VIX Forecasting*

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Abstract

The celebrated Black-Scholes model for valuing options uses a number of inputs– current stock price, risk-free interest rate, exercise price, time to maturity and volatility of returns. One critical input is the volatility of returns. Historical volatility is of little use as what is relevant is future volatility. Assuming efficient markets, a good source of volatility estimate is the implied volatility. Among the inputs to the Black-Scholes model all except volatility are known in advance. The output– the current call price is also known. Implied volatility is arrived at using the current call prices and all other inputs in the Black-Scholes formula to ascertain the volatility. This is a forward looking volatility estimate. We forecast volatility using VIX data obtained from CBOE. This paper adds value to extant literature by forecasting the revised VIX using a variety of forecasting tools like GARCH, EGARCH, APARCH, GJR and IGARCH. The EGARCH model is selected as it performs well on forecast accuracy. Using combinations of options, it is possible to trade volatility as if it were any other commodity, so that accurate predictions of future volatility give the forecaster the potential to make a more direct profit.

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I. Introduction

Volatility forecasting is an important task in financial markets. It has occupied the attention of academicians and practitioners over the past two decades. Its importance is visible in an array of areas like investment, security valuation, risk management and monetary policy making.

There are a number of motivations for this study. First, volatility as measured by the standard deviation or variance of returns, is often used as a crude measure of the total risk of financial assets. Second, the volatility of stock market prices enters directly into the Black-Scholes formula for deriving the prices of traded options. Although in the past, historical measures of volatility have been used as estimates of future volatility, there is a growing body of evidence that suggests the use of volatility predicted from more sophisticated time-series models will lead to more accurate options valuation (see, for example, Akigiray (1995); or Chu and Freund (1996)) Finally, using combinations of options, it is possible to trade volatility as if it were any other commodity, so that accurate predictions of future volatility give the forecaster the potential to make a more direct profit.

The literature on volatility forecasting is large and growing. Akigiray (1995), for example, finds the GARCH model superior to ARCH, exponentially weighted moving average and historical mean models for forecasting monthly US Stock index volatility. A similar result concerning the apparent superiority of GARCH is observed by West and Cho (1993) using one-step-ahead forecasts of dollar exchange rate volatility, evaluated using root-mean squared prediction errors. However, for long horizons the model behaves no better than its alternatives.Pagan and Schwert (1990) compare GARCH, EGARCH, Markov switching regime and three non-parametric models for forecasting monthly US Stock return volatilities.The EGARCH model followed by the GARCH models perform moderately; the remailing models produce very poor predictions. Frances and van Djik (1996) compare three members of the GARCH family (standard GARCH, QGARCH and the GJR model) for forecasting the weekly volatilities of various European stock market indices. They find that non-linear GARCH models were unable to beat the standard GARCH model. Brasilford and Faff (1996) find GJR and GARCH models slightly superior to various simpler models (random walk, historical mean, long term moving average etc.) for predicting Australian monthly stock index volatility. Brooks and Persand (2003) find that relative accuracies of various methods are highly sensitive to the measure used to evaluate them. Poon and Granger (2002) provide an extensive review of literature in this field. The conclusion arriving from a growing body this growing body of research is that forecasting volatility is a 'notoriously difficult task' (Brasilford and Faff 1996) although it appears that conditional heteroscedasticity models are among the best that are currently available.

Recent papers have also sought to compare predictive ability of volatility forecasts derived from market prices of traded options, with those generated using econometric models (Heynen and Kat 1994, Day and Lewis 1992). The general consensus appears to be that implied volatility forecasts are more accurate than those derived using pure time series analysis, but also that the latter still contain additional information not embedded in the implied values.

We add to the literature on volatility forecasting by attempting to forecast volatility using the revised VIX index from CBOE. Implied volatility is a preferred mode of forecasting volatility since Christensen and Hansen (2002) show that that implied volatility is an efficient forecast of realized return volatility. This study compares the performance of a number of forecasting procedures (GARCH, EGARCH, GJR, APARCH and IGARCH) in predicting the VIX volatility.

This paper is organized as follows. Section II discusses volatility and its properties. Section III delves into the importance of volatility estimation for option valuation. Section IV details the VIX construction methodology. Section V discusses the volatility models. Section VI details the forecasting model performance measures. Section VII describes the data used in the study. Section VIII describes the methodology followed. Section IX discusses the empirical results. Finally section X concludes the study along with the limitations of the study.

II. Volatility and its measures

Volatility is often calculated as the sample standard deviation (σ) which is the square root of variance, σ^2

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(R_i - \bar{R})^2 \tag{1}$$

Volatility can be seen as either (Dacoronga, Gençay, Müller, Olsen, and Pictel 2001):

- Realized volatility, also called historical volatility: determined by past observations;
- Model volatility: a virtual variable in a theoretical model such as GARCH or stochastic volatility (but there may be means to estimate this variable from data); and
- Implied volatility: a volatility forecast computed from market prices of derivatives such as options (see, for example Cox and Rubenstein (1985)) based on a model such as lognormal random walk assumed by Black and Scholes (1973).

In this paper we focus on *implied volatility.*

A number of stylized facts about volatility have emerged over the years. These include (Poon and Granger 2002, Engle and Patton 2001):

1. **Clustering**: Large changes in prices are followed by large changes, and small changes are followed by small changes.

2. **Mean reversion**: A period of high volatility will eventually give way to more normal volatility and similarly a period of low volatility will be followed by a rise.

3. **Asymmetry**: Positive and negative shocks do not have the same impact on volatility. This is often ascribed to a *leverage effect* and sometime to a *risk premium* effect. This asymmetric structure of volatility generates skewed distributions of forecast prices.

4. **Tail probabilities**: It is well established that the unconditional distribution of asset returns has heavy tails. Typical kurtosis estimates range from 4 to 50 indicating extreme non-normality.

III. Importance of volatility in option pricing

An option is the right but not an obligation to buy or sell an asset at a specified price (called strike or exercise price) on or before a specified date (called the strike or exercise date). An option to buy is called a call option while an option to sell is called a put option. If the option can be exercised only on a particular date it is referred to as a European option. However, if it can be exercised at any point in time on or before the exercise date it is termed as an American option. The Black-Scholes model for pricing European options on equity (Black and Scholes 1973) assumes the stock price has a lognormal distribution or the logarithm of stock prices has a normal distribution. Using a riskless hedge argument Black-Scholes proved that under certain assumptions, option pricing can be derived using a risk-neutral valuation relationship where all derivative assets generate only risk-free returns. Under this risk-neutral setting, investor risk preference and the required rate of return on stocks are irrelevant as far as pricing of derivatives is concerned. The Black-Scholes assumptions include constant volatility, σ, short sell with full use of proceeds, no transaction costs or taxes, divisible securities, no dividend before option maturity, no arbitrage, continuous trading and a constant risk-free rate, r.

The Black-Scholes formula for option pricing states that the option's price at time t is a function of S_t (the price of underlying security), X (the strike price), r (the risk free interest rate), T (time to option maturity) and σ(volatility of the underlying asset over time period from t to T). Given that S_t, X, r and T are observable, once the market has produced a price based on a transaction for the option, we could use a backward induction to derive the σ that market uses as an input. Such a volatility estimate is called *option implied volatility.*

Given that each asset can have only one σ, it is a well-known puzzle that options of the same time-to-maturity that differ in strikes appear to produce different implied volatility estimates for the same underlying asset. Volatility smile and smirk are names given to non-linear shapes of implied volatility plots.

IV. The VIX

In 1993, CBOE introduced the CBOE Volatility Index (VIX), constructed using the implied volatilities of eight different OEX option series so that, at any given time, it represented the implied volatility of a hypothetical at-the-money OEX option with exactly 30 days to expiration.

In September 2003, the CBOE introduced the new VIX. This series is a refined version based on the S&P 500 index option prices and incorporates information from the volatility "skew" by using a wider range of strike prices rather than just on-the-money series.

The generalized formula used in the construction of the new VIX calculation is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K}{K_i^2} \, e^{RT} \, Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \tag{2}$$

where,

σ is VIX/100; T is time to expiration; F is the forward index level derived from index option prices; K_i is the strike price of the i^{th} out-of-the-money option ΔK_i interval between strike prices – half the distance between the strike on either side of K_i; K_0 is the first strike below the forward index level, F; R is the risk-free interest rate to expiration and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i.

V. The Models

Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models have thus far been the most frequently applied class of time-varying volatility models. Since its introduction by Engle (1982) and subsequent generalization by Bollerslev (1986) this model has been extended in numerous ways which usually involves alternative formulations for the volatility process.

Volatility models are usually defined by their first two moments, the mean and the variance equation. The general notation for the mean equation of time varying volatility models is given by:

$$y_t = \mu_t + \sigma_t \varepsilon_t, \ \varepsilon_t \sim NID(0,1), \ t = 1, \ldots, T, \tag{3}$$

where y_t denotes the return series of interest and μ_t its conditional mean. The disturbance term ϵ_t is assumed to be *iid* with zero mean and unit variance. In addition the assumption of normality is added.

The functional forms of various models are summarized below (based on Laurent and Peters (2003)):

- ARCH(q): The ARCH(q) model can be expressed as:

$$\varepsilon_t = z_t \sigma_t$$
$$z_t \sim i.i.d. \ D(0,1)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \ \varepsilon_{t-1}^2$$

- GARCH(p,q): The GARCH(p,q) model can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where $D(.)$ is a probability density function with mean 0 and unit variance.

- EGARCH: The Exponential GARCH (EGARCH): model was introduced by Nelson (1991). Bollerslev and Mikkelsen (1996) re-express the model as follows:

$$\ln \sigma_t^2 = \omega + [1 - \beta(L)]^{-1} \ [1 + \alpha(L)] \ g(z_{t-1})$$

- GJR Model: This popular model is proposed by Glosten, Jagannathan, and Runkle (1993). Its generalized version is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i \ \varepsilon_{t-i}^2 + \gamma_i S_{t-i}^- \ \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j \ \sigma_{t-j}^2$$

where S_t^- os a dummy variable.

- IGARCH: In many high frequency time series applications, the conditional variance is estimated using a GARCH(p,q) process that exhibits a strong persistence, that is:

$$\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i \approx 1.$$

The conditional variance can be represented as a function of the squared residuals as:

$$\sigma_t^2 = \frac{\omega}{[1-\beta(L)]} + \{1-\phi(L)(1-L)(1-\beta(L)]^{-}1\}\varepsilon_t^2$$

- APARCH: This model has been introduced by Ding, Granger, and Engle (1993). The APARCH(p,q) model can be expressed as:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i(|\varepsilon_{t-1}| - \gamma_i\varepsilon_{t-i})^\delta + \sum_{j=1}^{p} \beta_j\sigma_{t-j}^\delta$$

VI. Measuring forecast errors

Comparing forecast performance of competing models is one of the most important aspects of any forecasting exercise. Little importance seem to have been given to comparing forecast models relative to the building of volatility models.

Popular evaluation measures used in literature include *Mean Error* (ME), *Mean Square Error* (MSE), *Root Mean Square Error* (RMSE), *Mean Absolute Error* (MAE), and *Mean Absolute Percent Error*(MAPE). Other less commonly used measures include *Mean logarithm of absolute error* (MLAE), Theil-Ustatistic and LINEX. Given that investors treat gains and losses differently, use of non-symmetric functions like LINEX is advisable but has not gained much popularity.

For the purpose of our study we use six measures, MSE, MAE, AMAPE, MedSE, R^2 and TIC. These are detailed in Appendix A.

VII. Data

The data is drawn from the CBOE website[1]. Daily data for the new VIX series is picked up for the period January 3, 1990 to October 31, 2003. Data for the period September 11,2001 to September 16, 2001 is unavailable due to the closure of the exchange and no adjustment is made for this. The data series contained three missing data points which were reconstructed by taking the

[1] `www.cboe.com/vix`

missing days' data to be the same as the previous day[2]. Thus a total of 3490 data points were available. The data was then split up into two parts:

- In-sample data: This data would be used to pick up the model(s) that have a high degree of predictability. The period from January 3, 1990 to May 11, 2000 is considered as in-sample period. This comprises 2619 data points and 75% of the total sample.
- Out-of-sample data: The models developed on in-sample data are tested for performance on the out-sample data. The period May 12, 2000 to October 31, 2003 is considered as out-of-sample period. This accounts for the balance 871 data points and 25% of the total sample.

The reason for the unequal split is that GARCH estimation procedure requires large sample sizes to produce good estimates.

We use the daily closing level of the VIX index and from the annualized VIX, which is expressed in terms of standard deviations, we calculate the daily VIX variance at time t as $\sigma^2_{IV,t} = VIX^2_t/252$ (based on Hal and Koopman (2000)). This assumes 252 trading days in a year. The main attraction of the VIX index is that it mitigates many of the problems which lead to biased implied volatility values.

VIII. Methodology

There are a number of procedures for system identification. We follow the procedure adopted by Näsström (2003). We first present some basic statistics and check for presence of autocorrelation and heteroscedasticity. The estimation is done only in the univariate case. The selection procedure is based on each model's ability to produce forecasts[3] (Näsström 2003, Zumbach 2003). The procedure is to first choose the number of lags (p,q) for the symmetric GARCH with a normal distribution and then to test for this specific (p,q) the different GARCH models. Finally when the model is chosen three different distributions for this model are tested and the parameters for the chosen GARCH model

[2] Approaches include

- using a *lumping methodology* wherein dates on which indices are not available are treated as having the same prices as the previous trading day;
- using a *spreading methodology* wherein indices between the trading days are averaged across all non-trading; and
- eliminate all index data for the day on which price of one or more indices in not available.

The second method changes the structure of the process whereas the third method leads to a loss of data

[3] Zumbach (2003) states "If the emphasis is to produce volatility forecasts, then estimating the parameters by minimising the forecast error is appropriate, whereas if the emphasis is on the data generating process, a log-likelihood estimate is better suited."

Table I
Q-Statistics on squared standardized residuals for VIX.

Q(10) = 17195.4 [0]
Q(15) = 22963.7 [0]
Q(20) = 28008.2 [0]

are presented. In the validation part we test to see if the correlation and heteroscedasticity has been removed from the data series and perform other tests to validate the model.

IX. Empirical results

Figure 1 gives us an indication of the pattern of the daily volatilities in VIX. It also provides us a density diagram of daily volatility which clearly indicates the non-normality of the data. This is affirmed by the Jarque-Bera statistic in Table III. The series is negatively skewed and exhibits leptokurtosis. Table III provides summary statistics for both the total sample as well as the in-sample periods. The effect of the large outliers in the full sample are illustrated by the very high values for the skewness and kurtosis coefficients.

From the plot of autocorrelations (ACF) and partial autocorrelations (PACF) in Figure 2 we can infer the presence of a serial correlation between the daily variances. The exponentially decaying nature of the ACF plot shows that the indicated model is an autoregressive one. The presence of linear dependence in daily volatility can be attributed to various market phenomena and anomalies. The presence of a common market factor, the speed of information processing by the market participants, day-of-the-week effects could contribute partially to the autocorrelations. Nonlinear dependence can be explained by the well documented fact of changing variances. Changing variances can also explain the high levels of kurtosis. Variance changes are often related to rate of information arrivals, level of trading activity, and corporate financial and operating leverage decisions. Based on the first 20 lags, no seasonality is apparent. The partial autocorrelation plot is used to identify the order of the autoregressive model. It is quite apparent that the first-order would give best results.

The indication from graphical plot that there exists correlation between data plots is tested with the Ljung-Box-test. The computed statistical values for the certain lags are presented together with the corresponding p-value in angle brackets. Results from the test are presented in Table I. The null hypothesis is that no serial correlation exists and this hypothesis is accepted when p-value is high. In this case we reject the hypothesis.

Table II
Engle's ARCH-test on raw series for VIX.

ARCH 1-2 test:	F(2,2612)= 9950.1 [0.0000]**
ARCH 1-5 test:	F(5,2606)= 4327.9 [0.0000]**
ARCH 1-10 test:	F(10,2596)= 2186.1 [0.0000]**

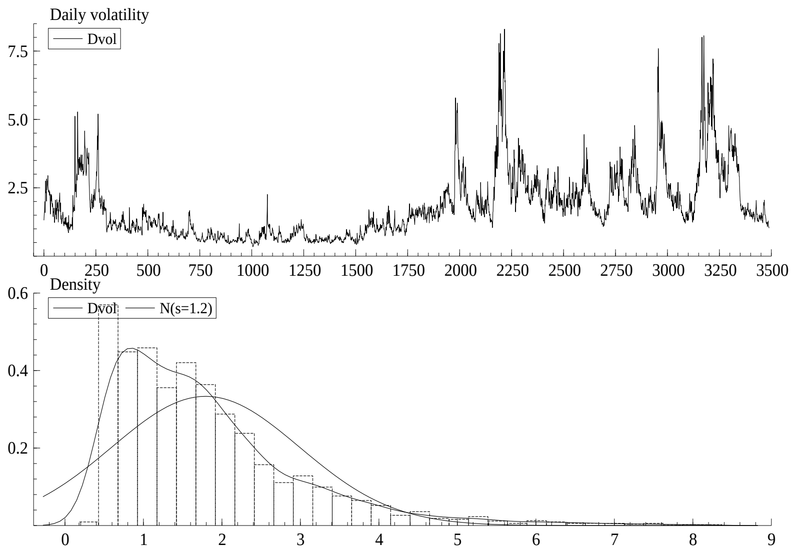

Figure 1
Daily volatilities and distribution

Engle's ARCH test is performed before estimation. This test supports that there there exists heteroscedasticity. P-values equal to zero for the time series. The results are presented in Table II.

Table III
Summary Statistics of the daily volatility on VIX from 01/03/90 to 10/31/03

Period	1990 to 2003	1990 to 2000
No. of Obs. T	3490	2619
Series	VIX $\sigma^2_{IV,t}$	VIX $\sigma^2_{IV,t}$
Mean	1.792528	1.524666
Standard Error	0.020254	0.020400
Median	1.523667	1.201429
Mode	0.538581	0.538581
Standard Deviation	1.196516	1.044018
Kurtosis	3.734009	6.329394
Skewness	1.67455	2.047533
Minimum	0.343953	0.343953
Maximum	8.302173	8.302173
Jarque-Bera	3649.5	6179.7
$\hat{\rho}_1$	0.97354	0.97171
$\hat{\rho}_{10}$	0.84207	0.83752
$\hat{\rho}_{20}$	0.74818	0.76012

We start our analysis by fitting an ARMA(0,0) model in the mean equation and GARCH(p,q) in the variance equation, assuming Gaussian distribution. We vary the parameters p and q of the GARCH model till we arrive at the best parameter estimates. The selection procedure is based on each model's ability to produce forecasts. Four different combinations of p and q are used. Table IV shows that GARCH(1,1) and GARCH (2,2) performs best on three out of six out-of-sample measures each. This indicates that model selection is highly sensitive to performance measures. For the purpose of further study we consider the GARCH(1,1) model in light of the fact that R^2's and MAE are very close to each other. This is substantiated by the first order partial autocorrelations.

We further test four different GARCH models. These models are described in Section V. In Table V we find that model EGARCH performs best. Therefore, EGARCH is used in future model selection.

Three different distributions are tested. The distributions are normal, student-t and skewed student-t distribution. A comparison of the results for the distributional assumptions is presented in Table VI. Clearly the Gauss (normal) distribution performs best. Hence we will use this distribution as our choice.

The best GARCH model in out-of-sample data is EGARCH(1,1). The parameters for the chosen model are presented in Table VII.

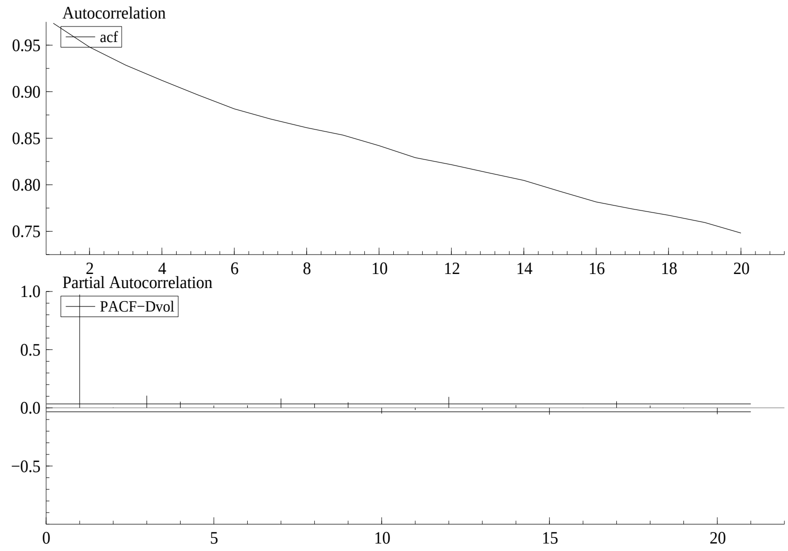

Figure 2
ACF and PACF plots of daily volatilities

Table IV
Selection of model for GARCH for VIX. Log likelihood as well as Akaike's information criterion are presented for different p and q. Six different forecast measures are also presented to determine which p and q perform best

(p,q)	(1,1)	(1,2)	(2,1)	(2,2)
Log	-1899.04	-1898.53	-1898.32	-1896.74
Akaike	1.4533	1.4536	1.4535	1.4530
R^2	0.587469	0.587441	0.589892	**0.590629**
MSE	**34.270**	34.460	34.950	34.710
MedSE	3.7880	3.7570	3.7370	**3.5810**
MAE	3.694	3.699	3.714	**3.690**
AMAPE	**0.6305**	0.6308	0.6311	0.6315
TIC	**0.4948**	0.6308	0.4968	0.4959

Table V
GARCH model selection for VIX. All models with $p = q = 1$

	GARCH	EGARCH	GJR	APARCH	IGARCH
Log	-1899.04	-1942.16	-1896.21	-1896.19	-1899.15
Akaike	1.4533	1.4877	1.4519	1.4526	1.4526
R^2	0.587469	0.379726	0.588423	**0.589587**	0.587283
MSE	34.270	**23.740**	38.920	39.430	32.880
MedSE	3.7880	**2.3540**	4.1650	4.1620	3.6700
MAE	3.694	**2.601**	3.909	3.929	3.627
AMAPE	0.6305	**0.5839**	0.6328	0.6338	0.6296
TIC	0.4948	**0.4810**	0.5097	0.5111	0.4899

Table VI
Distribution selection for VIX.

	Gauss	Student-t	Skewed Stud.-t
Log	-1942.16	-1972.21	-702.124
Akaike	1.4877	1.511423	0.542287
R2	0.379726	0.402848	**0.40924**
MSE	**23.740**	24.58	2.50E+05
MedSE	**2.3540**	2.493	2.37E+04
MAE	**2.601**	2.754	291.3
AMAPE	**0.5839**	0.5894	0.9792
TIC	0.4810	**0.4807**	0.9881

One part of the validation is already taken care of when the model was chosen on the basis of its forecasting ability. Other tests are performed to see if the autocorrelation in the series has been removed and also to test for heteroscedasticity.

For purposes of forecasting we look at 800 one-step ahead forecasts. Figure 3 gives a picture of the results. It indicates that most of the significant autocorrelation has been removed.

The Ljung-Box-Pierce Q-test (Lbq-test) in the validation part is used to test whether there still is a significant correlation in the standardized innovations or not. Table VIII provides the results on the Lbq-test. The test is made on z_t ($\frac{\varepsilon_t^2}{h_t^2} = z_t$) with lags of 10, 15 and 20. Each row in the table represents one test with a specific lag. H_0 is accepted, p-values are given within brackets.

Table VII
Estimated parameters for the EGARCH(1,1) model with Gauss distribution for VIX.

Parameter	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.989955	0.008778	112.8	0
Cst(V)	-2.52828	0.13685	-18.48	0
ARCH(Alpha1)	50.59693	188.9	0.2679	0.7888
GARCH(Beta1)	0.855719	0.018272	46.83	0
EGARCH(Theta1)	0.002379	0.008832	0.2694	0.7876
EGARCH(Theta2)	0.022714	0.084058	0.2702	0.787

Table VIII
Q-Statistics on squared standardized residuals for VIX.

Q(10) = 3.71709 [0.881704]
Q(15) = 4.57136 [0.983459]
Q(20) = 13.828 [0.740234]

The Nyblom test of stability indicates that the GARCH parameters are changing over time (Table IX).

The results from Engle's ARCH test are presented in Table X. This indicates that we have successfully removed the conditional heteroscedasticity.

To test whether the standardized residual is normally distributed or not a Jarque-Bera test is performed. The results from this test are presented in Table XI. Also presented are statistics pertaining to skewness and kurtosis. The results indicate that the standardized residuals are normally distributed.

The diagnostic test of Engle and Ng (1993) investigates possible misspecification of the conditional variance equation. The *sign bias test* examines the impact of positive and negative return shocks on volatility not predicted by the model under construction.The *negative (positive) size bias test* focusses on the different effects that large and small negative (positive) returns shocks have on volatility, which is not predicted by volatility model. Finally a joint test for these three tests is also provided. Table XII presents results of this test. t-statistics greater than 2 in absolute value provide an evidence of model misspecification. The model appears to be misspecified on positive sign test and the overall test thus indicating that some asymmetry does remain.

Table IX
Nyblom test statistic for VIX.

Joint Statistic of the Nyblom test of stability: 14.8236	
Individual Nyblom Statistics:	
Cst(M)	4.48386
Cst(V)	0.18950
ARCH(Alpha1)	0.64529
GARCH(Beta1)	1.05573
EGARCH(Theta1)	2.59706
EGARCH(Theta2)	0.42655

Table X
Engle's ARCH-test validation for VIX.

ARCH 1-2 test: F(2,2612) = 0.032960 [0.9676]
ARCH 1-5 test: F(5,2606) = 0.051733 [0.9983]
ARCH 1-10 test: F(10,2596)= 0.364790 [0.9617]

Table XI
Properties of standardized residuals for VIX.

	Statistic	t-Test	P-Value
Skewness	-0.03657	0.76444	0.4446
Excess Kurtosis	0.37379	3.9085	9.29E-05
Jarque-Bera	15.831	15.831	0.000365

Table XII
Diagnostic test based on the news impact curve for VIX standardized residuals.

	Test	Prob
Sign Bias t-Test	0.18658	0.85199
Negative Size Bias t-Test	1.02362	0.30602
Positive Size Bias t-Test	4.74405	0.00000
Joint Test for the Three Effects	26.45159	0.00001

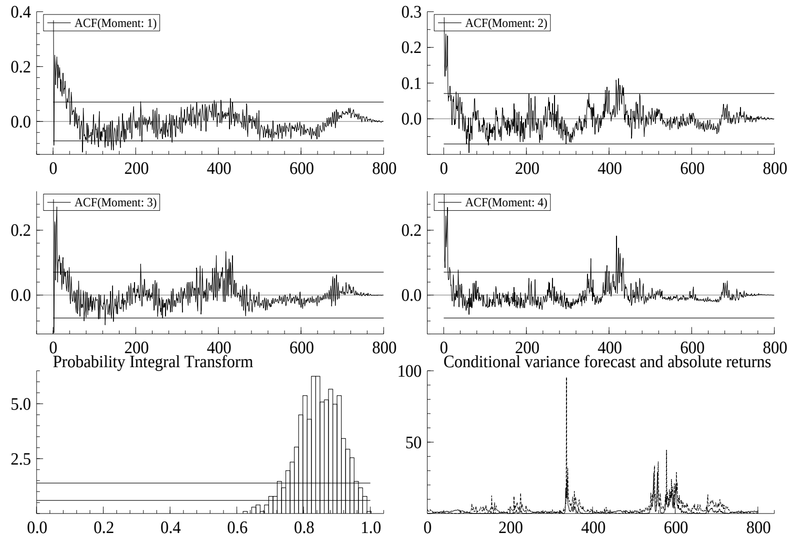

Figure 3
Forecast results

Overall, the results are indicative of the good performance of the forecasting methodology.

One interesting result is that in our study the EGARCH model performs best. A look at several past studies indicates the predominance of studies that find GARCH model to perform well. Table IX gives a sense of the nature of previous results.

X. Conclusions

Forecasting implied volatility is important as it enables a more accurate pricing of options. We test different (p,q) values in the symmetric case. These are then used as fixed p and q values for different GARCH models. In this case $p = q = 1$ appears to be the best out-of-sample choice. Different models are tested for these specific p and q values. The asymmetric EGARCH model performs best among the models that are tested. Different distributions for the specific EGARCH(1,1) model are then tested and estimated. There is no evidence that any model other than normal should be used. We note that model selection is highly sensitive to the choice of performance measures. Time dependent coefficients are a problem with this dataset. This implies that the coefficients will need to be re-estimated when new data becomes available.

Table XIII
Literature review of the performance of volatility measures in different countries

Country	Author(s)	Best Forecast Method	Other measures of volatility compared against
USA	Akigiray (1995)	GARCH (1,1)	Historical and EWMA
USA	Heynen (1995)	Stochastic volatility	Random walk, GARCH (1,1) and EGARCH (1,1)
UK	Dimson and Marsh (1990)	EWMA	Historical ARCH and GARCH
UK	Poon and Taylor (1992)	GARCH-M	ARCH and EWMA
UK	Corhay and Rad (1994)	GARCH (1,1)	Various orders and models of ARCH and GARCH
UK	Heynen (1995)	Stochastic volatility	Random walk, GARCH (1,1) and EGARCH (1,1)
UK	Vasilellis and Meade (1996)	GARCH	Historical and EWMA
Australia	Brasilford and Faff (1993)	GARCH (3,1)	Various orders of ARCH and GARCH
Australia	Heynen (1995)	Stochastic volatility	Random walk, GARCH (1,1) and EGARCH (1,1)
Australia	Brasilford and Faff (1996)	GJR-GARCH (1,1)	Historical, Random walk, 5 and 12 year moving average, Exponential smoothing, EWMA, GARCH (1,1), GARCH (3,1) and GJR-GARCH(3,1)
Canada	Calvet and Rahman (1995)	IGARCH	Various orders of ARCH and GARCH
Finland	Booth, Hatem, Virtanen, and Yli-Olli (1992)	GARCH (1,1)	Various orders of ARCH and GARCH
France	Corhay and Rad (1994)	GARCH (1,1)	Various orders and models of ARCH and GARCH
Germany	Corhay and Rad (1994)	GARCH (1,1)	Various orders and models of ARCH and GARCH
Holland	Corhay and Rad (1994)	GARCH (1,1)	Various orders and models of ARCH and GARCH
Hong Kong	Heynen (1995)	Stochastic volatility	Random walk, GARCH (1,1) and EGARCH (1,1)
Italy	Corhay and Rad (1994)	IGARCH (1,1)	Various orders and models of ARCH and GARCH
Japan	Tse (1991)	EWMA	Historical, ARCH and GARCH
Japan	Heynen (1995)	Stochastic volatility	Random walk, GARCH (1,1) and EGARCH (1,1)
Singapore	Tse and Tung (1992)	EWMA	Historical, ARCH and GARCH

Abstracted from Walsh and Tsou (1998)

Accurate forecasting of implied volatility can be useful in formulating derivative trading strategies. One of the most elegant trading strategies in derivatives trading is the volatility-dispersion strategy. Volatility dispersion trading is essentially a hedged strategy designed to take advantage of relative value differences in implied volatilities between an index and a basket of component stocks. It typically involves short option positions on an index, against which long option positions are taken on a set of components of the index. It is common to see a short straddle or near-ATM strangle on the index and long similar straddles or strangles on 30% to 40% of the stocks that make up the index. If maximum dispersion is realized, the strategy will make money on the long stock options and will lose very little on the short index options, since the latter would have moved very little. The strategy is typically a very low-premium strategy, with very low initial Δ[4] and typically a small net long $\mathcal{V}$ [5].

Some of the limitations of this study include:

- We focus exclusively on univariate methods. The multivariate case is not treated at all.
- Use of only daily data. We have not looked at how the models would behave on a higher frequency data.
- We have looked exclusively at GARCH models and have not looked at other methods such as neural networks or stochastic volatility.

[4]The delta (Δ) of an option is defined as the rate of change of the option price with respect to the price of the underlying asset

[5]The vega ($\mathcal{V}$) of a portfolio of derivatives is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset

APPENDIX

A. Evaluation measures

- Mean Squared Error(MSE)

  The MSE is calculated by:

$$\frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{\sigma}_t^2-\sigma_t^2)^2$$

- Median Squared Error(MedSE)

  The MedSE is:

$$\frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{\sigma}_{med}^2-\sigma_{med}^2)^2$$

- Mean Absolute Error (MAE)

  The MAE is:

$$\frac{1}{h+1}\sum_{t=S}^{S+h}|(\hat{\sigma}_t^2-\sigma_t^2)|$$

- Adjusted Mean Absolute Percentage Error (AMAPE)

$$\frac{1}{h+1}\sum_{t=S}^{S+h}\left|\frac{(\hat{\sigma}_t^2-\sigma_t^2)}{(\hat{\sigma}_t^2+\sigma_t^2)}\right|$$

- Theil's Inequality Coefficient (TIC)

$$\frac{\frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{\sigma}_t^2-\sigma_t^2)^2}{\sqrt{\frac{1}{h+1}\sum_{t+S}^{S+h}\hat{\sigma}_t^2+\frac{1}{h+1}\sum_{t+S}^{S+h}\sigma_t^2}}$$

  is a scale invariant measure that lies between 0 and 1 where 0 indicates perfect fit.

- Mincer-Zarnowitz R^2

  The R^2 is calculated by regressing $\hat{\sigma}_t^2$ on the ε_t^2 which can be formulated with the equation:

  $$\varepsilon_t^2 = \alpha + \beta\hat{\sigma}_t^2 + u_t$$

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