

$$\text{Let } \theta = 2\pi \cdot \left(\frac{ux}{M} + \frac{vy}{N} \right)$$

$$e^{-j\theta} = \overline{e^{j\theta}}$$

$$\text{DFT: } F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \cdot \exp(-j2\pi(\frac{ux}{M} + \frac{vy}{N}))$$

$$\text{IDFT: } f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp(j2\pi(\frac{ux}{M} + \frac{vy}{N}))$$

With conjugate of $F(u, v)$, we turn j into negative and get:

$$\begin{aligned} \overline{F(u, v)} &= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \cdot \exp(j2\pi(\frac{ux}{M} + \frac{vy}{N})) \\ &= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \cdot \exp(-j2\pi(\frac{(2M-u)x}{M} + \frac{(2N-v)y}{N})) \\ &= F(2M-u, 2N-v) \end{aligned}$$

Using IDFT:

$$\begin{array}{cccccc} 0 & 1 & 2 & \dots & M-1 \\ 0 & -1 & -2 & \dots & -M+1 & +2M. \end{array}$$

$$f'(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \overline{F(u, v)} \exp(j2\pi(\frac{ux}{M} + \frac{vy}{N}))$$

$$= \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(2M-u, 2N-v) \exp(j2\pi(\frac{ux}{M} + \frac{vy}{N}))$$

$$u = 2M - u' \quad u' = 0$$

$$= \frac{1}{MN} \sum_{u'=0}^{M+1} \sum_{v'=0}^{N+1} F(u', v') \exp(j2\pi(\frac{(2M-u')x}{M} + \frac{(2N-v')y}{N}))$$

$$u' = -u + 2M$$

$$= \frac{1}{MN} \sum_{u'=0}^{M+1} \sum_{v'=0}^{N+1} F(u', v') \exp(j2\pi(\frac{-u'x}{M} + \frac{-v'y}{N}))$$

$$= \frac{1}{MN} \sum_{u'=0}^{M+1} \sum_{v'=0}^{N+1} F(u', v') \exp(-j2\pi(\frac{u'x}{M} + \frac{v'y}{N}))$$

$$= \frac{1}{MN} \sum_{u'=0}^{M+1} \sum_{v'=0}^{N+1} F(u', v') \exp(j2\pi(\frac{-u'x}{M} + \frac{-v'y}{N}))$$

$$= \frac{1}{MN} \sum_{u'=0}^{M-1} \sum_{v'=0}^{N-1} F(u', v') \exp(j2\pi(\frac{u'(M-x)}{M} + \frac{v'(N-y)}{N}))$$

$$= f(M-x, N-y)$$

Therefore, applying IDFT to the conjugate of DFT of an

image flips the image along both x and y axes.