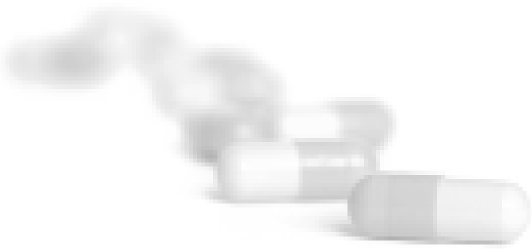

Instrumental Variables

MASTERS ECONOMETRICS: CROSS-SECTION

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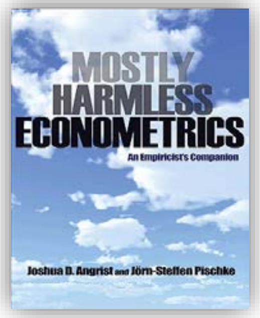

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Random Assignment

$$(y_0, y_1) \perp\!\!\!\perp D_i$$

Selection into treatment is random. So it should be correlated with X's or unobservables.

X's not needed, but could add to efficiency

- Comparing means

Selection on Observables

Cond on X's

$$(y_0, y_1) \perp\!\!\!\perp D_i | X$$

Controlling on X's takes care of selection bias.

Focus on X's so you are able to control correctly

Ensure overlap

- OLS or Matching

Selection on Unobservables

(Y_0, Y_1) not idependent of $D_i | X$

Some unobservables that determine treatment, also effect potential outcomes

Controlling on X's does not take care of all sample selection bias.

- IV's, RDD's, FE (and RCT's).

Outline

1. Potential outcome (revisited) ← add instrument
2. Homogenous Treatment Effects
   i. *Finding an Instrument*
   ii. *Estimating Treatment Effects*
3. Heterogenous Treatment Effects
   i. *Compliance Types*
   ii. *Identifying assumptions*
   iii. *Validity of assumptions*
4. RCT's with non-Compliance

POTENTIAL OUTCOMES

Potential Outcomes Approach

Y_i observed outcome of individual i

- Y_{0i} potential outcome of individual i is not treated (in control)
- Y_{1i} potential outcome of individual i is treated

D_i dummy variable denoting whether individual was treated / variable of interest

X_i set of characteristics or covariates / conditioning variables.

Y_0	Y_1	D	X
1	1	0	1
0	0	1	1
0	0	1	0
0	1	0	0

The relationship between the **observed outcome** and the **potential outcomes** can be set up in two ways:

1. The conventional way:

$$Y_i = \begin{cases} Y_{1i} \ if \ D_i = 1 \\ Y_{0i} \ if \ D_i = 0 \end{cases}$$

Control, not treated

Treated

2. The switching regression:

$$Y_i = Y_{0i} + (Y_{1i} - Y_{0i})D_i$$

causal effect

Potential Outcomes Approach

Potential outcomes and potential treatments both unobserved;
Data only includes $(Z, D, Y, \boldsymbol{X})$

Y_0	Y_1	D_0	D_1	X	Z
1	1	0	0	1	1
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	1	0	0

Y_i observed outcome of individual i

- Y_{0i} potential outcome of individual $\mathbf{i}$ is not treated (in control)
- Y_{1i} potential outcome of individual $\mathbf{i}$ is treated

D_i dummy variable denoting whether individual was treated / variable of interest

- D_{0i} potential outcome of individual $\mathbf{i}$ is instrument is turned off
- D_{1i} potential outcome of individual $\mathbf{i}$ is instrument is on

Z_i instrumental variable.

X_i set of characteristics or covariates / conditioning variables.

The relationship between the **observed outcome** and the **potential outcomes** can be set up in two ways:

1. The conventional way: $Y_i = \begin{cases} Y_{1i} & if\ D_i = 1 \\ Y_{0i} & if\ D_i = 0 \end{cases}$ and $D_i = \begin{cases} D_{1i} & if\ Z_i = 1 \\ D_{0i} & if\ Z_i = 0 \end{cases}$

2. The switching regression: $Y_i = Y_{0i} + (Y_{1i} - Y_{0i})D_i$ and $D_i = D_{0i}(1 - Z_i) + D_{1i}Z_i$

Example: Veterans

Angrist (1990) studied the effect of military veteran status on earnings. Each year from 1970 to 1972, random numbers were assigned to birth dates of 19-year-old American men. Those whose lottery numbers were below a certain cutoff were eligible for draft, while those with numbers above the cutoff could not be drafted.

This allows construction of a binary draft-eligible variable that is positively correlated with likelihood of serving.
In practice many eligible men were not drafted while many draft-ineligible men still volunteered for military service.

Drafted: $Z_i = \begin{cases} 1 & if\ eligable \\ 0 & if\ not\ eligable \end{cases}$

Went to war: $D_i = \begin{cases} D_{1i} & if\ Z_i = 1 \\ D_{0i} & if\ Z_i = 0 \end{cases}$ ← two potential treatment statuses

Future Earnings: $Y_i = \begin{cases} Y_{1i} & if\ D_i = 1 \\ Y_{0i} & if\ D_i = 0 \end{cases}$ ← two potential outcomes

> value of Z determines value of D, which in turn determines value of Y.

Homogenous Treatment Effects

Finding an Instrument

If we assume that the relationship between treatment and our outcome variable is linear, and that treatment effects are all constant then:

$$Y_i = \rho D_i + X_i + e_i$$

If exogeneity holds: OLS will give us an unbias measure of ρ

If exogeneity fails: OLS will not give us an unbias measure of ρ

If we however find a vaild instrument Z_i for D_i we may recover an unbias estimate of ρ .

Finding an Instrument

Instrument

A variable, Z_i, can be used as an instrument for D_i if Z_i satisfies both the exclusion restriction (independence assumption) and rank condition (relevance assumption).

Exclusion Restriction

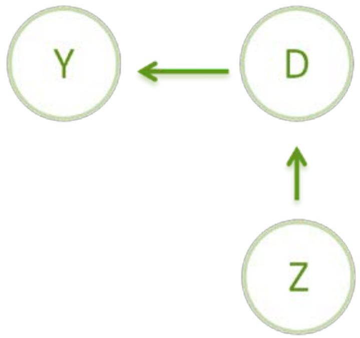
Z_i is uncorrelated with e_i

Z_i should have no effect on our outcome variable Y_i once we control for D_i. This will not be the case if Z_i is correlated with any of the unobserved factors in our error term.

Rank Condition

Z_i is correlated with D_i

Z_i should have an effect on treatment D_i.



Combining the two assumption:
Z_i is allowed to impact Y_i, but not directly, only through its affect on D_i.

Note: The Rank Condition is testable. The Exclusion Restriction is not. So you will have to convince people why you think it holds

Example: Effect of Class Attendance on Final Mark

Instrument needs to satisfy:

Exclusion restriction: ← not testable

Instrument should not affect the **final mark**, except through its effect on **class attendance**.

Relevance: ← testable

Instrument should have a significant effect on **class attendance**.

Possible instruments?

- Distance from living quarters to campus
- Quality of lecturer that student is appointed to
- Time of the day of the lecture you were dealt into
- Class conflicts

Example: Mother’s Smoking on Birth Weight

Instrument needs to satisfy:

Exclusion restriction:

Instrument should not affect the **birth weight**, except through its effect on **mothers smoking**.

Relevance:

Instrument should have a significant effect on **mother’s smoking**.

Possible instruments?

- Whether husband smokes?
- Price of cigarettes?
- Telling parents about impact?

Example: Effect of Policing on Crime

Instrument needs to satisfy:

Exclusion restriction:

Instrument should not affect the **crime**, except through its effect on **policing**.

Relevance:

Instrument should have a significant effect on **policing**.

Possible instruments?

- Local electoral cycles
- Proximity to doughnut shops

Estimating Treatment Effects

Once we have established that we have a valid instrument, there are a couple of ways we can use the instrument to recover the average treatment effect:

1. Two stage Least Squares

   Estimate $D_i = \pi_0 + \pi_1 Z_i + v_i$.

   Then regress Y_i on the predicted value D_i to recover τ.

```
regr veteran draftnr
predict veteran_hat
regr wage veteran_hat
```

2. Indirect Least Squares

   Regress D_i on Z_i to get $\hat{\pi}_1$

   Regress Y_i on D_i to get $\hat{\tau}$.

   The ratio of the two values gives us an estimate of the TE, $\tau_{ILS} = \frac{\tau}{\pi_1}$

```
regr veteran draftnr
regr wage veteran
di … / …
```

3. Wald-Estimator

   If D_i and Z_i are both binary, then we don't even have to run a regression

   The coefficient represent the difference in means.

$$\tau_{WALD} = \frac{\pi_z}{\gamma_z} = \frac{E[Y_i\,|Z = 1] - E[Y_i|Z_i = 0]}{E[D_i|Z = 1] - E[D_i|Z_i = 0]}$$

```
tab veteran draftnr
tab wage draftnr
```

Heterogenous Treatment Effect

Heterogenous Treatment Effects

When TE is heterogenous then the situations becomes slightly more complicated

The IV-estimates of before will now only be relevant for a certain subpopulation and will also require stronger assumptions than under homogenous TE's

Ideal

Y_0	Y_1	D_0	D_1	X	Z
1	1	0	0	1	1
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	1	0	0

Reality

Y	D	X	Z
1	0	1	1
0	1	1	1
0	1	0	0
1	0	0	0

Compliance Types

How individuals respond to draw of instrument in terms of choice of treatment divides population into 4 groups:

1. Always-Takers (AT) - *will take up treatment regardless of instrument* $(D_{0i} = 1 \,\&\, D_{1i} = 1)$
2. Never-Takers (NT) - *will not take up treatment regardless of instrument* $(D_{0i} = 0 \,\&\, D_{1i} = 0)$
3. Compliers (C) - *will only take up treatment if instrument is 1* $(D_{0i} = 0 \,\&\, D_{1i} = 1)$
4. Defiers (D) - *will only take up treatment if instrument is 0* $(D_{0i} = 1 \,\&\, D_{1i} = 0)$

Treat if D=0? Treat if D=1?

Unfortunately, we can not directly tell what type an individual is just by looking at D and Z, since we can only observe an individual under a single value of Z, not both.

Analogous to not observing both y_0 and y_1

Without any further assumptions, each combination Z and D corresponds to 2 potential compliance types:

	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	C/NT	D/NT
$D_i = 1$	D/AT	C/AT

can rule out 2 types for each combo

e.g: suppose we see someone who was drafted and who is at war, we can immediately tell that he is either a complier or a AT. He definitely wouldn't be a NT or a defier.

Identifying Assumptions
(required for LATE-theorem)

The LATE Theorem

If our instrument is:

1. randomly assigned
2. only affects our outcome through its effect on our endogenous variable
3. has a significant impact on take-up of treatment, and
4. does not deter anyone from treatment.

Assumption 1 and 2 deals with how the instrument is not allowed to impact Y_i (at least not directly),

Assumption 3 and 4 deals with how the instrument needs to impact treatment status, D_i.

Then we can use our instrument to estimate the local average treatment effect on our subsample of compliers.

$$\frac{E[Y_i\,|Z=1]-E[Y_i|Z_i=0]}{E[D_i|Z=1]-E[D_i|Z_i=0]}=E[Y_{1i}-Y_{0i}|D_{1i}>D_{0i}]$$

Wald-estimator

The LATE Theorem

If our instrument satisfies:

1. Random Assignment: $(Y_0, Y_1, D_0, D_1) \perp\!\!\!\perp Z$ before we had $(Y_0, Y_1) \perp\!\!\!\perp D$
2. Exclusion Restriction: $Y(D, Z) = Y(D)$
3. Instrument Relevance: $D_1 > D_0$ for at least some observations
4. Monotonicity: $D_1 \geq D_0$ for all observations

Then we can the instrument to estimate the local average treatment effect on our subsample of compliers.

$$\frac{E[Y_i \mid Z = 1] - E[Y_i \mid Z_i = 0]}{E[D_i \mid Z = 1] - E[D_i \mid Z_i = 0]} = E[Y_{1i} - Y_{0i} \mid D_{1i} > D_{0i}]$$

Identifying Assumptions

1. Random Assignment: $(Y_0, Y_1, D_0, D_1) \perp\!\!\!\perp Z$

before we had $(Y_0, Y_1) \perp\!\!\!\perp D|\boldsymbol{X}$

The instrument Z is uncorrelated to all four potential outcomes of y, and both potential outcomes of D

Our variables needs to be "as good as randomly distributed"

Distribution of potential outcomes
And potential take-up values should look same for those who were drafted and who were not

2. Exclusion Restriction: $Y(D, Z) = Y(D)$

If Z can affect Y only through D then Z adds no information about Y once its effect D has been taken into account.

We expect that $Y(D = 1, Z = 0) = Y(D = 1, Z = 1)$ and $Y(D = 0, Z = 0) = Y(D = 0, Z = 1)$.

Excludability helps us narrow down the set of potential outcome that we need to consider

being drafted has no effect on future earnings except through going to war

Identifying Assumptions

3. First Stage Power / Instrument Relevance

$D_1 > D_0$ for at least some observations

The instrument must affect treatment for at least some individuals.

This is a testable assumption, so no need for thought experiment.

The assumption requires $\mathrm{E}[D|Z=1] \neq E[D|Z=0]$

4. Monotonicity:

$D_1 \geq D_0$ for all observations

The instrument may not effect everyone, but those who are affected are all affected in the same direction.

Ask yourself: "Is there any reason why someone would accept treatment if $Z=0$ and not when $Z=1$?"

If assumption gets violated (if we have defiers), then the LATE becomes a weighted average of the actual TE experience by compliers and the opposite TE experienced by defiers.

Identifying Assumptions

Monotonicity assumption implies that there are no defiers in the sample.

	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	C/NT	NT
$D_i = 1$	AT	C/AT

If you did not go BUT WERE DRAFTED then you are NT

If you went to war BUT WERE NOT DRAFTED then you are AT

This means that:

All individuals with $Z = 0$ and $D = 1$ are always-takers and

All individuals with $Z = 1$ and $D = 0$ are never-takers.

Got $\mathrm{E}[Y|AT\ with\ Z = 0]$ & $\mathrm{E}[Y|NT\ with\ Z = 1]$

If instrument Z is **randomly assigned** then:

Ratio of AT's and expected outcome for AT's to the left and right would be identical

Ratio of NT's and expected outcome for NT's to the left and right would be identical

Back out ratio of compliers

Back out TE = $\mathrm{E}[Y|Compliers\ with\ Z = 0]\ -\ \mathrm{E}[Y|Complierswith\ Z = 1]$

Identifying Assumptions (not for exam)

Let π_c, π_n and π_a represent population proportions of compliers, never-takers and always-takers.

If $Z = 0$: then $\pi_n + \pi_c$ of population will choose $D = 0$ and π_a will choose $D = 1$.

This means that:

$$P(D = 0|Z = 0) = \pi_n + \pi_c$$
$$P(D = 1|Z = 0) = \pi_a$$

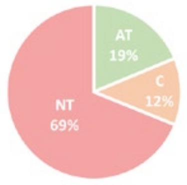
If $Z = 1$: then π_n of population will choose $w = 0$ and $\pi_c + \pi_a$ will choose $D = 1$.

This means that:

$$P(D = 0|Z = 1) = \pi_n$$
$$P(D = 1|Z = 1) = \pi_c + \pi_a$$

Can rewrite in terms of proportions:

$$\pi_a = P(D = 1|Z = 0)$$
$$\pi_c = P(D = 1|Z = 1) - P(D = 1|Z = 0)$$
$$\pi_n = P(D = 0|Z = 1) = 1 - P(D = 1|Z = 1)$$



Since $E(D|Z = 0)$ and $E(D|Z = 1)$ can be estimated from sample data, so can π_a, π_c and π_n.

Validity of Assumptions

Example: Veterans

Are identifying assumptions valid for military veteran example?

Random Assignment:

Random assignment of draft eligibility means independence assumption is very convincing.
Would not expect individuals who were born on low-lottery days to have different potential earnings distributions or potential take-up probabilities than high-lottery individuals.

distribution of never-takers, always-takers and compliers shouldn't vary by draft eligibility.

Exclusion restriction:

More reason to be concerned.
Draft eligible men were allowed to defer their military service if they wanted to proceed with their studies.
If some men decided to get more schooling because they had low lottery numbers and did not want to serve in the military, then this could have affected earnings via education attainment rather than via military service.

Monotonicity:

Is it possible that someone would have volunteered for the army (if draft ineligible) but decided not to because of draft eligibility? Seems unlikely.

Relevance:

Are there individuals who would not have gone to war if they didn't get picked, but ended up going just because they were eligible? Seems plausible. Testable in first-stage regression.

Example: Distance to Hospital

Can you use distance to nearest hospital as IV for effect of hospitalisation on health outcome?

Random Assignment:

Is the distributions of potential (treatment and no-treatment) health outcomes and potential take-up rates the same for those who live near and far from hospitals?

Exclusion restriction:

Living near or far from the hospital has no effect on an individual's health other than through the effect on the likelihood of hospitalisation.

Monotonicity:

No-one should be less inclined to visit a hospital because they live near one

Relevance:

At least some individuals must be more inclined to visit the hospital because they live near one.

Example: Cigarette Prices

Can you use cigarette price hike to measure the effect smoking during pregnancy on a child health?

Random Assignment:

Cigarette prices are not correlated to child health outcomes. Potential child health outcomes should be the same for those who live in areas where prices were and were not hiked.

Exclusion restriction:

Cigarette prices have no other indirect effect on a child's health other than through the effect of maternal smoking while pregnant.

Monotonicity:

None of the pregnant women should be more inclined to smoke due to a price hike of cigarettes

Relevance:

At least some of the women should be less inclined to smoke due to the price hike.

RCTs with non-Compliance

RCTs with non-compliance

Z = "the intent-to-treat", D = treatment

	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	5948	1915
$D_i = 1$	0	865

One-sided non-compliance:
Can't get treated unless z=1

We often end up with one-sided non-compliance when we conduct an RCT where we randomly offered treatment to a proportion of the sample.

> Everyone who were not offered treatment did not receive treatment but not
> But not everyone who were offered treatment took-up treatment
> Simply comparing those who took up treatment with those who did not would deliver a bias results, since there is still some self-selection into treatment.

We have two effects that we could be interested in:

1. Local Average Treatment Effect

Use the offer of treatment as an instrument for actually taking up treatment

LATE estimates the treatment effect for those who would take up treatment if they are offered treatment

2. Intent-to-Treat

Comparing the outcomes for those who were offered treatment to those who were not offered treatment

$$ITT = [Y_i|Z_i = 1] - [Y_i|Z_i = 0]$$

ITT estimates the total benefit of our random assignment (being offered treatment).

In many cases this may the actual intervention: e.g. offering conditional cash transfers or receiving reminder sms's