Linear Programming

Optimization in Systems and Control

SC42055

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LIST OF SYMBOLS

Descriptions of all parameters used are succinctly summarized in the table below.

Table 1: Parameters used to describe the linear optimization problem

Parameter description	Parameter	Value	Unit
Objective function	$f(x)$	[-]	€
Optimal objective function	$f^\star(x^\star)$	[-]	€
Model R cars sold	x_1	[-]	models
Manufacturing limitation model R	$x_{1,max}$	1000	model
Manufacturing limitation model R	$x_{1,min}$	1000	model
Model W cars sold	x_2	[-]	models
Manufacturing limitation model R	$x_{2,min}$	1250	model
Extra employees	x_3	[-]	employee
Employment limitation	$x_{3,max}$	72	employee
Model V cars sold	x_4	[-]	models
Manufacturing limitation model V	$x_{4,min}$	1500	model
Optimal argument	$x^\star$	[-]	models
Individual profit model R	P_1	25.000	$\frac{€}{\text{model}}$
Individual profit model W	P_2	30.000	$\frac{€}{\text{model}}$
Overall profit	P_{tot}	[-]	$\frac{€}{\text{model}}$
Selling price model R	s_1	55.000	$\frac{€}{\text{model}}$
Selling price model W	s_2	75.000	$\frac{€}{\text{model}}$
Manufacturing costs model R (labor excluded)	$c_{m,1}$	30.000	$\frac{€}{\text{model}}$
Manufacturing costs model W (labor excluded)	$c_{m,2}$	45.000	$\frac{€}{\text{model}}$
Salary costs (all employees together)	C_h	372.000	€
Salary costs with variable amount of employees	$C_{h,new}$	[-]	€
Amount of available employees	N_w	108	employee
Avarage employee salary	S_w	3450	$\frac{€}{\text{employee}}$
Battery cells needed per model R produced	bc_1	4000	$\frac{\text{battery cells}}{\text{model}}$
Battery cells needed per model W produced	bc_2	6000	$\frac{\text{battery cells}}{\text{model}}$
Battery cells needed per model V produced	bc_4	2000	$\frac{\text{battery cells}}{\text{model}}$
Maximum amount of battery cells produced	bc_{max}	9 mln	$\frac{\text{battery cells}}{model}$
Maximum amount of battery cells produced new contract	$bc_{max,new}$	12 mln	$\frac{\text{battery cells}}{model}$
Manufacturing time model R	ht_1	10	$\frac{\text{hours}}{\text{model}}$
Manufacturing time model W	ht_2	15	$\frac{\text{hours}}{\text{model}}$
Manufacturing time model V	ht_4	8	$\frac{\text{hours}}{\text{model}}$
Maximum available manufacturing time	ht_{max}	17280	hours
Maximum available manufacturing time new contract	$ht_{max,new}$	[-]	hours
Needed storage space model R	ar_1	10	$\frac{m^2}{\text{model}}$
Needed storage space model W	ar_2	12	$\frac{m^2}{\text{model}}$
Needed storage space model V	ar_4	12	$\frac{m^2}{\text{model}}$
Maximum storage space available	ar_{max}	24000	m^2

1
Linear Programming

1.1. The standard form

Before we explicitly explain the minimalization problem, we introduce the standard optimization problem we aim to solve:

$$\boxed{\begin{aligned} \underset{x_1,x_2}{\text{minimize}} \quad & -(P_1x_1 + P_2x_2 - C_h) \\ \text{subject to} \quad & x_1, x_2 \geq 0 \\ & Ax \leq b \end{aligned}} \tag{1.1}$$

More explicitly we can expand the constraints to

$$\underbrace{\begin{bmatrix} bc_1 & bc_2 \\ ht_1 & ht_2 \\ ar_1 & ar_2 \end{bmatrix}}_{\text{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{x}} \leq \underbrace{\begin{bmatrix} bc_{max} \\ ht_{max} \\ ar_{max} \end{bmatrix}}_{\text{b}} \tag{1.2}$$

Now that we have formulated the standard minimization problem we can explain where it is originated from. For conciseness reasons, all subscript 1 belong to the model R and 2 to model W (and 4 for the model V). Furthermore, descriptions for all the parameters defined can be found back in chapter .

1.1.1. Maximizing profit

First the variables where we minimize over (x) are the amount of models sold. In order to maximize the overall profit, P_{tot}, we aim to minimize the overall profit times -1 (objective function, f_x). The overall profit and the objection function are defined as:

$$P_{tot} = P_1x_1 + P_2x_2 - C_h \tag{1.3a}$$

$$f_x = -P_{tot} = -(P_1x_1 + P_2x_2 - C_h) \tag{1.3b}$$

where P represents the individual model profit defined as,

$$P = s - c_m \tag{1.4}$$

where s is the selling price and c_m its manufacturing costs (without considering the employment salaries). C_h represents the labor costs and can be calculated as follows,

$$C_h = N_w S_w \tag{1.5}$$

where N_w is the amount of workers and S_w is the salary per worker. Both are constant within the first subexercises.

1.1.2. Constraints

The constraints can be divided into three 1-dimensional constraints as described in equation 2.3. The **first constraint** gives rise to the maximum amount of ***battery cells*** to be developed. bc_i represents the amount of battery cells needed per model. bc_{max} is the maximum amount of battery cells that can be produced per month.The **second constraint** gives rise to the maximum amount **manufacturing time**. Here, ht_i represents the amount of hours needed to manufacture the model and ht_{max} the total hours available for labor, ht_{max} is calculated by,

$$ht_{max} = N_w H_w \tag{1.6}$$

where H_w represents the amount of hours available per employee. The **third constraint** has to do with the maximum ***storage space*** available. Here, ar_i represents the total storage space needed to store the model for the remaining of the month. ar_{max} is the maximum storage space available to the company. All constraints are logically summarized in the A and b matrices.

1.2. OPTIMAL MANUFACTURE NUMBERS

Using the function *linprog* [1] in MATLAB it is found that the optimal number of cars to be manufactured is 1728 model R cars and 0 model W cars:

$$x^{\star(1)} = \begin{bmatrix} 1728 \\ 0 \end{bmatrix} \tag{1.7}$$

1.3. A LIMITING CONSTRAINT

Inequality constraints are in most cases limiting a linear optimization problem. Furthermore, one often sees that some linear inequality constraints restrict the optimization much more than others do. In many cases, some inequality constraint are on the verge of what is allowed. For example, if a scalar inequality constraints reads as $a \leq 2$ and the actual value of a turns out to be 2 in the optimal case, we can state that this inequality constraint is (one of the) limiting constraints. Calculating the left hand side (depending on the optimal argument(s)) of the inequality constraints and compare them with the value of the constraint itself (fixed value) can give us an insight what constraints are (most) limiting the optimization problem. We did that for all three constraints 2.3:

$$\underbrace{Ax^{\star(1)} = \begin{bmatrix} bc_1 & bc_2 \\ ht_1 & ht_2 \\ ar_1 & ar_2 \end{bmatrix} \begin{bmatrix} 1728 \\ 0 \end{bmatrix} = \begin{bmatrix} 6.912.000 \\ 17.280 \\ 17.280 \end{bmatrix}}_{\text{depending on the optimal arguments } x^{\star(1)}} \leq \underbrace{\begin{bmatrix} bc_{max} \\ ht_{max} \\ ar_{max} \end{bmatrix} = \begin{bmatrix} 9.000.000 \\ 17.280 \\ 24.000 \end{bmatrix}}_{\text{actual value of the constraint}} \tag{1.8}$$

Here we can clearly see that the limiting factor in our case is the amount of available manufacturing hours since the hours needed to manufacture the cars is exactly equal to the maximum amount of available manufacturing hours. The other constraints (battery cell production and storage space) turn out to live relatively far from their maximum values and are therefor less limiting.

1.4. THE OPTIMAL BENEFIT

The optimal benefit, according to the algorithm, in this situation is €43mln and is easily calculated by the objective function evaluated at $x^{\star(1)}$:

$$f^{\star(1)} = f\left(x^{\star(1)}\right) = -\left(\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} x^{\star(1)} - C_h\right) = -4.36 \cdot 10^7 \tag{1.9}$$

By taking the opposite sign one retrieves the total profit mentioned above.

[1] https://nl.mathworks.com/help/optim/ug/linprog.html

2
Production Limitation: Model R

2.1. Introduction of the new limitation

By limiting the maximum amount of model R's produced to 1000 per month ($x_{1,max}$), we have to slightly adjust the optimization problem found in 1. By adding the new constraint to 1.1 we find the new optimization problem:

$$\boxed{\begin{aligned} \underset{x_1,x_2}{\text{minimize}} \quad & -(P_1x_1 + P_2x_2 - C_h) \\ \text{subject to} \quad & x_1, x_2 \geq 0 \\ & A_R x \leq b_R \end{aligned}}$$
$$\underbrace{\begin{bmatrix} bc_1 & bc_2 \\ ht_1 & ht_2 \\ ar_1 & ar_2 \\ 1 & 0 \end{bmatrix}}_{A_R} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{x}} \leq \underbrace{\begin{bmatrix} bc_{max} \\ ht_{max} \\ ar_{max} \\ x_{1,max} \end{bmatrix}}_{b_R} \tag{2.1}$$

Since we added a linear (inequality) constraint the optimization problem remains linear and is therefor solvable using linear programming tools.

2.2. The optimal amount of models to be manufactured

The optimal number of models manufactured in this case is found to be 1000 model R cars and 485 model W cars:

$$x^{\star(2)} = \begin{bmatrix} 1000 \\ 485 \end{bmatrix} \tag{2.2}$$

One immediately observes that the optimal amount of model R's to be manufactured is equal to its limit. Furthermore, evaluating the constraints, by following the same recipe as in section 1.3, makes clear that a new limiting factor is the newly introduced constraint:

$$\underbrace{A_R x^{\star(2)} = \begin{bmatrix} bc_1 & bc_2 \\ ht_1 & ht_2 \\ ar_1 & ar_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1000 \\ 485 \end{bmatrix} = \begin{bmatrix} 6.912.000 \\ 17.280 \\ 15.824 \\ 1000 \end{bmatrix}}_{\text{depeding on the optimal arguments } x^{\star(2)}} \leq \underbrace{\begin{bmatrix} bc_{max} \\ ht_{max} \\ ar_{max} \end{bmatrix} = \begin{bmatrix} 9.000.000 \\ 17.280 \\ 24.000 \\ 1.000 \end{bmatrix}}_{\text{actual value of the constraint}} \tag{2.3}$$

2.3. The optimal benefit

The optimal benefit according to the algorithm is €39mln in this case and is easily calculated by the objective function evaluated at $x^{\star(2)}$:

$$f^{\star(2)} = f\left(x^{\star(2)}\right) = -\left(\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} x^{\star(2)} - C_h\right) = -3.9 \cdot 10^7 \tag{2.4}$$

By taking the opposite sign one retrieves the total profit mentioned above. Clearly, the limit on the number of model R cars sold results in a lower overall profit. This can be easily clarified since adding a new constraint will never increase the performance. Thereby, since we introduced a constraint that turned out to be limiting, this will in most cases result in a higher objective value (read: lower overall profit).

3
NEW VARIABLE: EMPLOYEES

Formulate the new optimization problem in order to maximize the profit. Is it possible to formulate it as a single LP problem?

3.1. FORMULATING THE NEW OPTIMIZATION PROBLEM

We now introduce a new variable x_3, representing the number of extra employees. This has a huge impact on the optimization problem since adding this variable will lead to including non-linear constraints. The new optimization is defined as

$$\boxed{\begin{array}{ll} \underset{x_1,x_2,x_3}{\text{minimize}} & -\left(P_1x_1+P_2x_2-C_{h,new}(x_3)\right) \\ \text{subject to} & x_1,x_2,x_3 \geq 0 \\ & Cons_w \leq b_w \end{array}}$$
$$\underbrace{\begin{bmatrix} bc_1x_1+bc_2x_2 \\ \left(ht_1-\frac{x_3}{12}\right)x_1+\left(ht_2-\frac{x_3}{12}\right)x_2 \\ ar_1x_1+ar_2x_2 \\ x_1 \\ x_3 \end{bmatrix}}_{Cons_w} \leq \underbrace{\begin{bmatrix} bc_{max,new} \\ (N_w+x_3)\,h_w \\ ar_{max,new} \\ x_{1,max} \\ x_{3,max} \end{bmatrix}}_{b_w} \tag{3.1}$$

Clearly, the optimization problem is not linear anymore, since the third constraint is non-linear (x_1x_3 and x_2x_3 terms). How we manipulated the optimization problem in the form defined above can be found back in the following subsections.

3.1.1. NEW OBJECTIVE FUNCTION

Since the amount of employees is variable, the manufacturing costs due to labor is variable as well. This can be easily calculated:

$$C_{h,new} = (N_w + x_3)\,S_w \tag{3.2}$$

3.1.2. TIME CONSTRAINT

Since the amount of employees determines the manufacturing time per model we have to change these constraints. Clearly ht_1 and ht_2 become dependent on the amount of extra employees x_3. For every extra employee the manufacturing time decreases by 5 minutes ($\frac{1}{12}h$): $h_t \rightarrow h_t - \frac{x_3}{12}$. Thereby, the total amount of available manufacturing hours increases thus also the actual value of the constraint increases for an increasing amount of employees. Therefore the constraint is adjusted to

$$\left(ht_1-\frac{x_3}{12}\right)x_1+\left(ht_2-\frac{x_3}{12}\right)x_2 \leq (N_w+x_3)\,h_w \tag{3.3}$$

3.1.3. BATTERY CELL AND STORAGE SPACE IMPROVEMENTS

More battery cells can be produced and more space is available and are now limited to $bc_{max,new}$ and $ar_{max,new}$ respectively.

3.1.4. EMPLOYMENT CONSTRAINT

The maximum amount of extra employees is limited to $x_{3,max}$. All constraints are logically summarized in $Cons_w$ and b_w.

3.2. IMPLEMENTING THE THE NEW VARIABLE

The new variable changes the linear optimization problem to a non-linear optimization problem. By applying a trick we can manage to break the non-linear optimization problem in multiple linear optimization problem: we let the amount of extra employees vary from 0 to 72 (in integer steps logically) and find the optimal amount of extra employees resulting in the largest overall profit by solving the following linear optimization problems

$$\boxed{\begin{array}{ll} \underset{x_1,x_2}{\text{minimize}} & -\left(P_1x_1 + P_2x_2 - (N_w + x_3)\,S_w\right) \\ \text{subject to} & x_1, x_2 \geq 0 \\ & A_{wf}x \leq b_{wf} \end{array}}$$

$$\underbrace{\begin{bmatrix} bc_1 & bc_2 \\ ht_1 - \frac{x_3}{12} & ht_2 - \frac{x_3}{12} \\ ar_1 & ar_2 \\ 1 & 0 \end{bmatrix}}_{A_{wf}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \underset{A_{wf}}{\leq} \underbrace{\begin{bmatrix} bc_{max,new} \\ (N_w + x_3)\,h_w \\ ar_{max,new} \\ x_{1,max} \end{bmatrix}}_{b_{wf}} \tag{3.4}$$

for various x_3. This is implemented in matlab using a well known for loop.

3.3. THE OPTIMAL AMOUNT OF MODELS TO BE MANUFACTURED AND EXTRA EMPLOYEES

The optimal number of models manufactured by solving the multipe linear optimization problems in this case is found to be 1000 model R cars and 1333 model W cars:

$$x^{\star(3)} = \begin{bmatrix} 1000 \\ 1333 \end{bmatrix} \tag{3.5}$$

One immediately observes that the optimal amount of model R's to be manufactured is again equal to its limit. Furthermore, evaluating the constraints, following the same recipe as in section 1.3, makes clear that a limiting factor is again the constraint on the number of model R cars, which was introduced in the previous chapter. In this case we have a second limiting factor, namely the number of available battery cells.

$$\underbrace{A_R x^{\star(3)} = \begin{bmatrix} bc_1 & bc_2 \\ ht_1 & ht_2 \\ ar_1 & ar_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1000 \\ 1333 \end{bmatrix} = \begin{bmatrix} 12.000.000 \\ 16.000 \\ 26.000 \\ 1000 \end{bmatrix}}_{\text{depeding on the optimal arguments } x^{\star(3)}} \leq \underbrace{\begin{bmatrix} bc_{max,new} \\ ht_{max} \\ ar_{max,new} \\ x_{3,max} \end{bmatrix} = \begin{bmatrix} 12.000.000 \\ 28.800 \\ 31.000 \\ 1.000 \end{bmatrix}}_{\text{actual value of the constraint}} \tag{3.6}$$

The optimal number of workers associated with this benefit is 144.

3.4. THE OPTIMAL BENEFIT

The optimal benefit according to the algorithm is €$6,45 \cdot 10^7$ in this case. The optimal benefit is higher than it has been before.

The optimal benefit according to the algorithm is €64,5mln in this case and is easily calculated by the objective function evaluated at $x^{\star(3)}$:

$$f^{\star(3)} = f\left(x^{\star(3)}\right) = -\left(\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} x^{\star(3)} - C_h\right) = -6,45 \cdot 10^7 \tag{3.7}$$

The above result implies that there will be a strong growth in profit, when comparing this to the €39mln profit in the previous chapter.

4
New Variable: model V

4.1. Is it beneficial to build the model V?

Introducing a possible new model V will slightly adjust the optimization problem as follows:

$$\boxed{\begin{array}{ll} \underset{x_1,x_2,x_4}{\text{minimize}} & -(P_1 x_1 + P_2 x_2 + P_4 x_4 - (N_w + x_3)\, S_w) \\ \text{subject to} & x_1, x_2, x_4 \geq 0 \\ & A_V x \leq b_V \end{array}}$$
$$\underbrace{\begin{bmatrix} bc_1 & bc_2 & bc_4 \\ ht_1 - \frac{x_3}{12} & ht_2 - \frac{x_3}{12} & ht_4 \\ ar_1 & ar_2 & ar_4 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{A_V} \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} \leq \underbrace{\begin{bmatrix} bc_{max,new} \\ (N_w + x_3)\, h_w \\ ar_{max,new} \\ -x_{1,min} \\ -x_{2,min} \\ -x_{4,min} \end{bmatrix}}_{b_{wf}} \tag{4.1}$$

Optimization of this linear optimization problem results in the following optimal arguments:

$$x^{\star(4)} = \begin{bmatrix} 1500 \\ 1000 \\ 0 \end{bmatrix} \tag{4.2}$$

This clearly shows that the maximal profit is reached when the production of the model V is kept to zero. This result is achieved while having a lower bound on the model R and W. These lower bounds are based on the contracts.

4.2. Can the contracts be satisfied?

The contracts can be satisfied, while even achieving a profit of €67mln. This is the highest profit of all the cases up to this point. Therefore it seems to be a good move for Edison automotive to not produce the model V at this point and try to sell 1500 model R cars and 1000 model W cars.

5
MATLAB CODE

5.1. CHOOSING THE LINPROG-ALGORITHM

Based on the MATLAB guide to choosing an algorithm[1] for the linprog function and the more extensive MATLAB description page for Linear Programming Algorithms [2] the choice is made to use the dual-simplex algorithm within the linprog function.

The dual-simplex algorithm is an algorithm for small to medium sized problems and is regularly faster than the interior-point and the interior-point-legacy algorithms, which are the two alternatives when using linprog.

5.2. THE ACTUAL MATLAB CODE

```
%% Linear programming assignment
% Course: SC42055 Optimization in Systems and Control
% Jacob Lont, 4424409 and Casper van Engelenburg, 4237080

%% Always first pull from Github before making any changes

% E1, E2 and E3 are parameters changing from 0 to 18 for each group
    according to the sum of
% the last three numbers of the student IDs:
% E1 = Da1 + Db1, E2 = Da2 + Db2, E3 = Da3 + Db3,
% where Da;3 is the right-most digit of one student and Db;3 is the
    right-most digit of the other
% student.

EE1 = 4+0; EE2 = 0+8; EE3 = 9+0;

num_employees = 100 + EE2;
hours_per_month = 160;
month_salary = 3000 + 50*EE3;
salary_costs = num_employees*month_salary; % Company costs on employees

storage_space = (15+EE3)*10^3; % m^2 storage stock space available
space_R = 10; % m^2 taken by a model R car
space_W = 12; % m^2 taken by a model W car

% The model R and model W are respectively being sold for 55000e and
    75000e.
price_R = 55*10^3; % Euro
price_W = 75*10^3; % Euro
```

[1] https://nl.mathworks.com/help/optim/ug/choosing-the-algorithm.html
[2] https://nl.mathworks.com/help/optim/ug/linear-programming-algorithms.html

```
% the cost of manufacturing the cars without considering the worker
    salaries is 30000e for the
% model R and 45000e for the model W.
costs_R = 30*10^3; % Euro without worker salaries
costs_W = 45*10^3; % Euro without worker salaries

%% 2 Formulating LP problem using linprog.m

% CONCISE FORMULATION:
% -----------------------------------------
%  min  -fx(x1,x2)
% x1,x2
%
% st    x1,x2 integer (including 0) (or: x1,x2 in R+)
%       Ax <= b : inequality constraint
% -----------------------------------------
%
% EQUATIONS EXPLICITLY:
% fx = P1*x1 + P1*x2 - Cm : objective value
% A = [c1 c2; H1 H2; A1 A2]
% x = [x1 x2]'
% b = [ctot Htot Atot]'
%
% PARAMETERS
% x1(x2): amount of model R(W) sold per month
% P1(P2): price of model R(W) (excluding manufacturing costs)
% Cm: manufacturing costs (solid number in this question)
% c1(c2): battery cells needed to manufacture 1 model R (W)
% H1(H2): hours needed to manufacture 1 model R (W)
% A1(A2): storage needed to store 1 model R (W)
% ctot: maximum amount of battery cell production p/month
% Htot: maximum amount of available hours p/month
% Atot: maximum amount of available storage space

%parameter set

%price
P1 = price_R - costs_R;
P2 = price_W - costs_W;
P = -[P1 P2]';
Cm = salary_costs;

%battery cells
c1 = 4E3;
c2 = 6E3;
ctot = (5+EE1)*1E6;

%manufacturing hours
H1 = 10;
H2 = 15;
Htot = hours_per_month*num_employees;

%storage
A1 = space_R;
A2 = space_W;
Atot = storage_space;
```

```
%objective, inequality constraints into concise matrix notation
A = [c1 c2; H1 H2; A1 A2];
b = [ctot Htot Atot]';

%bounds on x and initial condition and equality constraints
lb = [0 0]';
ub = []';
x0 = [0 0]';
Ae = [];
be = [];

%linprog
options = optimoptions('linprog','Algorithm','dual-simplex');
[x, fval, flag] = linprog(P, A, b, Ae, be, lb, ub, x0, options);

profit2 = -fval - Cm  % Minus because it was a maximization problem
    initially


% RESULTS:

%  Optimal benefit:
% profit2 =
%    4.3573e+07
%  Number of cars:
%  x =
%         1728
%            0
% Limiting constraint: Htot = 17280, which is equal to 1728*10 hours



%% 3 A change in the market
ub = [1000 inf]';
[x, fval, flag] = linprog(P, A, b, Ae, be, lb, ub, x0, options);
profit3 = -fval - Cm  % Minus because it was a maximization problem
    initially

% RESULTS:
%  Optimal benefit:
% profit3 =
%     39187400
%  Number of cars:
%  x =
%     1000
%     485.3333 = 485



%% 5 further increase the profits of the company.
% for each additional worker that the company hires, the time required
    to
% manufacture each car is reduced by 5 minutes.
% this time reduction is limited to 6 hours, i.e. the maximum number of
% extra workers is limited to 72
```

```
ctot = (8+EE1)*1E6;
Atot = (22+EE3)*10^3;
ub = [1000 inf]';

for extra_employees = 0:72

    num_employees_total = num_employees + extra_employees;
    salary_costs = num_employees_total*month_salary; % Company costs on
        employees
    Cm = salary_costs;

    %manufacturing hours
    H1 = 10-(1/12)*extra_employees;
    H2 = 15-(1/12)*extra_employees;
    Htot = hours_per_month*num_employees_total;

    %objective, inequality constraints into concise matrix notation
    A = [c1 c2; H1 H2; A1 A2];
    b = [ctot Htot Atot]';

    [x, fval, flag] = linprog(P, A, b, Ae, be, lb, ub, x0, options);

    % Store the data for analysis
    results(1,extra_employees+1) = num_employees_total;
    results(2,extra_employees+1) = -fval - Cm;
    results(3,extra_employees+1) = x(1);
    results(4,extra_employees+1) = x(2);

end

% Find the minimum of all these cases
[profit5, max_index] = max(results(2,:))  % Minus because it was a
   maximization problem initially
opt_num_workers = results(1,max_index);   % Optimal number of workers
x_max = [results(3,max_index),results(4,max_index)]; % Number of cars
   at the optimum

% RESULTS:
%  Optimal number of workers:
%       opt_num_workers = 144
%   Optimal benefit:
%       profit5 = 64503200
%  Number of cars:
%       x =
%          1000 model R
%          1333 model W



%% 6 Succes and a new model car!
% Because of the success, the 1000 limit on the model R is no longer in
    place
% and they have signed contracts to produce at least 1250 model R and
% 1000 model W per month.

c3 = 2E3;   % Battery cells for model V
```

```
A3 = 8;      % m^2 space taken by model V

extra_employees = opt_num_workers-num_employees;
H1 = 10-(1/12)*extra_employees;
H2 = 15-(1/12)*extra_employees;
H3 = 8;      % 8 hours independently of the number of workers

price_V = 45E3; % Euro
costs_V = 45E3; % Euro

P1 = price_R - costs_R;
P2 = price_W - costs_W;
P3 = price_V - costs_V;
P = -[P1 P2 P3]';

Htot = hours_per_month*opt_num_workers;
ctot = (8+EE1)*1E6;
Atot = (22+EE3)*10^3;

%objective, inequality constraints into concise matrix notation
A = [c1 c2 c3; H1 H2 H3; A1 A2 A3];
b = [ctot Htot Atot]';

%bounds on x and initial condition and equality constraints
lb = [1250 1000 0]';     % implementing the contract constraints
ub = []';
x0 = [0 0]';
Ae = [];
be = [];

%linprog
options = optimoptions('linprog','Algorithm','dual-simplex');
[x, fval, flag] = linprog(P, A, b, Ae, be, lb, ub, x0, options);

profit6 = -fval - Cm  % Minus because it was a maximization problem
    initially

% RESULTS:
%   Optimal benefit:
%       profit5 = 66879000
%  Number of cars:
%       x =
%          1500 model R
%          1000 model W
%          0    model V


% is it economically beneficial to build the new model V?
%  No, it is not. It is better to stick to the model R and W

% If not, can Edison Automotive at least satisfy the new two contracts
    on
% the model W and model R?
% Yes, Edison Automotive can satisfy the new two contracs. See the
    results.
```