int_\pi^\infty f(u).\cos\lambda u.du$$

$$= \int_0^\pi \frac{\pi}{2}\cos\lambda u\,du + 0$$

$$= \frac{\pi}{2}\left(\frac{\sin\lambda y}{\lambda}\right)_0^\pi$$

$$= \frac{\pi}{2\lambda}\sin\lambda\pi$$

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16 The Fourier cosine transform $F_c(\lambda)$ of $f(x) = e^{-x}$, $x>0$.

A). $\frac{2}{1-\lambda^2}$ B). $\frac{1}{1-\lambda^2}$

C). $\frac{2}{1+\lambda^2}$ D). $\frac{1}{1+\lambda^2}$

Sol: D.

Explaination

$$F_c(\lambda) = \int_0^\infty f(u) \cdot \cos\lambda u\,du$$

$$= \int_0^\infty e^{-u} \cos\lambda u\,du.$$

$a=1$
$b=\lambda$

$$= \left\{\frac{e^{-u}}{1+\lambda^2}\left[-\cos\lambda u + \lambda\sin\lambda u\right]\right\}_0^\infty$$

$$= -\frac{1}{1+\lambda^2}[-1] = \underline{\underline{\frac{1}{1+\lambda^2}}}$$

18 $f(x) = e^{-kx}$, $x>0$, then Fourier cosine transform $F_c(\lambda)$ of $f(x)$ is given by

A). $-\frac{k}{k^2+\lambda^2}$ B). $\frac{k}{k^2+\lambda^2}$

C). $\frac{\lambda}{k^2+\lambda^2}$ D). $\frac{1}{k^2+\lambda^2}$

Explaination

Sol: B

$$F_c(\lambda) = \int_0^\infty f(u) \cos\lambda u \cdot du$$

$$= \int_0^\infty e^{-ku} \cos\lambda u\,du$$

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$$= \left\{ \frac{e^{-kx}}{k^2+\lambda^2} \left[-k\cos\lambda u + \lambda\sin\lambda u \right] \right\}_0^\infty$$

$a = -k$
$b = \lambda$.

$$= \frac{-e^0}{k^2+\lambda^2}\left[-k\right] = \frac{k}{k^2+\lambda^2}$$

18 If $f(x) = 1$, $x > 0$, then $F_c(\lambda)$ of $f(x)$ is

A). $\frac{\sin\lambda}{\lambda}$ B). $\frac{\cos\lambda}{\lambda}$

C). $\frac{\cos 2\lambda}{\lambda}$ D). $\frac{\sin 2\lambda}{\lambda}$

Sol: A

Explaination

$$F_c(\lambda) = \int_0^\infty f(u).\cos\lambda u.du$$

$$= \int_0^1 \cos\lambda u du$$

$$= \left(\frac{\sin\lambda y}{\lambda}\right)_0^1 = \frac{\sin\lambda}{\lambda}$$

19 If $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ then $F_c(\lambda)$ is

A) $\frac{1}{2}\left[\frac{-\cos(\lambda+1)u}{(\lambda+1)} - \frac{\sin(1-\lambda)y}{1-\lambda}\right]_0^\pi$ B) $\frac{1}{2}\left[\frac{\cos(\lambda+1)y}{(\lambda+1)} - \frac{\sin(1-\lambda)y}{1-\lambda}\right]_0^\pi$

C) $\frac{1}{2}\left[\frac{-\cos(\lambda+1)y}{\lambda} - \frac{\cos(1-\lambda)y}{1-\lambda}\right]_0^\pi$ D) $\frac{1}{2}\left[\frac{\sin(\lambda+1)y}{\lambda+1} - \frac{\cos(1-\lambda)y}{1-\lambda}\right]_0^\pi$

Sol:

Explaination

$$F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u.du$$

$$= \int_0^\pi f(u)\cos\lambda u du + \int_\pi^\infty f(u)\cos\lambda u.du$$

$$= \int_0^{\pi} \sin u \cos \lambda u \cdot du \qquad \sin(1+\lambda)u + \sin(1-\lambda)u$$

$$= \frac{1}{2}\int_0^{\pi} \sin(1+\lambda)u + \sin(1-\lambda)u$$

$$= \frac{1}{2}\left[\frac{-\cos(1+\lambda)u}{1+\lambda} - \frac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$$

$$= -2\cos u \cos \lambda B$$

$\boxed{\text{Ans: C}}$

20 The Fourier transform $F_c(\lambda)$ of $f(x) = \begin{cases} \cos x, & 0 < x \\ 0, & x > \pi \end{cases}$

A). $\frac{1}{2}\left[\frac{\sin(1-\lambda)u}{1-\lambda} - \frac{\cos(1+\lambda)u}{1+\lambda}\right]_0^{\pi}$ B). $\frac{1}{2}\left[-\frac{\cos(1+\lambda)u}{1+\lambda} - \frac{\sin(1-\lambda)u}{1-\lambda}\right]$

C). $\frac{1}{2}\left[\frac{-\cos(1+\lambda)u}{1+\lambda} - \frac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ D). $\frac{1}{2}\left[\frac{\sin(1+\lambda)u}{1+\lambda} - \frac{\sin(1-\lambda)u}{1-\lambda}\right]_0$ Sol:

<u>Explanation</u>

$$F_c(\lambda) = \int_0^{\infty} f(u)\cos\lambda u \cdot du$$

$$= \int_0^{\pi} f(u)\cdot\cos\lambda u\cdot du + \int_{\pi}^{\infty} f(u)\cos\lambda u\, du$$

$$= \int_0^{\pi} \cos u\cdot\cos\lambda u\cdot du + 0$$

$$= \frac{1}{2}\int_0^{\pi}\left[\cos(1-\lambda)u + \cos(1+\lambda)u\right] du$$

$$= \frac{1}{2}\left[\frac{\sin(1-\lambda)u}{1-\lambda} + \frac{\sin(1+\lambda)u}{1+\lambda}\right]_0^{\pi}$$

$\boxed{\text{Ans. D}}$

21 The Fourier transform $F_c(\lambda)$ of $f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}$ is

A). $\frac{1}{2}\left[\frac{\sin(\lambda+1)a}{\lambda+1} - \frac{\sin(\lambda-1)a}{\lambda-1}\right]$ B). $\frac{1}{2}\left[\frac{\sin(\lambda-1)a}{\lambda-1} - \frac{\sin(\lambda+1)a}{\lambda+1}\right]$

C). $\frac{1}{2}\left[\frac{\sin(\lambda+1)a}{\lambda+1} + \frac{\sin(\lambda-1)a}{\lambda-1}\right]$ D). $\frac{\sin(\lambda+1)a}{\lambda+1}$

Explaination

$$F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u \cdot du$$

$$= \int_0^a f(u)\cos\lambda u \cdot du + \int_a^\infty f(u)\cos\lambda u\, du$$

$$= \int_0^a \cos u \cdot \cos\lambda u \cdot du + 0$$

$$= \frac{1}{2}\int_0^a [\cos(1-\lambda)u + \cos(1+\lambda)u]\, du$$

$$= \frac{1}{2}\left[\frac{\sin(1-\lambda)u}{1-\lambda} + \frac{\sin(1+\lambda)u}{1+\lambda}\right]_0^a$$

$$= \frac{1}{2}\left[\frac{\sin(1-\lambda)a}{1-\lambda} + \frac{\sin(1+\lambda)a}{1+\lambda}\right]$$

$$= \frac{1}{2}\left[\frac{\sin(1+\lambda)a}{1+\lambda} + \frac{\sin(\lambda-1)a}{\lambda-1}\right]$$

Ans : C.

Fourier sine transform $(0, \infty)$.

22 Find Fourier sine transform of $f(x) = e^{-mx}$, $m > 0$, $x > 0$ is.

A). $\frac{\lambda}{m^2+\lambda^2}$ B). $\frac{m}{m^2+\lambda^2}$

C). $\frac{1}{m^2+\lambda^2}$ D). $\frac{-m}{m^2+\lambda^2}$

Explaination

$$F_c(\lambda) = \int_0^{\infty} f(u) \sin\lambda u \cdot du$$

$$= \int_0^{\infty} e^{-mu} \sin\lambda u . du.$$

$$= \left\{ \frac{e^{-mu}}{m^2+\lambda^2} \left[-m\sin\lambda u - \lambda\cos\lambda u \right] \right\}_0^{\infty}$$

$$= \frac{-1}{m^2+\lambda^2}\left[-\lambda\right] = \underline{\underline{\frac{\lambda}{m^2+\lambda^2}}}$$

$\boxed{\text{Ans : A}}$

23 The fourier sine transform $F_s(\lambda)$ of $f(x) = \begin{cases} \pi/2 \\ 0 \end{cases}$

A). $\frac{\pi}{2}\left[\frac{1-\cos\lambda\pi}{\lambda}\right]$ B). $\frac{\pi}{2}\left[\frac{\cos\lambda\pi-1}{\lambda}\right]$

C). $\frac{\pi}{2}\left[\frac{1-\cos\lambda\pi}{\lambda}\right]$ D). $\left(\frac{\cos\lambda\pi}{\lambda}\right)$

Explaination

$$F_s(\lambda) = \int_0^{\infty} f(u) \cdot \sin\lambda u \cdot du$$

$$= \int_0^{\pi} f(u) \cdot \sin\lambda u \cdot du + \int_{\pi}^{\infty} f(u) \cdot \sin\lambda u . du$$

$$= \int_0^{\pi} \frac{\pi}{2} \sin\lambda u . du + 0$$

$$= \frac{\pi}{2}\left[\frac{-\cos\lambda y}{\lambda}\right]_0^{\pi}$$

$$= \frac{\pi}{2}\left[\frac{1-\cos\lambda\pi}{\lambda}\right]$$ //

24 The fourier sine transform $F_s(\lambda)$ of

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

A). $\left(\frac{\cos \lambda \pi - 1}{\lambda} \right)$

B). $\left(\frac{1 - \cos \lambda}{\lambda} \right)$.

c). $\left(\frac{1 - \sin \lambda}{\lambda} \right)$

D). $\left(\frac{\cos \lambda \pi}{\lambda} \right)$.

Explanation.

$$F_s(\lambda) = \int_0^\infty f(u) \sin \lambda u . du$$

$$= \int_0^1 f(u) \sin \lambda u . du + \int_1^\infty f(u) \sin \lambda u . du$$

$$= \int_0^1 \sin \lambda u . du + 1$$

$$= \left[\frac{-\cos \lambda y}{\lambda} \right]_0^1 = \frac{-\cos \lambda}{\lambda} + \frac{1}{\lambda}$$

$$= \frac{1 - \cos \lambda}{\lambda}$$

Ans: B.

25 The fourier sine transform $F_s(\lambda)$ of $f(x)$

is $= \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

A. $\frac{\lambda \sin \lambda + \cos \lambda - 1}{\lambda^2}$

B $\frac{\cos \lambda - \lambda \sin \lambda - 1}{\lambda^2}$.

C. $\frac{\cos \lambda - \lambda \sin \lambda + 1}{\lambda^2}$

D. $\frac{\lambda \sin \lambda + 1}{\lambda^2}$.

Explaination

$$F_S(\lambda) = \int_0^\infty f(u) \sin\lambda u \, du$$

$$= \int_0^1 f(u) \sin\lambda u \cdot du + \int_1^\infty f(u) \cdot \sin\lambda u \cdot du .$$

$$= \int_0^1 x \sin\lambda u \cdot du + 0$$

$$= \left\{ u\left(\frac{-\cos\lambda y}{\lambda}\right) - (1) \int \frac{-\cos\lambda y}{\lambda} du \right\}_0^\infty$$

$$= \left[-\frac{u\cos\lambda y}{\lambda} + \frac{\sin\lambda y}{\lambda^2} \right]_0^\infty$$

$$= \frac{-\cos\lambda}{\lambda} + \frac{\sin\lambda}{\lambda^2} .$$

$$= \frac{\sin\lambda - x\cos\lambda}{\lambda^2}$$

Ans: C

6 If $f(x) = \begin{cases} x^2, & 0<x<1 \\ 0, & x>1 \end{cases}$ then fourier transform $F_S(\lambda)$ of $f(x)$ is given by

A). $\dfrac{-\lambda^2\cos\lambda + 2\lambda\sin\lambda + 2(\cos\lambda - 1)}{\lambda^3}$

B) $\dfrac{\lambda^2\cos\lambda + 2\lambda\sin\lambda + 2(\cos\lambda - 1)}{\lambda^3}$

c) $\dfrac{\lambda^2\cos\lambda - 2\lambda\sin\lambda + 2(\cos\lambda - 1)}{\lambda^3}$

d) $\dfrac{\lambda^2\cos\lambda - 2\lambda\sin\lambda - 2(\cos\lambda - 1)}{\lambda^3}.$

Explaination

$$F_s(\lambda) = \int_0^\infty f(u)\ \sin\lambda u \cdot du$$

$$= \int_0^1 f(u) \sin\lambda u \cdot du + \int_0^\infty f(u) \cdot \sin\lambda u \cdot du$$

$$= \int_0^1 x^2 \sin\lambda u \cdot du + 0.$$

$$= \left\{ \frac{x^2 \cos\lambda u}{\lambda} + \int 2u \frac{\cos\lambda u}{\lambda} du \right\}_0^1$$

$$= \left\{ \frac{u^2 \cos\lambda y}{\lambda} + \frac{2u \sin\lambda u}{\lambda^2} - \int \frac{2 \sin\lambda y}{\lambda^2} du \right\}_0^1$$

$$= \left[\frac{u^2 \cos\lambda y}{\lambda} + \frac{2u \sin\lambda y}{\lambda^2} + \frac{2\cos\lambda y}{\lambda 3} \right]_0^1$$

$$= \frac{-\cos\lambda}{\lambda} + \frac{2\sin\lambda}{\lambda 2} + \frac{2\cos\lambda}{\lambda 3} - \frac{2}{\lambda 3}$$

$$= \frac{-\cos\lambda}{\lambda} + \frac{2\sin\lambda}{\lambda^2} + \frac{2(\cos\lambda - 1)}{\lambda^3}$$

$$= \frac{-\lambda^2\cos\lambda + 2\lambda\sin\lambda + 2(\cos\lambda - 1)}{\lambda 3.}$$

$\boxed{\text{Ans: A.}}$

27 The Fourier sine transform $F_s(\lambda)$ of ... is given by.

A). $\frac{3\lambda}{1+\lambda^2}$ B). $\frac{\lambda}{1-\lambda^2}$

C). $\frac{\lambda}{1+\lambda^2}$ D). $\frac{\lambda}{1-\lambda^2}$

Explaination

$$F_s(\lambda) = \int_0^\infty f(u) \sin \lambda u . du$$

$$= \int_0^\infty e^{-x} \sin \lambda u \, du$$

$a = -1$
$b = \lambda$.

$$= \left\{ \frac{e^{-x}}{1+\lambda^2} \left[-\sin \lambda x - \lambda \cos \lambda u \right] \right\}_0^\infty$$

$$= \frac{-1}{1+\lambda^2} \times -\lambda = \frac{\lambda}{1+\lambda^2}$$

$\boxed{\text{Ans: } \frac{\lambda}{1+\lambda^2} = C}$

28 If $f(x) = e^{-kx}$, $x > 0$, $k > 0$, then Fourier sine transform $F_s(\lambda)$ of $f(x)$ is given by

A). $\frac{\lambda}{k^2+\lambda^2}$ B). $\frac{k}{k^2+\lambda^2}$

C) $\frac{1}{k^2+\lambda^2}$ D). $\frac{-k}{k^2+\lambda^2}$

Explaination

$$F_s(\lambda) = \int_0^\infty f(u) \cdot \sin\lambda u \, du$$

$$= \int_0^\infty e^{-ku} . \sin\lambda u . du$$

$$= \left\{ \frac{e^{-ku}}{k^2+\lambda^2} \left[-\sin\lambda u - \lambda\cos\lambda u \right] \right\}_0^\infty$$

$$= \frac{-1}{k^2+\lambda^2} \left[-\lambda \right] = \underline{\underline{\frac{\lambda}{k^2+\lambda^2}}}$$

Ans. A

29 The Fourier sine transform $F_s(\lambda)$ of $f(x) = e^{-|x|}, 0 < x < \infty$ is.

A). $\frac{\lambda}{1+\lambda^2}$ B). $\frac{1}{1+\lambda^2}$

C). $\frac{1}{1-\lambda^2}$ D). $\frac{-1}{1+\lambda^2}$

Explanation

$$F_s(\lambda) = \int_0^\infty f(u) \sin\lambda u \, du$$

$$= \int_0^\infty e^{-x} \sin\lambda u \, du$$

$$= \left\{ \frac{e^{-x}}{1+\lambda^2} \left[-\sin\lambda u - \lambda\cos\lambda u \right] \right\}_0^\infty$$

$$= \frac{-1}{1+\lambda^2} \left[-\lambda \right] = \underline{\frac{\lambda}{1+\lambda^2}}$$

Ans. A

C. Inverse Fourier transform

33 The inverse Fourier transform $f(x)$ defined in $-\infty < x < \infty$ of $F(\lambda)$ is.

A). $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\lambda)\, e^{i\lambda x}\, d\lambda$

B). $\frac{2}{\pi}\int_{-\infty}^{\infty} F(\lambda)\, e^{-i\lambda x}\, d\lambda$

C). $\frac{1}{2\pi}\int_{-\infty}^{0} F(\lambda)\, e^{ix}\, d\lambda$

D) $\frac{1}{2\pi}\int_{0}^{\infty} F(\lambda)\, e^{i\lambda x}\, dx.$

Ans: A.

34 The inverse fourier transform, $f(x)$ defined in $-\infty < x < \infty$ of $F(\lambda) = \left[\frac{1-i\lambda}{1+\lambda^2}\right]$ is

A). $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}\right] d\lambda.$

B). $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2} + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}\right] d\lambda$

C). $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2} + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}\right] d\lambda$

D). $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\cos\lambda x + \lambda\sin\lambda x}{1-\lambda^2} + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1-\lambda^2}\right] d\lambda$

Explaination

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\lambda)\, e^{i\lambda x}\, d\lambda.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{1-\lambda i}{1+\lambda^2}\right) e^{i\lambda x}\, d\lambda.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[\left(\frac{1-\lambda i}{1+\lambda^2}\right)(\cos\lambda x + i\sin\lambda x)\right] d\lambda$$

$$= \frac{1}{2\pi(1+\lambda^2}\int_{-\infty}^{\infty} \left[\cos\lambda x + i\sin\lambda x - i\lambda\cos\lambda x + \lambda\sin\lambda x\right] d\lambda$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+\lambda^2}\left[\cos\lambda x + \lambda\sin\lambda x\right] + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}$$

$\boxed{\text{Ans . B}}$

35 The inverse Fourier transform $f(x)$ defined in $-\infty < x < \infty$ of $F(\lambda) = \pi\left[\frac{1-i\lambda}{1+\lambda^2}\right]$ is.

A). $\frac{1}{2}\int_{0}^{\infty}\left[\frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2} + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}\right] d\lambda.$

B]. $\frac{1}{2}\int_{-\infty}^{\infty}\left[\frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2} + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}\right] d\lambda$

C] $\frac{1}{2}\int_{-\infty}^{\infty}\left[i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1+\lambda^2}\right] d\lambda$

D] $\frac{1}{2}\int_{-\infty}^{\infty}\left[\frac{\cos\lambda x + \lambda\sin\lambda x}{1-\lambda^2} + i\,\frac{-\lambda\cos\lambda x + \sin\lambda x}{1-\lambda^2}\right] d\lambda.$

<u>Explaination</u>

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\lambda)\, e^{i\lambda x}\, d\lambda.$$

42 In the fourier integral representation $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\pi\frac{(1-i\lambda)}{1+\lambda^2}\right)e^{i\lambda x}d\lambda$

$= 0, \; x<0$ $\quad F(\lambda)$ is

$\; e^{-x}, \; x>0,$

A) $\frac{1+\lambda^2}{1-i\lambda}$ B) $\frac{\sin\lambda}{1+\lambda^2}$

C) $\frac{\cos\lambda}{1+\lambda^2}$ D) $\pi\left(\frac{1-i\lambda}{1+\lambda^2}\right)$

Explaination

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\pi\left(\frac{1-i\lambda}{1+\lambda^2}\right)e^{-i\lambda x}\,d\lambda$$

$$F(\lambda) = \frac{\pi(1-i\lambda)}{(1+\lambda^2)}$$

$\boxed{\text{Ans : D}}$

~~If the fourier integral representation of f(x) is~~

$$\cancel{\frac{2}{\pi}\int_0^{\infty}\frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda} = \begin{cases} \cancel{1, \; |x|<1} \\ \cancel{0, \; |x|>1} \end{cases}$$ ~~then~~

~~value of i~~

43 For the fourier sine integral representation

$$e^{-x}\cos x = \frac{2}{\pi}\int_0^{\infty}\frac{\lambda^3}{\lambda^4+4}\sin\lambda x\,d\lambda, \; F_s(\lambda) \text{ is}$$

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A) $\frac{\lambda}{\lambda^4+4}$ B) $\frac{\lambda^3}{\lambda^4+4}$ C) $\frac{\lambda^4+4}{\lambda^3}$ D) $\frac{1}{\lambda^4+4}$.

Explaination

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\lambda^3}{\lambda^4+4} \sin\lambda x \, d\lambda.$$

where $F_s(\lambda) = \frac{\lambda^3}{\lambda^4+4}$

$\boxed{\text{Ans: B}}$

74 For the Fourier cosine integral representation

$$\frac{2}{\pi}\int_0^\infty \frac{\cos\frac{\pi\lambda}{2}}{1-\lambda^2} \cos\lambda x \, d\lambda = \begin{cases} \cos x, & |x| \leq \frac{\pi}{2} \\ 0, & |x| > \frac{\pi}{2} \end{cases}, \text{ then } \ldots$$

cosine transform $F_c(\lambda)$ is

A) $\frac{1-\lambda^2}{\cos\frac{\pi\lambda}{2}}$ B) $\frac{\sin\frac{\pi\lambda}{2}}{1-\lambda^2}$

C) $\frac{\cos\frac{\pi\lambda}{2}}{1-\lambda^2}$ D) $\frac{\cos\frac{\pi\lambda}{2}}{1+\lambda^2}$

Explaination

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\cos\frac{\pi\lambda}{2}}{1-\lambda^2} \cos\lambda x \, d\lambda$$

$$F_c(\lambda) = \frac{\cos\frac{\pi\lambda}{2}}{1-\lambda^2}$$

$\boxed{\text{Ans: C.}}$

47 For the Fourier sine integral representation

$\frac{6}{\pi}\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2+1)(\lambda^2+4)} d\lambda = e^{-x} - e^{-2x}$, $x>0$, $F_s(\lambda)$ is

A). $\frac{(\lambda^2+1)(\lambda^2+4)}{3\lambda}$

B). $\frac{\lambda}{(\lambda^2+1)(\lambda^2+4)}$

c). $\frac{3\lambda}{(\lambda^2+1)(\lambda^2+4)}$

D) $\frac{\lambda \sin \lambda x}{(\lambda^2+1)(\lambda^2+4)}$

Explaination

$$f(x) = \frac{6}{\pi}\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2+1)(\lambda^2+4)} d\lambda$$

$$F_s(\lambda) = \frac{\lambda \sin \lambda x}{(\lambda^2+1)(\lambda^2+4)}$$

Ans : D

48 For the Fourier sine integral representation

$\frac{2}{\pi}\int_0^\infty \frac{2\lambda \sin \lambda x}{\lambda^4+4} d\lambda = e^{-x}\sin x$, $x>0$, $F_s(\lambda)$ is

A). $\frac{\lambda^4+4}{2\lambda \sin \lambda x}$

B). $\frac{2\lambda}{\lambda^4+4}$

c) $\frac{2\lambda \sin \lambda x}{\lambda^4+4}$

D). $\frac{2\lambda \cos \lambda x}{\lambda^4+4}$

Explaination

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{2\lambda \sin \lambda x}{\lambda^4+4} d\lambda$$

$$F_s(\lambda) = \frac{2\lambda \sin \lambda x}{\lambda^4+4}$$

D]. Application of Fourier Integral

52 If the Fourier integral representation of $f(x)$ is $\frac{2}{\pi}\int_0^{\infty} \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda = \begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$ then value of integral $\int_0^{\infty} \frac{\sin\lambda}{\lambda}\, d\lambda$ is

A]. $\frac{\pi}{4}$ B] $\frac{\pi}{2}$ C] 0 D] 1

Explaination

Since $\frac{2}{\pi}\int_0^{\infty} \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda = \begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$

put $x=0$

$$\Rightarrow \frac{2}{\pi}\int_0^{\infty} \frac{\sin\lambda}{\lambda}\, d\lambda = 1$$

$$\Rightarrow \int_0^{\infty} \frac{\sin\lambda}{\lambda}\, d\lambda = \frac{\pi}{2}$$

$\boxed{\text{Ans : B}}$

53 If the Fourier integral representation of $f(x)$ is $\frac{1}{\pi}\int_0^{\infty} \frac{\cos\lambda x + \cos(\lambda\pi - x)}{1-\lambda^2}\, d\lambda = \begin{cases} \sin x, & 0<x<\pi \\ 0, & x<0 \ \& \ x>\pi \end{cases}$

then the value of the integral $\int_0^{\infty} \frac{\cos\frac{\lambda\pi}{2}}{1-\lambda^2}\, d\lambda$ is

A]. $\frac{\pi}{4}$ B]. 1 C] 0 D] $\frac{\pi}{2}$

Explaination

$\because \frac{1}{\pi}\int_0^\infty \frac{\cos\lambda x + \cos(\lambda(\pi - x))}{1-\lambda^2} d\lambda = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x < 0 \end{cases}$

put $x = \frac{\pi}{2}$.

$$\frac{1}{\pi}\int_0^\infty \frac{\cos\lambda\frac{\pi}{2} + \cos\lambda\left(\pi - \frac{\pi}{2}\right)}{1-\lambda^2} d\lambda = \sin \pi/2$$

$$\int_0^\infty \frac{\cos\lambda\frac{\pi}{2} + \cos\lambda\left(\frac{\pi}{2}\right)}{1-\lambda^2} d\lambda = \pi$$

$$\int_0^\infty \frac{2\cos\lambda\frac{\pi}{2}}{1-\lambda^2} = \pi$$

$$\int_0^\infty \frac{\cos\lambda\frac{\pi}{2}}{1-\lambda^2} = \frac{\pi}{2}$$

$\boxed{\text{Ans: D.}}$

56. Given that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$. then Fourier Sine transform $F_s(\lambda)$ of $f(x) = \frac{1}{x}, x>0$ is given by

A]. π B] $\frac{\pi}{4}$ C] $\frac{\pi}{2}$ D] $-\pi$.

Explaination

$$F_s(\lambda) = \int_0^\infty f(u) \sin\lambda u \, du$$

$$= \int_0^\infty \frac{1}{u} \sin\lambda u \, du$$

$$= \frac{\pi}{2}$$

Ans : C

57 For the fourier cosine transform $\int_0^\infty \left(\frac{1-\cos u}{u^2}\right) \cos\lambda u . du = \begin{cases} \frac{\pi}{2}(1-\lambda), & 0<\lambda<1 \\ 0, & \lambda>1 \end{cases}$, the value of integral $\int_0^\infty \frac{\sin^2 z}{z^2}$ is

A]. 1   B] $\frac{\pi}{2}$

C]. 0   D] $\frac{\pi}{4}$.

Explaination

Since $\int_0^\infty \left(\frac{1-\cos u}{u^2}\right) \cos\lambda u . du = \begin{cases} \frac{\pi}{2}(1-\lambda), & 0<\lambda<1 \\ 0, & \lambda>1 \end{cases}$

put $\lambda = 0$.

$\because \sin^2 x = \frac{1-\cos 2x}{2}$

$$\int_0^\infty \left(\frac{1-\cos u}{u^2}\right) du = \frac{\pi}{2}.$$

$$\int_0^\infty \frac{2\sin^2 u/2}{u^2}\, du = \frac{\pi}{2}$$

put $\frac{u}{2} = z$.

$du = 2dz$

$u^2 = 4\cancel{dz}z^2$.

$$\int_0^\infty \frac{2\sin^2 z}{4z^2} \cdot 2dz = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin^2 z}{z^2}\, dz = \frac{\pi}{2}$$

Ans : B

58 For the Fourier sine integral representation $\frac{2}{\pi}\int_0^\infty \left(\frac{1-\cos\lambda}{\lambda}\right) \sin\lambda x \, d\lambda = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$, the value of integral $\int_0^\infty \frac{\sin^3 t}{t}\, dt$ is.

A]. $\frac{\pi}{2}$ B] 1 C] 0 D] $\frac{\pi}{4}$

Explaination :

since $\frac{2}{\pi}\int_0^\infty \left(\frac{1-\cos\lambda}{\lambda}\right) \sin\lambda x \, d\lambda = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

put $x = 1/2$.

$$\frac{2}{\pi}\int_0^\infty \frac{2\sin^2 \frac{\lambda}{2}}{\lambda} \sin\frac{\lambda}{2} \cdot d\lambda = 1$$

$$\int \sin^3 \frac{\lambda}{2}\, d\lambda = \frac{\pi}{4}$$