NEWTON RAPHSON METHOD

This method is one of the most powerful method and well known method, used for finding a root of f(x)=o,the formula many be derived in many ways the simplest way to derive this formula is by using the first two termsin Taylor series expansion of the form,

$$f(x_{n+1}) = f(x_n)(x_{n+1} \text{-} x_n)\, f'(x_n)$$

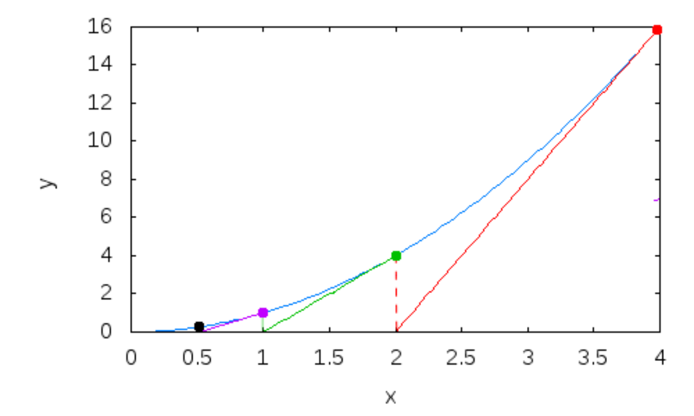
Setting $f(x_{n+1})$ =o gives

Thus on simplification we get

$$x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$$

Geometrical Interpretation

- Let the curve f(x)=o meet the x-axis at α,it means α is one of the original root of the f(x)=x =o.Let $x_0$be the point near the root α of the equation f(x)=o then the equation of tangent $P[x_0, f(x_0)]\ is$
- $y - f(x_0) = f'(x_0)(\mathrm{x} - x_0)$
- This cut the x axis at x_1= $x_0 - \dfrac{f(x_0)}{f\prime(x_0)}$



Reference from [CC BY-SA]

- This is the first approximation of the root α.If $P[x_1, f(x_1)]$ is the corresponding point to x_1 on the curve then the tangent at $P_1 is$
- $y - f(x_1) = f'(x_1)(\mathrm{x} - x_1)$
- This cut the x axis at $x_2 = x_1 - \frac{f(x_1)}{f\prime(x_1)}$
- This is the second approximation to the root α.Repeating this process we will get the root α with better approximations quite rapidly.

Remarkable Notes

1. When f′(x) is very large .i.e. Slop is very large then h will be small and hence the root can be calculated in even less time.
2. If we choose the initial approximation $x_0$close to the root then we get the root of the equation very quickly.
3. The process will evidently fail if f′(x) $= 0$ is in the neighbourhood of the root. In such cases the Regula falsie method should be used.
4. If the initial approximation to the root is not given, choose two say a and b such that f(a) and f(b) are of opposite signs if |f(a)|<|f(b)| then take a as the initial approximation.

Descriptive Examples

1.Find the real root of the equation $x^3 + 2x - 5 = 0$ by applying Newton-Raphson method at the end of fifth iteration.

Answer: **Step:1**

Given $f(x) = x^3 + 2x - 5$

$$f'(x) = 3x^2 + 2$$

$$f''(x) = 6x$$

Since $f(0) = -5, f(1) = -2, f(2) = 7$

A root lies between 1 and 2.Since $f''(x) = 6x$ which is positive for interval (1,2) hence we choose $x_0 = 2$ as $f(x_0)\, f''(x_0) > 0$

Step:2

BY Newton-Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{7}{14} = 1.5$$

Then $\mathrm{f}(x_1) = 1.375, \mathrm{f}'(x_1) = 8.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.5 - \frac{1.375}{8.75} = 1.343$$

Then $\mathrm{f}(x_2) = 0.1083, \mathrm{f}'(x_2) = 7.411$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 1.343 - \frac{0.1083}{7.411} = 1.329$$

Then $\mathrm{f}(x_3) = 0.1083, \mathrm{f}'(x_3) = 7.2987$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 1.329 - \frac{0.0053}{7.2987} = 1.3283$$

Then $\mathrm{f}(x_4) = 0.00227, \mathrm{f}'(x_4) = 7.29314$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$x_5 = 1.3283 - \frac{0.00227}{7.29314} = 1.3283$$

Hence required root is 1.3283

2.Find by Newton-Raphson method $3x - cosx - 1 = 0$ correct to four decimal places.

Given $f(x) = 3x - cosx - 1$

$$f'(x) = 3 + sinx$$

Since $f(0) = -2, f(1) = 1.4597,$
A root lies between 0 and 1.Since $f''(x) = cosx$ which is positive for interval (0,1) hence we choose
$x_0 = 0.6(near\ to\ 1)$ as $f(x_0)\, f''(x_0) > 0$
Step:2

BY Newton-Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.6 - \frac{3(0.6) - \cos(0.6) - 1}{3 + \sin(0.6)} = 0.6071$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.6071 - \frac{3(0.6071) - \cos(0.6071) - 1}{3 + \sin(0.6071)} = 0.607$$

2.Find by Newton-Raphson method $sinx - xcosx = 0$.Assume initial guess value for $x = \frac{3\pi}{2}$,correct up to five decimal places

Step:1

Given $f(x) = sinx - xcosx$

$f'(x) = cosx - cosx + xsinx = xsinx$

Given initial $x_0 = \frac{3\pi}{2}$

Step:2

BY Newton-Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = \frac{3\pi}{2} - \frac{\sin\left(\frac{3\pi}{2}\right) - \left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}\sin\left(\frac{3\pi}{2}\right)} = 4.50018$$

$$x_2 = x_1 - \frac{f(x_1)}{f\prime(x_1)}$$

$$x_2 = 4.5001824 - \frac{\sin(4.50018) - (4.50018)\cos(4.50018)}{4.50018\sin(4.50018)} = 4.4934195$$

$$x_3 = x_2 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = 4.4934195 - \frac{\sin(4.4934195) - (4.5001824)\cos(4.4934195)}{4.4934195\sin(4.493195)}$$
$$= 4.4934195$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 4.4934095 - \frac{\sin(4.4934095) - (4.4934095)\cos(4.4934095)}{(4.4934095)\sin(4.4934095)}$$

$$= 4.4934095$$

Convergence of Newton Raphson Scheme.

$$Newton - Raphon\ formula\ is\ of\ the\ form$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$Consider = g(x) = x - \frac{f(x)}{f'(x)}$$

Differentiating= $g'(x) = \frac{f(x)f''(x)}{[f'(x)]^2}$

and for convergence we *require that* , $\left|\frac{f(x)f''(x)}{[f'(x)]^2}\right| < 1$