R2. Model fitting and evaluation

2020 - 2021 Skagit River steelhead forecast.

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This is version 0.20.12.21.	

Requirements

All analyses require the R software (v3.4.3) for data retrieval, data processing, and summarizing model results, and the JAGS software (v4.2.0) for Markov chain Monte Carlo (MCMC) simulation. Please note that some of the R code below may not work with older versions of JAGS due to some changes in the ways that arrays are handled.

We also need a few packages that are not included with the base installation of R, so we begin by installing them (if necessary) and then loading them.

```
if(!require("here")) {
  install.packages("here")
  library("here")
if(!require("readr")) {
  install.packages("readr")
  library("readr")
}
if(!require("rjags")) {
  install.packages("rjags")
  library("rjags")
}
if(!require("loo")) {
  install.packages("loo")
  library("loo")
if(!require("ggplot2")) {
  install.packages("ggplot2")
  library("ggplot2")
}
if(!require("coda")) {
  install.packages("coda")
  library("coda")
}
if(!require("shinystan")) {
  install.packages("shinystan")
  library("shinystan")
}
if(!require("R2jags")) {
  install.packages("R2jags")
  library("R2jags")
if(!require("dclone")) {
  install.packages("dclone")
  library("dclone")
}
if(!require("snow")) {
  install.packages("snow")
  library("snow")
}
## set directory locations
datadir <- here("data")</pre>
jagsdir <- here("jags")</pre>
analdir <- here("analysis")</pre>
savedir <- here("analysis/cache")</pre>
```

We also need a couple of helper functions.

```
## better round
Re2prec <- function(x, map = "round", prec = 1) {
    ## 'map' can be "round", "floor", or "ceiling"</pre>
```

```
## 'prec' is nearest value (eg, 0.1 means to nearest tenth; 1 gives normal behavior)
  if(prec<=0) { stop("\"prec\" cannot be less than or equal to 0") }</pre>
  do.call(map,list(x/prec))*prec
}
## wrapper function to fit JAGS models & rearrange output
fit_jags <- function(model, data, params, inits, ctrl, dir = jagsdir) {</pre>
  jm <- jags.model(file.path(jagsdir, model),</pre>
                    data,
                    inits.
                    ctrl$chains,
                    ctrl$burn,
                    quiet = TRUE)
  return(coda.samples(jm, params, ctrl$length, ctrl$thin))
#alternative wrapper to fit model in parallel; one chain per core
fit_jags2<-function(model,data,params,inits,ctrl,dir=jagsdir){</pre>
  # jm<-jaqs.parallel(data=data,</pre>
                       inits=inits.
  #
                       parameters.to.save=params,
  #
                       model.file = file.path(jagsdir, model),
                       n.chains = ctrl $chains,
                       n.iter = ctrl$length,
  #
                       n.burnin = ctrl$burn,
  #
                       n.thin = ctrl\$thin,
                       DIC = TRUE.
  # )
  # write.csv(jm$BUGSoutput$summary,file.path(savedir,paste(model,".csv",sep="")))
  # jm<-jags.parfit(data=data,</pre>
                       inits=inits,
  #
                       parameters.to.save=params,
  #
                       model.file = file.path(jagsdir, model),
  #
                       n.chains = ctrl\$chains,
                       n.iter = ctrl$length,
                       n.burnin = ctrl\$burn,
                       n.thin = ctrl\$thin,
  #
  #
                       DIC = TRUE,
  # )
  # return(as.mcmc.list(as.mcmc(jm)))
  cl <- makeCluster(3, type = "SOCK")</pre>
  inits2 <- jags.fit(data=data,</pre>
                      params=params,
                      model=file.path(jagsdir, model),
                      inits=inits,
                      n.chains=ctrl$chains,
                      n.adapt = 0,
                      n.update = 0,
                      n.iter = 0)$state(internal = TRUE)
  jm <- jags.parfit(cl=cl,</pre>
                     data = data,
                     params = params,
                     model = file.path(jagsdir, model),
```

User inputs

We begin by supplying values for the following parameters, which we need for model fitting and evaluation.

```
## first & last years of fish data
yr_frst <- 1978</pre>
yr_last <- 2020</pre>
## min & max adult age classes
age_min <- 3
age_max <- 8
## years (if any) of age-comp to skip; see below
age skip <- 0
## number of years ahead for run forecasts from the most recent year of data
n fore <- 1
## number of recent year forecasts
n_forecasts <- 10
## first year of 1 step ahead forecast
yr_begin <- 2011</pre>
## last year of 1 step ahead forecast
yr_end <- 2020</pre>
## upper threshold for Gelman {\mathfrak C} Rubin's potential scale reduction factor (Rhat).
Rhat_thresh <- 1.1</pre>
```

Next we specify the names of three necessary data files containing the following information:

- 1. observed total number of adult spawners (escapement) by year;
- 2. observed age composition of adult spawners by year;
- 3. observed total harvest by year;

```
## 1. file with escapement data
## [n_yrs x 2] matrix of obs counts; 1st col is calendar yr
```

```
fn_esc <- "skagit_sthd_esc.csv"

## 2. file with age comp data
## [n_yrs x (1+A)]; 1st col is calendar yr
fn_age <- "skagit_sthd_age.csv"

## 3. file with harvest data
## [n_yrs x 2] matrix of obs catch; 1st col is calendar yr
fn_harv <- "skagit_sthd_catch.csv"</pre>
```

Loading the fish data

Here we load in the first three data files and do some simple calculations and manipulations. First the spawner data:

```
## escapement
dat_esc <- read_csv(file.path(datadir, fn_esc))
## years of data
dat_yrs <- dat_esc$year

## number of years of data
n_yrs <- length(dat_yrs)

## log of escapement
ln_dat_esc <- c(log(dat_esc$escapement),rep(NA,n_fore))</pre>
```

Next the age composition data:

```
## age comp data
dat_age <- read_csv(file.path(datadir, fn_age))</pre>
## num of age classes
A <- age_max - age_min + 1
# ## drop year col & first age_min+age_skip rows
# dat_age <- dat_age[-(1:(age_min+age_skip)),-1]
# ## add row(s) of NA's for forecast years
# if(n_fore > 0) {
   dat_age <- rbind(dat_age,</pre>
#
                      matrix(0, n_fore, A,
#
                              dimnames =list(n_yrs+seq(n_fore),
#
                                             colnames(dat age))))
# }
# ## total num of age obs by cal yr
# dat_age[,"sum"] <- apply(dat_age, 1, sum)</pre>
# ## row indices for any years with no obs age comp
# idx_NA_yrs <- which(dat_age$sum<A, TRUE)</pre>
# ## replace 0's in yrs w/o any obs with NA's
\# dat_age[idx_NA_yrs,(1:A)] \leftarrow NA
# ## change total in yrs w/o any obs from 0 to A to help dmulti()
\# dat_age[idx_NA_yrs,"sum"] \leftarrow A
```

```
# ## convert class
# dat_age <- as.matrix(dat_age)</pre>
```

And then the harvest data:

```
## harvest
dat_harv <- read_csv(file.path(datadir, fn_harv))
## drop year col & first age_max rows
#dat_harv <- c(dat_harv$catch,rep(NA,n_fore))</pre>
```

Loading the covariates

Our analysis investigates 5 covariates as possible drivers of the population's instrinic growth rate:

- 1. Maximum river discharge in winter;
- 2. Minimum river discharge in summer;
- 3. North Pacific Gyre Oscillation;

All of the covariates are contained in the file /data/skagit_sthd_covars.csv. We will load and then standardize them to have zero-mean and unit-variance.

```
dat_cvrs <- read_csv(file.path(datadir, "skagit_sthd_covars.csv"))
## drop year col
# dat_cvrs <- dat_cvrs[,-1]
# ## transform the covariates to z-scores
# scl_cvrs <- as.matrix(scale(dat_cvrs))
# ## total number of covariates
# n_cov <- dim(dat_cvrs)[2]</pre>
```

Specifying models in JAGS

Now we can specify the model in JAGS. We fit a total one model, which we outline below, based on a beverton holt process model with covariates.

Beverton-Holt with covars

```
cat("
    model {

    ##-----
    ## PRIORS
    ##-----

## alpha = intrinsic productivity
    alpha ~ dnorm(0,0.001) T(0,);
    mu_BH_a <- log(alpha);
    E_BH_a <- mu_BH_a + sigma_r/(2 - 2*phi^2);</pre>
```

```
## strength of dens depend
beta_inv ~ dnorm(0, 1e-9) T(0,);
beta <- 1/beta inv;</pre>
## covariate effects
for(i in 1:n_cov) { gamma[i] ~ dnorm(0,0.01) }
## AR(1) coef for proc errors
#phi ~ dunif(-0.999,0.999);
#phi <- 0;</pre>
phi_prior ~ dbeta(2,2);
phi <- phi_prior*2-1;</pre>
#phi ~ dunif(0,0.999);
## innovation in first year
innov_1 ~ dnorm(0,tau_r*(1-phi*phi));
## process variance for recruits model
sigma_r ~ dnorm(0, 2e-2) T(0,);
tau_r <- 1/sigma_r;</pre>
## obs variance for spawners
tau_s <- 1/sigma_s;</pre>
sigma_s ~ dnorm(0, 0.001) T(0,);
## unprojectable early recruits;
## hyper mean across all popns
Rec_mu ~ dnorm(0,0.001);
## hyper SD across all popns
Rec_sig ~ dunif(0,100);
## precision across all popns
Rec_tau <- pow(Rec_sig,-2);</pre>
## multipliers for unobservable total runs
ttl_run_mu ~ dunif(1,5);
ttl_run_tau ~ dunif(1,20);
## get total cal yr returns for first age_min yrs
for(i in 1:(age_min+age_skip)) {
ln_tot_Run[i] ~ dnorm(ttl_run_mu*Rec_mu,Rec_tau/ttl_run_tau);
tot_Run[i] <- exp(ln_tot_Run[i]);</pre>
## maturity schedule
## unif vec for Dirch prior
theta <-c(1,10,10,5,1,1)
## hyper-mean for maturity
pi_eta ~ ddirch(theta);
## hyper-prec for maturity
pi_tau ~ dnorm(0, 0.01) T(0,);
for(t in 1:(n_yrs-age_min+n_fore)) { pi_vec[t,1:A] ~ ddirch(pi_eta*pi_tau) }
## estimated harvest rate
for(t in 1:(n_yrs+n_fore)) { h_rate[t] ~ dunif(0,1) }
```

```
## LIKELIHOOD
##-----
## predicted recruits in BY t
covar[1] <- inprod(gamma,mod_cvrs[1,]);</pre>
ln_BH_a[1] <- mu_BH_a + covar[1];</pre>
E_{\ln_{c}} = \ln_{c} - \ln_{c} + \ln_{c} = \ln_{c} - \ln_{c} + \ln_{c} = 
tot_ln_Rec[1] ~ dnorm(E_ln_Rec[1],tau_r);
res_ln_Rec[1] <- tot_ln_Rec[1] - E_ln_Rec[1];</pre>
w[1] <- phi * innov_1 + res_ln_Rec[1];
## median of total recruits
tot_Rec[1] <- exp(tot_ln_Rec[1]);</pre>
## R/S
ln_RS[1] <- tot_ln_Rec[1] - ln_Sp[1];</pre>
## brood-yr recruits by age
for(a in 1:A) {
Rec[1,a] <- tot_Rec[1] * pi_vec[1,a];</pre>
}
## brood years 2:(n_yrs-age_min)
for(t in 2:(n_yrs-age_min+n_fore)) {
## predicted recruits in BY t
covar[t] <- inprod(gamma, mod_cvrs[t,]);</pre>
ln_BH_a[t] <- mu_BH_a + covar[t];</pre>
E_{\ln Rec[t]} \leftarrow \ln_BH_a[t] + \ln_Sp[t] - \log(1 + beta*Sp[t]) + phi*res_ln_Rec[t-1];
tot_ln_Rec[t] ~ dnorm(E_ln_Rec[t],tau_r);
res_ln_Rec[t] <- tot_ln_Rec[t] - E_ln_Rec[t];</pre>
w[t] <- phi * res_ln_Rec[t-1] + res_ln_Rec[t];</pre>
## median of total recruits
tot_Rec[t] <- exp(tot_ln_Rec[t]);</pre>
## R/S
ln_RS[t] <- tot_ln_Rec[t] - ln_Sp[t];</pre>
## brood-yr recruits by age
for(a in 1:A) {
Rec[t,a] <- tot_Rec[t] * pi_vec[t,a];</pre>
} ## end t loop over year
## get predicted calendar year returns by age
## matrix Run has dim [(n_yrs-age_min) x A]
## step 1: incomplete early broods
## first cal yr of this grp is first brood yr + age_min + age_skip
for(i in 1:(age_max-age_min-age_skip)) {
## projected recruits
for(a in 1:(i+age_skip)) {
Run[i,a] <- Rec[(age_skip+i)-a+1,a];</pre>
```

```
## imputed recruits
for(a in (i+1+age_skip):A) {
lnRec[i,a] ~ dnorm(Rec_mu,Rec_tau);
Run[i,a] <- exp(lnRec[i,a]);</pre>
}
## total run size
tot_Run[i+age_min+age_skip] <- sum(Run[i,1:A]);</pre>
## predicted age-prop vec for multinom
for(a in 1:A) {
age_v[i,a] <- Run[i,a] / tot_Run[i+age_min];</pre>
## multinomial for age comp
dat_age[i,1:A] ~ dmulti(age_v[i,1:A],dat_age[i,A+1]);
lp_age[i] <- logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]);</pre>
## step 2: info from complete broods
## first cal yr of this grp is first brood yr + age_max
for(i in (A-age_skip):(n_yrs-age_min-age_skip+n_fore)) {
for(a in 1:A) {
Run[i,a] <- Rec[(age_skip+i)-a+1,a];</pre>
## total run size
tot_Run[i+age_min+age_skip] <- sum(Run[i,1:A]);</pre>
## predicted age-prop vec for multinom
for(a in 1:A) {
age_v[i,a] <- Run[i,a] / tot_Run[i+age_min];</pre>
}
## multinomial for age comp
dat_age[i,1:A] ~ dmulti(age_v[i,1:A],dat_age[i,A+1]);
lp_age[i] <- ifelse(i < n_yrs-age_min-age_skip+n_fore, logdensity.multi(dat_age[i,1:A],age_v[i,1:A]</pre>
## get predicted calendar year spawners
## first cal yr is first brood yr
for(t in 1:(n_yrs+n_fore)) {
## obs model for spawners
#Sp[t] <- max(10,tot_Run[t] - dat_harv[t]);</pre>
est_harv[t] = ifelse(t > n_yrs,1,h_rate[t] * tot_Run[t]);
dat_harv[t] ~ dlnorm(log(est_harv[t]), 20);
Sp[t] = tot_Run[t] - est_harv[t];
ln_Sp[t] \leftarrow log(Sp[t]);
ln_dat_esc[t] ~ dnorm(ln_Sp[t], tau_s);
```

```
lp_esc[t] <- ifelse(t < n_yrs + 1,logdensity.norm(ln_dat_esc[t],ln_Sp[t], tau_s),0);
} ## end model description

", file=file.path(jagsdir, "IPM_BH_cov_AR.txt"))</pre>
```

Beverton-Holt with covars

```
cat("
model {
 ##----
  ## PRIORS
  ##----
  ## alpha = intrinsic productivity
  alpha ~ dnorm(0,0.001) T(0,);
  mu_BH_a <- log(alpha);</pre>
  E_BH_a \leftarrow mu_BH_a + sigma_r/(2 - 2*phi^2);
  ## strength of dens depend
  beta_inv ~ dnorm(0, 1e-9) T(0,);
  beta <- 1/beta_inv;</pre>
  ## covariate effects
  for(i in 1:n_cov) { gamma[i] ~ dnorm(0,0.01) }
  ## AR(1) coef for recruitment residual
  #phi ~ dunif(-0.999,0.999);
  #phi <- 0;</pre>
  phi_prior ~ dbeta(2,2);
  phi <- phi_prior*2-1;</pre>
  #phi ~ dunif(0,0.999);
  ## MA(1) coef recruitment residual
  theta_res_prior ~ dbeta(2,2);
  theta_res <- theta_res_prior*2-1;</pre>
  #theta res \sim dunif(0,0.999);
  ## innovation in first year
  #innov_1 ~ dnorm(0,tau_r*(1-phi*phi));#AR1
  innov_1 ~ dnorm(0,(1-phi^2)/((1+2*phi*theta_res+theta_res^2)*sigma_r^2));#AR1MA1
  ## process variance for recruits model
  sigma_r ~ dnorm(0, 2e-2) T(0,);
  tau_r <- 1/sigma_r;</pre>
  ## obs variance for spawners
  tau_s <- 1/sigma_s;</pre>
  sigma_s ~ dnorm(0, 0.001) T(0,);
  ## unprojectable early recruits;
```

```
## hyper mean across all popns
Rec_mu ~ dnorm(0,0.001);
## hyper SD across all popns
Rec sig ~ dunif(0,100);
## precision across all popns
Rec_tau <- pow(Rec_sig,-2);</pre>
## multipliers for unobservable total runs
 ttl_run_mu ~ dunif(1,5);
 ttl_run_tau ~ dunif(1,20);
## get total cal yr returns for first age_min yrs
for(i in 1:(age_min+age_skip)) {
      ln_tot_Run[i] ~ dnorm(ttl_run_mu*Rec_mu,Rec_tau/ttl_run_tau);
      tot_Run[i] <- exp(ln_tot_Run[i]);</pre>
}
## maturity schedule
## unif vec for Dirch prior
theta <-c(1,10,10,5,1,1)
## hyper-mean for maturity
pi eta ~ ddirch(theta);
## hyper-prec for maturity
pi_tau ~ dnorm(0, 0.01) T(0,);
for(t in 1:(n_yrs-age_min+n_fore)) { pi_vec[t,1:A] ~ ddirch(pi_eta*pi_tau) }
## estimated harvest rate
for(t in 1:(n_yrs+n_fore)) { h_rate[t] ~ dunif(0,1) }
##----
## LIKELIHOOD
##-----
## predicted recruits in BY t
covar[1] <- inprod(gamma,mod_cvrs[1,]);</pre>
ln_BH_a[1] <- mu_BH_a + covar[1];</pre>
E_{n_{ec}[1]} \leftarrow n_{BH_a[1]} + n_{Sp[1]} - \log(1 + beta*Sp[1]) + phi * innov_1 + theta_res * 0;
tot_ln_Rec[1] ~ dnorm(E_ln_Rec[1], tau_r);
res_ln_Rec[1] <- tot_ln_Rec[1] - E_ln_Rec[1];</pre>
w[1] <- phi * innov_1 + theta_res * 0 + res_ln_Rec[1]</pre>
## median of total recruits
tot_Rec[1] <- exp(tot_ln_Rec[1]);</pre>
ln_RS[1] <- tot_ln_Rec[1] - ln_Sp[1];</pre>
## brood-yr recruits by age
for(a in 1:A) {
  Rec[1,a] <- tot_Rec[1] * pi_vec[1,a];</pre>
## brood years 2:(n_yrs-age_min)
for(t in 2:(n_yrs-age_min+n_fore)) {
  ## predicted recruits in BY t
```

```
covar[t] <- inprod(gamma, mod_cvrs[t,]);</pre>
  ln_BH_a[t] <- mu_BH_a + covar[t];</pre>
  #-----
 #version 4; more similar to AR1 original model
 E_{n_{ec}[t]} \leftarrow n_BH_a[t] + n_Sp[t] - \log(1 + beta*Sp[t]) + phi * w[t-1] + theta_res * res_ln_Rec[t]
 tot ln Rec[t] ~ dnorm(E ln Rec[t], tau r);
 res ln Rec[t] <- tot ln Rec[t] - E ln Rec[t];
 w[t] <- phi * w[t-1] + theta_res * res_ln_Rec[t-1] + res_ln_Rec[t];
 ## median of total recruits
 tot_Rec[t] <- exp(tot_ln_Rec[t]);</pre>
 ## R/S
 ln_RS[t] <- tot_ln_Rec[t] - ln_Sp[t];</pre>
 ## brood-yr recruits by age
 for(a in 1:A) {
    Rec[t,a] <- tot_Rec[t] * pi_vec[t,a];</pre>
 }
} ## end t loop over year
## get predicted calendar year returns by age
## matrix Run has dim [(n_yrs-age_min) x A]
## step 1: incomplete early broods
## first cal yr of this grp is first brood yr + age_min + age_skip
for(i in 1:(age_max-age_min-age_skip)) {
 ## projected recruits
 for(a in 1:(i+age_skip)) {
    Run[i,a] <- Rec[(age_skip+i)-a+1,a];</pre>
 ## imputed recruits
 for(a in (i+1+age_skip):A) {
    lnRec[i,a] ~ dnorm(Rec_mu,Rec_tau);
    Run[i,a] <- exp(lnRec[i,a]);</pre>
 ## total run size
 tot_Run[i+age_min+age_skip] <- sum(Run[i,1:A]);</pre>
 ## predicted age-prop vec for multinom
 for(a in 1:A) {
    age_v[i,a] <- Run[i,a] / tot_Run[i+age_min];</pre>
 }
 ## multinomial for age comp
 dat_age[i,1:A] ~ dmulti(age_v[i,1:A],dat_age[i,A+1]);
 lp_age[i] <- logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]);</pre>
## step 2: info from complete broods
## first cal yr of this grp is first brood yr + age_max
for(i in (A-age_skip):(n_yrs-age_min-age_skip+n_fore)) {
 for(a in 1:A) {
    Run[i,a] <- Rec[(age_skip+i)-a+1,a];</pre>
 }
```

```
## total run size
    tot_Run[i+age_min+age_skip] <- sum(Run[i,1:A]);</pre>
    ## predicted age-prop vec for multinom
    for(a in 1:A) {
      age_v[i,a] <- Run[i,a] / tot_Run[i+age_min];</pre>
    ## multinomial for age comp
    dat_age[i,1:A] ~ dmulti(age_v[i,1:A],dat_age[i,A+1]);
    #lp_age[i] <- logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]);</pre>
    lp_age[i] <- ifelse(i < n_yrs-age_min-age_skip+n_fore,</pre>
    logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]),0)
  ## get predicted calendar year spawners
  ## first cal yr is first brood yr
  for(t in 1:(n_yrs+n_fore)) {
    ## obs model for spawners
    # Sp[t] <- max(10,tot_Run[t] - dat_harv[t]);
    est_harv[t] = h_rate[t] * tot_Run[t];
    dat_harv[t] ~ dlnorm(log(est_harv[t]), 20);
    Sp[t] = tot_Run[t] - est_harv[t];
    ln_Sp[t] <- log(Sp[t]);</pre>
    ln_dat_esc[t] ~ dnorm(ln_Sp[t], tau_s);
    lp_esc[t] <- ifelse(t < n_yrs + 1,logdensity.norm(ln_dat_esc[t],ln_Sp[t], tau_s),0);</pre>
} ## end model description
 ", file=file.path(jagsdir, "IPM_BH_cov_MA1_AR1.txt"))
```

Beverton-Holt with covars

```
cat("
model {

##-----

## PRIORS

##-----

## alpha = intrinsic productivity
alpha ~ dnorm(0,0.001) T(0,);
mu_BH_a <- log(alpha);
E_BH_a <- mu_BH_a + sigma_r/(2 - 2*phi^2);

## strength of dens depend
beta_inv ~ dnorm(0, 1e-9) T(0,);
beta <- 1/beta_inv;

## covariate effects
for(i in 1:n_cov) { gamma[i] ~ dnorm(0,0.01) }

## AR(1) coef for recruitment residual</pre>
```

```
#phi ~ dunif(-0.999,0.999);
#phi <- 0;</pre>
phi_prior ~ dbeta(2,2);
phi <- phi_prior*2-1;</pre>
#phi ~ dunif(0,0.999);
## innovation in first year
innov_1 ~ dnorm(0,tau_r*(1-phi*phi));#AR1
## process variance for recruits model
sigma_r ~ dnorm(0, 2e-2) T(0,);
tau_r <- 1/sigma_r;</pre>
## obs variance for spawners
tau_s <- 1/sigma_s;</pre>
sigma_s ~ dnorm(0, 0.001) T(0,);
## unprojectable early recruits;
## hyper mean across all popns
Rec_mu ~ dnorm(0,0.001);
## hyper SD across all popns
Rec_sig ~ dunif(0,100);
## precision across all popns
Rec_tau <- pow(Rec_sig,-2);</pre>
## multipliers for unobservable total runs
 ttl_run_mu ~ dunif(1,5);
 ttl_run_tau ~ dunif(1,20);
## get total cal yr returns for first age_min yrs
for(i in 1:(age_min+age_skip)) {
      ln_tot_Run[i] ~ dnorm(ttl_run_mu*Rec_mu,Rec_tau/ttl_run_tau);
      tot_Run[i] <- exp(ln_tot_Run[i]);</pre>
}
## maturity schedule
## unif vec for Dirch prior
theta \leftarrow c(1,10,10,5,1,1)
## hyper-mean for maturity
pi_eta ~ ddirch(theta);
## hyper-prec for maturity
pi_tau ~ dnorm(0, 0.01) T(0,);
for(t in 1:(n_yrs-age_min+n_fore)) { pi_vec[t,1:A] ~ ddirch(pi_eta*pi_tau) }
## estimated harvest rate
for(t in 1:(n_yrs+n_fore)) { h_rate[t] ~ dunif(0,1) }
##-----
## LIKELIHOOD
##-----
## predicted recruits in BY t
covar[1] <- inprod(gamma,mod_cvrs[1,]);</pre>
ln_BH_a[1] <- mu_BH_a + covar[1];</pre>
E_{n_{ec}[1]} < n_{BH_a[1]} + n_{Sp[1]} - \log(1 + \beta_{Sp[1]}) + phi * innov_1;
```

```
tot_ln_Rec[1] ~ dnorm(E_ln_Rec[1], tau_r);
res_ln_Rec[1] <- tot_ln_Rec[1] - E_ln_Rec[1];</pre>
w[1] <- phi * innov_1 + res_ln_Rec[1];</pre>
## median of total recruits
tot_Rec[1] <- exp(tot_ln_Rec[1]);</pre>
ln_RS[1] <- tot_ln_Rec[1] - ln_Sp[1];</pre>
## brood-yr recruits by age
for(a in 1:A) {
  Rec[1,a] <- tot_Rec[1] * pi_vec[1,a];</pre>
## brood years 2:(n_yrs-age_min)
for(t in 2:(n_yrs-age_min+n_fore)) {
  ## predicted recruits in BY t
  covar[t] <- inprod(gamma, mod_cvrs[t,]);</pre>
  ln_BH_a[t] <- mu_BH_a + covar[t];</pre>
  tot_ln_Rec[t] ~ dnorm(E_ln_Rec[t], tau_r);
  res_ln_Rec[t] <- tot_ln_Rec[t] - E_ln_Rec[t];</pre>
  w[t] <- phi * w[t-1] + res_ln_Rec[t];</pre>
  ## median of total recruits
  tot_Rec[t] <- exp(tot_ln_Rec[t]);</pre>
  ## R/S
  ln_RS[t] <- tot_ln_Rec[t] - ln_Sp[t];</pre>
  ## brood-yr recruits by age
  for(a in 1:A) {
    Rec[t,a] <- tot_Rec[t] * pi_vec[t,a];</pre>
} ## end t loop over year
## get predicted calendar year returns by age
## matrix Run has dim [(n_yrs-age_min) x A]
## step 1: incomplete early broods
## first cal yr of this grp is first brood yr + age_min + age_skip
for(i in 1:(age_max-age_min-age_skip)) {
  ## projected recruits
  for(a in 1:(i+age_skip)) {
    Run[i,a] <- Rec[(age_skip+i)-a+1,a];</pre>
  ## imputed recruits
  for(a in (i+1+age_skip):A) {
    lnRec[i,a] ~ dnorm(Rec_mu,Rec_tau);
    Run[i,a] <- exp(lnRec[i,a]);</pre>
  }
  ## total run size
  tot_Run[i+age_min+age_skip] <- sum(Run[i,1:A]);</pre>
  ## predicted age-prop vec for multinom
  for(a in 1:A) {
```

```
age_v[i,a] <- Run[i,a] / tot_Run[i+age_min];</pre>
    }
    ## multinomial for age comp
    dat_age[i,1:A] ~ dmulti(age_v[i,1:A],dat_age[i,A+1]);
    lp_age[i] <- logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]);</pre>
  ## step 2: info from complete broods
  ## first cal yr of this grp is first brood yr + age_max
  for(i in (A-age_skip):(n_yrs-age_min-age_skip+n_fore)) {
    for(a in 1:A) {
      Run[i,a] <- Rec[(age_skip+i)-a+1,a];</pre>
    ## total run size
    tot_Run[i+age_min+age_skip] <- sum(Run[i,1:A]);</pre>
    ## predicted age-prop vec for multinom
    for(a in 1:A) {
      age_v[i,a] <- Run[i,a] / tot_Run[i+age_min];</pre>
    }
    ## multinomial for age comp
    dat_age[i,1:A] ~ dmulti(age_v[i,1:A],dat_age[i,A+1]);
    #lp_age[i] <- logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]);</pre>
    lp_age[i] <- ifelse(i < n_yrs-age_min-age_skip+n_fore,</pre>
    logdensity.multi(dat_age[i,1:A],age_v[i,1:A],dat_age[i,A+1]),0)
  ## get predicted calendar year spawners
  ## first cal yr is first brood yr
  for(t in 1:(n_yrs+n_fore)) {
    ## obs model for spawners
    # Sp[t] <- max(10,tot_Run[t] - dat_harv[t]);
    est_harv[t] = h_rate[t] * tot_Run[t];
    dat_harv[t] ~ dlnorm(log(est_harv[t]), 20);
    Sp[t] = tot_Run[t] - est_harv[t];
    ln_Sp[t] \leftarrow log(Sp[t]);
    ln_dat_esc[t] ~ dnorm(ln_Sp[t], tau_s);
    lp_esc[t] <- ifelse(t < n_yrs + 1,logdensity.norm(ln_dat_esc[t],ln_Sp[t], tau_s),0);</pre>
} ## end model description
 ", file=file.path(jagsdir, "IPM_BH_cov_AR_resid.txt"))
```

Fitting the models and generating the one year ahead forecasts

Before fitting the model in JAGS, we need to specify the MCMC control parameters.

```
## 1. MCMC control params
mcmc_ctrl <- list(</pre>
```

```
chains = 4,
  length = 200000, #5e5,
  burn = 100000, #2e5,
 thin = 100 \# 400
## total number of MCMC samples after burnin
mcmc_samp <- mcmc_ctrl$length*mcmc_ctrl$chains/mcmc_ctrl$thin</pre>
## empty list for fits
n_{mods} <-3
## empty list for fits
mod_fits <- vector("list", n_mods*(n_forecasts+1))</pre>
models <- c("IPM_BH_cov_MA1_AR1","IPM_BH_cov_AR","IPM_BH_cov_AR_resid")</pre>
## counter to index fitted jags models (33 in total: 3 models x 11 1 year ahead forecasts including upc
## return year)
t <- 1
for(n in 1:n_mods){
  ## counter to index data to feed model for year specific forecasts
  ## first forecast will be for 10 years prior to the most recent return year;
  ## last forecast will be current forecast for the upcoming return year
  c <- 0
  #n <-2
  model <- models[n]
  for(i in 1:(n forecasts+1)){
    if(file.exists(file.path(savedir, paste(model,"_","y",i,".rds",sep = "")))) {
      mod_fits[[t]] <- readRDS(file.path(savedir, paste(model,"_","y",i,".rds",sep = "")))</pre>
      c <- c + 1
      t < -t + 1
  } else { ## else, fit & save
        ## cnt & time stamp
        cat("Count =", t, "; Time =", round(((proc.time()-timer_start)/60)["elapsed"], 1), "\n",
            file="cnt_time.txt", append=TRUE)
        #range of years. Last year in range
        dat_yrs <- seq(yr_frst,(yr_last - n_forecasts + c),1)</pre>
        ## number of years of data
        n_yrs <- length(dat_yrs)</pre>
        ## get first & last years
        yr_frst_forecast <- min(dat_yrs)</pre>
        yr_last_forecast <- max(dat_yrs)</pre>
        ## get escapement data
        dat_esc_forecast <- dat_esc[which(dat_esc$year %in% dat_yrs),]</pre>
        ## log of escapement
        ln_dat_esc <- c(log(dat_esc_forecast$escapement),rep(NA,n_fore))</pre>
```

```
## get age data
dat_age_forecast <- dat_age[which(dat_age$year %in% dat_yrs),]</pre>
## drop year col & first age min+age skip rows
dat_age_forecast <- dat_age_forecast[-(1:(age_min+age_skip)),-1]</pre>
## add row(s) of NA's for forecast years
if(n fore > 0) {
  dat_age_forecast <- rbind(dat_age_forecast,</pre>
                             matrix(0, n_fore, A,
                                     dimnames = list(n_yrs+seq(n_fore),colnames(dat_age_forecast)
}
## total num of age obs by cal yr
dat_age_forecast[,"sum"] <- apply(dat_age_forecast, 1, sum)</pre>
## row indices for any years with no obs age comp
idx_NA_yrs <- which(dat_age_forecast$sum<A, TRUE)</pre>
## replace 0's in yrs w/o any obs with NA's
dat_age_forecast[idx_NA_yrs,(1:A)] <- NA</pre>
## change total in yrs w/o any obs from 0 to A to help dmulti()
dat_age_forecast[idx_NA_yrs,"sum"] <- A</pre>
## convert class
dat_age_forecast <- as.matrix(dat_age_forecast)</pre>
## get harvest data
dat harv forecast <- dat harv[which(dat harv$year %in% dat yrs),]
## drop year col & first age_max rows
dat_harv_forecast <- c(dat_harv_forecast$catch,rep(NA,n_fore))</pre>
## get covariate data
dat_cvrs_forecast <- dat_cvrs[which(dat_cvrs$year <= yr_last + n_fore - age_min),1:4]</pre>
## drop year col
dat_cvrs_forecast <- dat_cvrs_forecast[,-1]</pre>
## transform the covariates to z-scores
scl_cvrs_forecast <- scale(dat_cvrs_forecast)</pre>
## total number of covariates
n_cov <- dim(dat_cvrs_forecast)[2]</pre>
## ----jags setup-----
## 1. Data to pass to JAGS
dat_jags <- list(dat_age = dat_age_forecast,</pre>
                 ln_dat_esc = ln_dat_esc,
                  dat_harv = dat_harv_forecast,
                  A = A
                  age_min = age_min,
                  age_max = age_max,
                  age_skip = age_skip,
                 n_yrs = n_yrs,
                 n_fore = n_fore)
## 2. Model params/states for JAGS to return
      These are specific to the process model,
##
      so we define them in 'par_jags' below.
```

```
if(model == "IPM_BH_cov_AR" | model == "IPM_BH_cov_AR_resid"){
  init_vals_cov <- function() {</pre>
  list(alpha = 5,
       beta_inv = exp(mean(ln_dat_esc, na.rm = TRUE)),
       gamma = rep(0, 3),
       pi_tau = 10,
       pi_eta = rep(1,A),
       pi_vec = matrix(c(0.01, 0.35, 0.47, 0.15, 0.01, 0.01),
                        n_yrs-age_min+n_fore, A,
                        byrow = TRUE),
       Rec_mu = log(1000),
       Rec_sig = 0.1,
       tot \ln Rec = rep(\log(1000), n \text{ yrs - age min + n fore}),
       phi_prior = 0.5,
       innov_1 = 0)
  }
  ## params/states to return
  par_jags<- c("alpha", "E_BH_a", "ln_BH_a",</pre>
                 "beta",
                 "gamma"
                 "Sp", "Rec", "tot_ln_Rec", "ln_RS",
                 "pi_eta", "pi_tau",
                 "sigma_r", "sigma_s", "w", "res_ln_Rec",
                 "lp_age","lp_esc","phi"
}else{
  init_vals_cov <- function() {</pre>
    list(alpha = 5,
         beta_inv = exp(mean(ln_dat_esc, na.rm = TRUE)),
         gamma = rep(0, 3),
         pi_tau = 10,
         pi_eta = rep(1,A),
         pi_vec = matrix(c(0.01, 0.35, 0.47, 0.15, 0.01, 0.01),
                          n_yrs-age_min+n_fore, A,
                          byrow = TRUE),
         Rec_mu = log(1000),
         Rec_sig = 0.1,
         tot_ln_Rec = rep(log(1000), n_yrs - age_min + n_fore),
         phi_prior = 0.75,theta_res_prior = 0.75,
         innov 1 = 0)
    }
  ## params/states to return
  par_jags <- c("alpha","E_BH_a","ln_BH_a",</pre>
                 "beta",
                 "gamma",
                 "Sp", "Rec", "tot_ln_Rec", "ln_RS",
                 "pi_eta", "pi_tau",
                 "sigma_r", "sigma_s", "res_ln_Rec", "w", "theta_res", "phi",
                 "lp_age","lp_esc"
```

```
}#endif
        ## set of multi-covariate models
        cset <- colnames(scl_cvrs_forecast)</pre>
        dat jags$n cov <- length(cset)</pre>
        dat_jags$mod_cvrs <- scl_cvrs_forecast[, cset]</pre>
        ## fit model & save it
        \# mod\_fits[[t]] \leftarrow fit\_jags(paste(model,".txt",sep = ""), dat\_jags, par\_jags,
                                       init_vals_cov, mcmc_ctrl)
        mod_fits[[t]]<-fit_jags2(model=paste(model,".txt",sep = ""),</pre>
                             data=dat_jags,
                             params=par_jags,
                             inits=init_vals_cov,
                             ctrl=mcmc_ctrl
        saveRDS(mod_fits[[t]], file.path(savedir,paste(model,"_","y",i,".rds",sep = "")))
        c < - c + 1
        t <- t + 1
    }## end if
  }##next forecast year(i)
}## next model(n)
```

Save the output for all of the forecasts.

```
# save(mod_fits, file = file.path(savedir, "forecasts.rda"))
```

Model diagnostics

Here is a table of the Gelman & Rubin statistics (R_{hat}) for the estimated parameters. Recall that we set an upper threshold of 1.1, so values larger than that deserve some additional inspection.

```
colnames(summary_stats2)[1:4]<-c("rhat","neff","mean","sd")
write.csv(summary_stats2,file.path(savedir,"Summary_stats_AR1_MA1.csv"))</pre>
```

Model diagnostics

Here is a table of the Gelman & Rubin statistics (R_{hat}) for the estimated parameters. Recall that we set an upper threshold of 1.1, so values larger than that deserve some additional inspection.

```
## params of interest
par_conv <- c("alpha","beta",paste0("gamma[",seq(3),"]"),</pre>
               "sigma_r", "sigma_s", "pi_tau", "phi", paste0("pi_eta[", seq(A-1), "]"))
## Gelman-Rubin
\#gelman.diag(mod\_fits[[2]][,par\_conv])
## Autocorrelation
# t(round(autocorr.diag(mod_fits[[2]][,par_conv],
                         lags = seg(mcmc_ctrl$thin, 4*mcmc_ctrl$thin, mcmc_ctrl$thin),
#
                         relative=FALSE), 2))
## Use ShinyStan to look at effective draws, Gelman-Rubin, Autocorrelation
fit_bh_cov_AR <- readRDS(file.path(savedir,"IPM_BH_cov_AR_y11.rds"))</pre>
my_sso <- launch_shinystan(as.shinystan(fit_bh_cov_AR))</pre>
summary_stats1<-data.frame(lapply(c("rhat", "neff", "mean", "sd", "quantiles"), function(x) retrieve(my_sso,
colnames(summary_stats1)[1:4]<-c("rhat", "neff", "mean", "sd")</pre>
write.csv(summary_stats1,file.path(savedir,"Summary_stats_AR.csv"))
```

Model diagnostics AR1 recruitment residuals

Here is a table of the Gelman & Rubin statistics (R_{hat}) for the estimated parameters. Recall that we set an upper threshold of 1.1, so values larger than that deserve some additional inspection.

```
## params of interest
par_conv <- c("alpha","beta",paste0("gamma[",seq(3),"]"),</pre>
               "sigma_r", "sigma_s", "pi_tau", "phi", paste0("pi_eta[", seq(A-1), "]"))
## Gelman-Rubin
#gelman.diag(mod_fits[[3]][,par_conv])
## Autocorrelation
# t(round(autocorr.diag(mod_fits[[3]][,par_conv],
                         lags = seq(mcmc_ctrl$thin, 4*mcmc_ctrl$thin, mcmc_ctrl$thin),
                         relative=FALSE), 2))
#
## Use ShinyStan to look at effective draws, Gelman-Rubin, Autocorrelation
fit_bh_cov_AR_resid <- readRDS(file.path(savedir,"IPM_BH_cov_AR_resid_y11.rds"))</pre>
my_sso <- launch_shinystan(as.shinystan(fit_bh_cov_AR_resid))</pre>
summary_stats3<-data.frame(lapply(c("rhat", "neff", "mean", "sd", "quantiles"), function(x) retrieve(my_sso,
colnames(summary_stats3)[1:4]<-c("rhat", "neff", "mean", "sd")</pre>
write.csv(summary_stats3,file.path(savedir, "Summary_stats_AR_resid.csv"))
```

Model selection

```
tot_mods <- n_forecasts*n_mods
# get escapement data
dat_esc_forecast <- dat_esc[which(dat_esc$year %in% seq(yr_begin,yr_end,1)),]</pre>
## get harvest data
dat_harv_forecast <- dat_harv[which(dat_harv$year %in% seq(yr_begin,yr_end,1)),]</pre>
## observed terminal run size
obs_trs <- dat_esc_forecast$escapement + dat_harv_forecast$catch
mod_res<- do.call("rbind", mod_fits)</pre>
c <-1
pred_trs <- NULL</pre>
for(n in 1:n_mods){
  #n <- 1
  pred_esc <- NULL</pre>
  for(i in 1:(n_forecasts)){
    #i <- 1
  p_dat <- mod_res[[c]][,grep("Sp", colnames(mod_res[[c]]))]</pre>
  p_dat <- round(median(p_dat[,dim(p_dat)[2]]))</pre>
  pred_esc[i] <- p_dat</pre>
  c <- c+1
  }
  pred_trs_mod <- pred_esc + dat_harv_forecast$catch</pre>
  pred_trs <- cbind(pred_trs,pred_trs_mod)</pre>
  \#names(pred\_trs) \leftarrow paste(models[n], "\_", "pred\_trs", sep = "")
}
colnames(pred_trs) <- models</pre>
## compute model performance statistics
Error <- obs_trs - pred_trs</pre>
SE <- Error<sup>2</sup>
PE <- Error/obs_trs
APE <- abs(PE)
RMSE <- apply(SE,2,function(x){sqrt(mean(x))})</pre>
MPE <- apply(PE,2,function(x){mean(x)})</pre>
MAPE <- apply(APE,2,function(x){mean(x)})</pre>
model_selection <- cbind(RMSE,MPE,MAPE)</pre>
model_selection <- data.frame(model_selection)</pre>
model_selection[,"RMSE_weight"] <- (1/model_selection[,"RMSE"])/sum(1/model_selection[,"RMSE"])</pre>
```

```
## extract median 2020 forecast from each model
n_yrs <- length(dat_yrs)</pre>
fit_bh_cov_MA1_AR1 <- readRDS(file.path(savedir, "IPM_BH_cov_MA1_AR1_y11.rds"))</pre>
fit_bh_cov_AR1 <- readRDS(file.path(savedir, "IPM_BH_cov_AR_y11.rds"))</pre>
fit_bh_cov_resid <- readRDS(file.path(savedir, "IPM_BH_cov_AR_resid_y11.rds"))</pre>
mod_res_MA1_AR1 <- do.call("rbind", fit_bh_cov_MA1_AR1)</pre>
mod_res_AR1 <- do.call("rbind", fit_bh_cov_AR1)</pre>
mod_res_AR1_resid <- do.call("rbind", fit_bh_cov_resid)</pre>
p_dat_AR1MA1 <- mod_res_MA1_AR1[,grep("Sp", colnames(mod_res_MA1_AR1))]</pre>
p_{dat_AR1MA1_2020} \leftarrow round(quantile(p_{dat_AR1MA1_n, yrs+n_fore), probs = c(0.025, 0.5, 0.975)))
p_dat_AR1 <- mod_res_AR1[,grep("Sp", colnames(mod_res_AR1))]</pre>
p_{dat_AR1_2020} \leftarrow round(quantile(p_{dat_AR1_n_yrs_n_fore),probs = c(0.025,0.5,0.975)))
p_dat_AR1_resid <- mod_res_AR1_resid[,grep("Sp", colnames(mod_res_AR1_resid))]</pre>
mod res AR1 resid 2020 <- round(quantile(p dat AR1 resid[,n yrs+n fore],probs = c(0.025,0.5,0.975)))
model_selection[,"2020_forecast"] <- c(p_dat_AR1MA1_2020[2],p_dat_AR1_2020[2],mod_res_AR1_resid_2020[2]
weighted_forecast <- sum(model_selection[,"2020_forecast"]*model_selection[,"RMSE_weight"])</pre>
print(model_selection)
##
                             RMSE
                                          MPE
```

```
## RMSE MPE MAPE RMSE_weight 2020_forecast
## IPM_BH_cov_AR 2339.910 -0.1004097 0.3727213 0.4351989 5094
## IPM_BH_cov_AR_resid 3627.144 0.3329551 0.3376668 0.2807516 2158
```

Via loo() and compare() with full table of results. Note that elpd_diff will be negative (positive) if the expected predictive accuracy for the first (second) model is higher.

```
LOOIC <- vector("list", n_mods)
model_indices \leftarrow c(11,22,33)
## extract log densities from JAGS objects
for(i in 1:n_mods) {
  #i <- 1
  ## convert mcmc.list to matrix
  tmp_lp <- as.matrix(mod_fits[[model_indices[i]]])</pre>
  ## extract pointwise likelihoods
  tmp_lp <- tmp_lp[,grepl("lp_", colnames(tmp_lp))]</pre>
  ## if numerical underflows, convert -Inf to 5% less than min(likelihood)
  if(any(is.infinite(tmp_lp))) {
    tmp_lp[is.infinite(tmp_lp)] <- NA</pre>
    tmp_min <- min(tmp_lp, na.rm = TRUE)</pre>
    tmp_lp[is.na(tmp_lp)] <- tmp_min * 1.05</pre>
  }
  ## calculate LOOIC
```

```
LOOIC[[i]] <- loo(tmp_lp)
}
## compute pseudo weights
model_weights <- loo_model_weights(LOOIC, method = "pseudobma",optim_method = "BFGS", optim_control = 1
## LOOIC for all data
tbl LOOIC <- round(loo compare(x = LOOIC), 2)
rownames(tbl_L00IC) <- sub("model", "", rownames(tbl_L00IC))</pre>
tbl_LOOIC <- tbl_LOOIC[order(as.numeric(rownames(tbl_LOOIC))), ]</pre>
tbl_LOOIC \leftarrow cbind(model = c("B-H", "B-H", "B-H"),
                   error = c("MA1_AR1","AR1","AR1_resid"),
                   as.data.frame(tbl_LOOIC),pseudo_bma_weight = as.matrix(model_weights))
tbl_LOOIC[order(tbl_LOOIC[,"looic"]), ]
##
    model
              error elpd_diff se_diff elpd_loo se_elpd_loo p_loo se_p_loo looic se_looic
      B-H
                         0.0 0.00 -395.32 48.96 138.77 10.38 790.64
## 2
      B-H AR1_resid
                          -3.7
                                 5.07 -399.02
                                                     49.15 143.49
                                                                      11.74 798.04
                                                                                      98.30
                               5.33 -399.72
      B-H MA1 AR1
                          -4.4
                                                      48.96 142.71
                                                                      12.32 799.43
                                                                                      97.92
##
   pseudo_bma_weight
## 2
            0.6800957
## 3
            0.1789055
## 1
            0.1409988
## best model
best_i <- which(tbl_LOOIC[,"looic"] == min(tbl_LOOIC[,"looic"]))</pre>
best_fit <- mod_fits[[best_i]]</pre>
```

These results show that the Beverton-Holt model with AR1 error has the lowest LOOIC value. All results will be derived from model averaging based on pseudo bayesian model average weights.

Model Selection Via Approximate leave-future-out cross validation

Description here

```
# # m=1
# N <- 43
# L <- 20
# k_thres <- 0.7
# approx_elpds_1sap <- rep(NA, N)
#
# # more stable than log(sum(exp(x)))
# log_sum_exp <- function(x) {
# max_x <- max(x)
# max_x + log(sum(exp(x - max_x)))
# }
# # more stable than log(mean(exp(x)))
# log_mean_exp <- function(x) {
# log_sum_exp(x) - log(length(x))</pre>
```

```
# }
#
# # compute log of raw importance ratios
# # sums over observations *not* over posterior samples
# sum_log_ratios <- function(loglik, ids = NULL) {</pre>
# if (!is.null(ids)) loglik <- loglik[, ids, drop = FALSE]
#
  rowSums(loglik)
# }
#
# # for printing comparisons later
# rbind_print <- function(...) {</pre>
# round(rbind(...), digits = 2)
# }
# #extract pontwise log likelihoods
# tmp_lp <- as.matrix(mod_fits[[m]])</pre>
# ## extract pointwise likelihoods
# tmp_lp <- tmp_lp[,grepl("lp_", colnames(tmp_lp))]</pre>
# ## if numerical underflows, convert -Inf to 5% less than min(likelihood)
# if(any(is.infinite(tmp_lp))) {
# tmp_lp[is.infinite(tmp_lp)] <- NA</pre>
# tmp_min <- min(tmp_lp, na.rm = TRUE)</pre>
# tmp_lp[is.na(tmp_lp)] \leftarrow tmp_min * 1.05
# }
#
# tmp_lp<-tmp_lp[,grepl("esc", colnames(tmp_lp))]</pre>
# #qet yrs assoc
\# names_loglik<-data.frame(strsplit(colnames(tmp_lp), "\\[/\\]"))
# yrnames<-as.numeric(names_loglik[2,])</pre>
# loglik <- matrix(NA,ncol=N,nrow=dim(tmp_lp)[1])</pre>
# for(i in 1:N){
  if(!is.null(ncol(tmp_lp[,yrnames==i]))){
      loglik[,i] = apply(tmp_lp[,yrnames==i],1,sum)
   }else(loglik[,i] = tmp_lp[,yrnames==i])
# }
#
# # initialize the process for i = L
# past <- 1:L
# oos <- L + 1
\# \# df_past \leftarrow df[past, , drop = FALSE]
\# \# df_{oos} \leftarrow df[c(past, oos), , drop = FALSE]
# # fit_past <- update(fit, newdata = df_past, recompile = FALSE)
# #loglik <- log_lik(fit_past, newdata = df_oos, oos = oos)</pre>
# approx_elpds_1sap[L + 1] <- log_mean_exp(loglik[, oos])</pre>
# # iterate over i > L
# i_refit <- L
# refits <- L
# ks <- NULL
# for (i in (L + 1):(N - 1)) {
# past <- 1:i
# oos <-i+1
```

```
#
   # df_past <- df[past, , drop = FALSE]</pre>
   \# df_{oos} \leftarrow df[c(past, oos), drop = FALSE]
#
#
   # loglik <- log_lik(fit_past, newdata = df_oos, oos = oos)</pre>
#
  logratio <- sum_log_ratios(loglik, (i_refit + 1):i)</pre>
   psis_obj <- suppressWarnings(psis(logratio))</pre>
#
  k <- pareto_k_values(psis_obj)</pre>
   ks \leftarrow c(ks, k)
   # if (k > k_thres) {
#
#
       # refit the model based on the first i observations
#
   \# i_refit <- i
#
   # refits <- c(refits, i)</pre>
#
        fit_past <- update(fit_past, newdata = df_past, recompile = FALSE)</pre>
    \# \quad loglik \gets log\_lik(fit\_past, newdata = df\_oos, oos = oos)
#
#
   # approx_elpds_1sap[i + 1] <- log_mean_exp(loglik[, oos])</pre>
#
   # } else {
#
      lw <- weights(psis_obj, normalize = TRUE)[, 1]</pre>
#
      approx_elpds_1sap[i + 1] <- log_sum_exp(lw + loglik[, oos])</pre>
#
# }
# approx_elpd_1sap <- sum(approx_elpds_1sap, na.rm = TRUE)</pre>
# print(paste("approx LFO =" ,approx_elpd_1sap))
# print(ks)
```