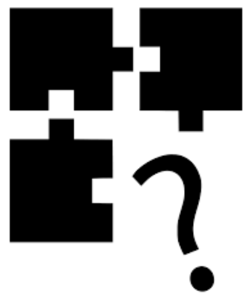
A GAME THEORETIC APPROACH TO DEADLINE ADHERENCE

Microeconomics 871 Essay



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1. Introduction

Various theories of the relationship between lecturer and student show that a student's actions are often influenced by the behaviour of their lecturer (Conrad, 1983), (Conrad, 1991), (Liu & Shi, 2017). Identity-based motivation theory posits that a relationship of trust and leniency between student and lecturer causes both to better reach their goals, increases performance, and improves mental health (Liu & Shi, 2017). Lecturers often require students to complete tasks, including essays, before a deadline. Game theoretic models provide a useful framework to analyse the interactions between a student and lecturer (Osborne, 2004) since it offers insights into when a student may choose to miss a deadline and when a lecturer will give an extension, in the presence of incomplete information.

The paper considers the strategic interaction of students and lecturers in the context of dynamic games of incomplete information. A student is required to submit an assignment with by a certain deadline, but experiences a crisis and must choose whether to hand in either on time or late. Both players have continuous type spaces and a discrete set of actions.

The remainder of the essay is organised as follows. section 2 discusses the literature on games of incomplete information with applications. Section 3 presents our model of deadline adherence. Section 4 analyses the solutions to the game. Section 5 provides an extension of the game and the final section 6 concludes.

2. Games of Incomplete Information

When making decisions, agents often do not have full information (Trabelsi, 2020). Von Neumann & Morgenstern (1944: 30) first used the term *incomplete information* to describe a model in which parts of the normal form structure was unspecified. However, Von Neumann and Morgenstern deemed further research into such a model unimportant (Myerson, 2004). Luce & Adams (1956) disagreed and extended on the incomplete information literature by assuming that each player has a perception of the payoff function of the other player. A player's perception does not have be a true reflection of the other player's payoffs, but it forms the basis of the player's strategy. Harsanyi (1995) developed a general analytical framework for games of incomplete information, which made it practical to solve the games.

Following Harsanyi (1995) approach, players have less than full information about each others payoff functions and, based on Bayesian methodology, both players have expectations in the form of subjective probability distributions. Both players estimate the probability that the other player is of a certain type, subject to the available information. To solve the model, the game of incomplete information will be reinterpreted as a game with complete and imperfect information, through transforming its basic mathematical structure by adding nature as a third player.

3. A Model of Deadline Adherence

The model of deadline adherence sheds some light on how students think about assignment submissions, and how lecturers respond when assignments are submitted late. In the model, a student receives an assignment with a due date set by the lecturer. While the student is working on the assignment, she experiences a crisis and therefore spends less time on the assignment. She has two options: she can hand in the assignment on time or she can hand in late. If she hands in on time, she will get a payoff of $a - c$, where a is her potential pre-crisis mark, and c is the negative academic impact of the crisis. Submitting late gives the student the opportunity to recover some of the cost of the crisis on her final mark. If the lecturer gives her a penalty for late submission, her payoff is $a - \beta c - m$, where m is the percentage point penalty subtracted from her final mark. If the lecturer decides the waive the penalty, the student gets a payoff of $a - \beta c$. The parameter $\beta \in [0, 1]$ represents the type of the student. A low β suggests a strong academic resiliency, whereas a high β suggests low resiliency. The student observes her own type but not the lecturer's type. The lecturer decided whether to give a penalty, m, if a student submits late. The lecturer has disutility from giving a penalty since the student submitted late due to a crisis. The magnitude of her disutility depends on the magnitude of the penalty, m, , and the lecturer's degree of empathy. The parameter $\delta \in [0, 1]$ denotes the lecturer's penalty, describing her type. A higher δ implies that a lecturer is more empathetic. The lecturer observes her own type but not the student's type. If the lecturer does not impose a penalty, he derives benefit from helping a student through a difficult time academically, yielding the payoff δc. The lecturer knows that by waving the penalty, he may be encouraging this student, and other students to hand in late in the future. The lecturer would rather deter late hand-ins, and receives a negative payoff d for not deterring[1] late hand-ins.

The lecturer and student both have continuous type spaces, which are assumed to be independently and randomly chosen by nature at the start of the game from a uniform distribution[2]: $\delta \sim Uniform(0, 1)$ and $\beta \sim Uniform(0, 1)$. The parameters a, c, m & d are all common knowledge. Each player needs to choose her action based on her own type, their beliefs of the other player's type, and the values of parameters a, c, m & d. Figure 3.1 shows the game in extensive form[3] and figure 3.1 shows a summary of the game's parameters.

[1]This deterrent parameter relates to the literature on games of repeated interaction and reputations (Clark & Montgomery, 1998).

[2]A uniform distribution puts equal chance on any of the outcomes between 0 and 1 happening. Other more realistic probability distributions have been omitted for ease of interpretation. Assuming a normal distribution would give a similar interpretation of how changes in parameters affect the outcomes but would needlessly complicate the game.

[3]The simultaneous form game can be found in the appendix 6.1

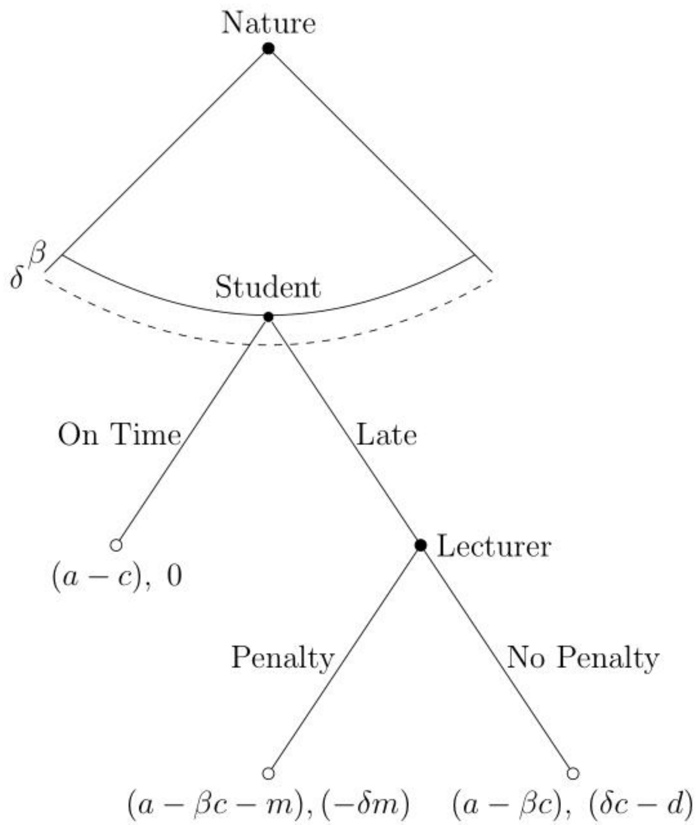

Figure 3.1: This game is dynamic, where nature first chooses the student's and lecturer's types. The dashed line represents information only known by the lecturer and the solid line represents information only known by the student. Then the student moves, deciding to hand in on time or late after experiencing a crisis. If the student hands in late, the lecturer decides to impose a penalty or not.

Parameter	Explanation	Restriction
a	Potential assignment mark	$0 \leq a \leq 1$
c	Cost of crisis to assignment mark	$0 < c \leq 1$
β	Student's type: level of resiliency	$\beta \sim Uniform(0,1)$
m	Mark penalty	$0 < m \leq 1$
δ	Lecturer's type: level of empathy	$\delta \sim Uniform(0,1)$
d	Detterent	$0 < d \leq 1$

Table 3.1: Game Parameters

4. Results and Discussion

We solve for the players' best responses to understand how the lecturer and student will make their decisions given their beliefs. The best response for the student would be to hand in on time if the expected payoff from handing in on time is higher than the expected payoff of submitting late. Defining p as the probability that the lecturer will give a penalty, a student should hand in on time where:

$$\beta > \frac{c - mp}{c}$$

The right hand side is a constant[4]. This implies that there is some threshold value of β, for which a student should hand in on time. If the student believes that the lecturer will give no penalty (i.e. $p = 0$), then she should only hand in on time if $\beta > \frac{c}{c} \Rightarrow \beta > 1$. Since β lies between 0 and 1, the inequality will never hold and she should always hand in late. Intuitively, a student can never do worse by handing in late if there is no penalty[5] but she will do better to hand in late if she is resilient in any way ($\beta < 1$) and can partially recover from the crisis.

However, if the student believes that the lecturer will give a penalty with some positive probability ($p > 0$) then her decision to hand in on time depends on her level of resiliency, the magnitude of the crisis and the size of the penalty. As the cost of the crisis increases, the threshold value increases and the student becomes more likely to hand in late, unless her she has a very low resiliency. If the mark penalty is high, the threshold value is smaller and the student is more likely to hand in on time, unless is very resilient (i.e., β is very low).). We analyse the lecturer's best response rule similarly. The lecturer should impose a penalty where:

$$\delta < \frac{d}{c + m}$$

The threshold value for the lecturer is the ratio between the deterrent factor and the sum of the cost of the crisis and the penalty mark. If the deterrent factor is high relative to the cost of crisis and the penalty mark, then the lecturer is more likely to give a penalty. If the cost of the crisis large and the mark penalty is high relative to the deterrent factor, the lecturer is more likely to give no penalty. A highly empathetic lecturer is more likely to waive the penalty.

While best response analysis is useful, solving for the Bayesian Nash Equilibrium (BNE) is necessary if one wants to understand the outcome of the game and the players' strategies. The full derivations of the solution concepts can be found in the appendix (6).

[4] Since c and m are known, and p is a belief the student holds.

[5] Handing in late is a weakly dominant strategy.

After observing her private type, the student chooses the following hand-in pattern:

$$s_\beta^*(\beta) = \begin{cases} \text{On time} & \text{if} \quad \beta > 1 - \frac{md}{c^2+mc} \\ \text{Late} & \text{if} \quad \beta \leq 1 - \frac{md}{c^2+mc} \end{cases}$$

And the lecturer's BNE strategy is given by:

$$s_\delta^*(\delta) = \begin{cases} \text{Penalty} & \text{if} \quad \delta < \frac{d}{c+m} \\ \text{No penalty} & \text{if} \quad \delta \geq \frac{d}{c+m} \end{cases}$$

When both players play their equilibrium strategies neither has an incentive to deviate, resulting in a BNE. We would interpret the student's BNE strategy similarly to her best response, however, her strategy profile only depends on her own type and no longer on her beliefs about the lecturer's type. The lecturer's best response function and equilibrium strategy profile are the same because the lecturer observes whether the student hands in late or not and therefore has no need to hold beliefs about when the student will hand in.

5. Extension

We extend the game above to account for grade inflation. The lecturer has an incentive to inflate a student's grade since a higher grade is associated a more favorable evaluation from the student. Favorable evaluations are linked to an increase in salary and likelihood of promotion for the lecturer (Goswami & Mumit, 2018). However, inflating a grade is ethically wrong therefore the lecturer incurs cost γ if she decides to inflate. If the lecturer leaves the mark unchanged, she occurs an empathy cost δ. A student experiences a cost of ω for asking the lecturer and the lecturer experiences a cost of ϕ of being bothered by the student . If the lecturer decides not to penalize the student, then he will never choose to inflate a students mark. Knowing this, a student that hands in late and receives no penalty will always accept his mark. If a student hands in on time, a lecture will always leave the mark unchanged at the request of the student, therefore the student will always accept the mark. If the lecturer decides to penalize a late student, the lecturer will choose to inflate a student's mark if the ethical cost is smaller than the empathy cost the lecturer experiences;

$$\gamma < \delta m$$

If the ethical cost the lecturer experiences is larger than the empathy cost the lecturer experiences for giving a penalty, then the lecturer is strict. Otherwise, the lecturer is lenient. Therefore, a strict lecturer will not inflate a student mark, but a lenient lecture will (Franz, 2010).

A summary of the game's parameter's and restrictions are shown in table 5.1 and the game is represented in Figure 5.1

Table 5.1: Extended Game Parameters

Parameter	Explanation	Restrictions
ω	Exogenous cost of asking for the lecturer for higher mark	$0 < \omega \leq 1$
γ	Ethical cost of inflating students mark	$0 < \delta \leq 1$
ϕ	Cost of annoyance lecture experience for student bothering him	$0 <\phi \leq 1$
x	Additional marks that student get when lecturer decides to inflates	$1 <$ x ≤ 2

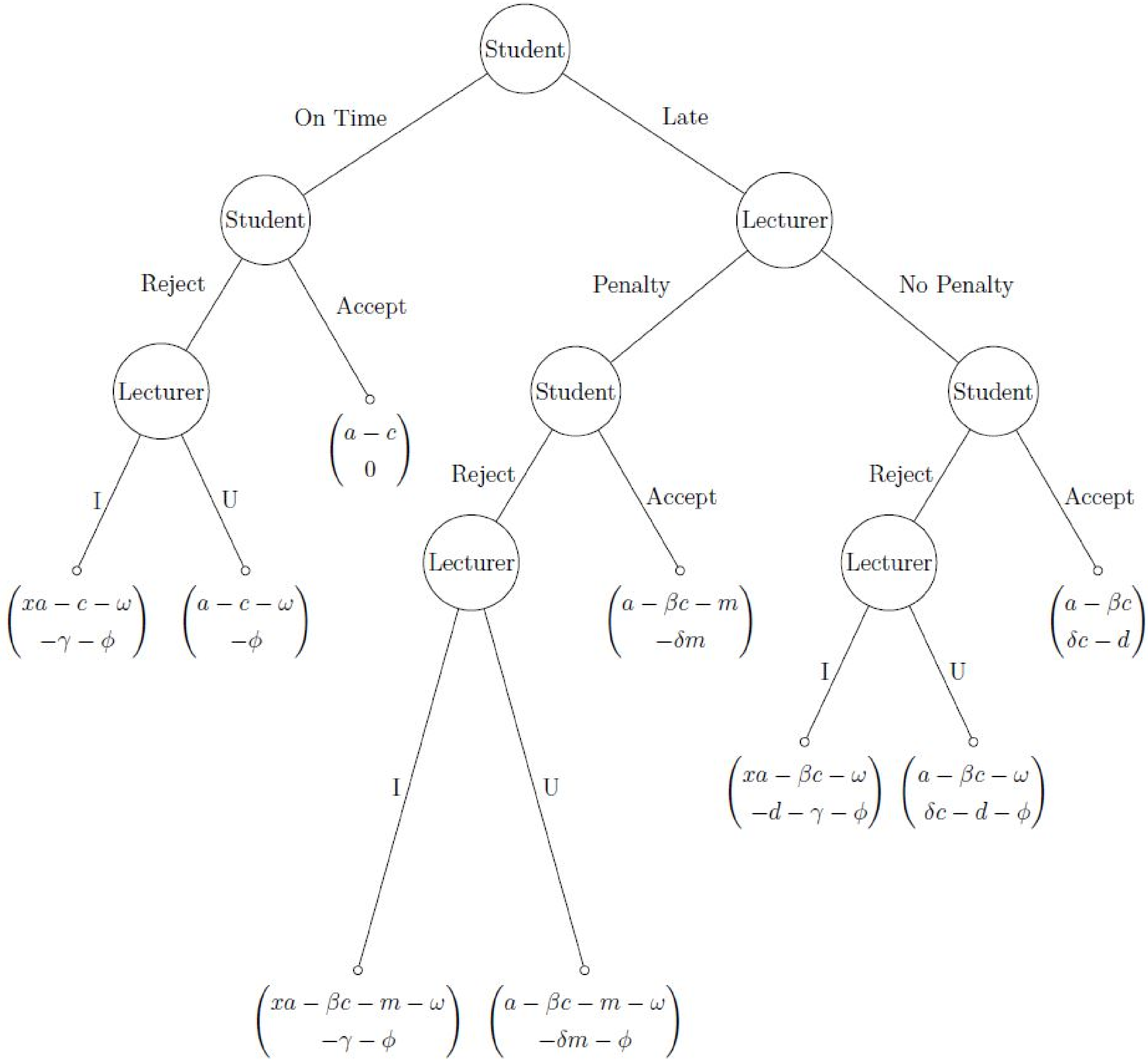

Figure 5.1: This is an extended version of the game presented in Figure 3.1. To account for grade inflation, the student gets to accept the grade the lecturer game him, or reject the grade and ask the lecturer for a better mark. The lectuerer can then decided to inflate (I) the students mark or leave the mark unchanged (U).

6. Conclusion

The results of the model are intuitive and provide a useful insight into how student's think about handing in assignments, and how lecturers respond to late submissions. Whether a lecturer penalizes a student for late submission depends largely on the magnitude of the crisis the student is facing, relative to the lecturer's own degree of empathy. The student chooses whether to hand in late depending on the inconvenience to the lecturer, the size of the penalty, the magnitude of the crisis, and their ability to bounce back from an academic setback.

One shortcoming of the model is that is assumes c is common knowledge. Lecturers often have too many students to have a personal relationship with each one to know if they are experiencing a crisis. Another shortcoming is that the lecturer's benefit from giving no penalty does not account the benefit of no penalty to the student. More resilient students would fare well to hand in late since they recover from the crisis easily. However, if the lecturer's decision to penalise a student also depends on the student's resiliency, it would deter students with low resiliency from taking an extension, allowing them to focus on their other work due later in the term.

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Appendix A

	Penalty	No Penalty
On Time	$a - c,\ 0$	$a - c,\ 0$
Late	$a - \beta c - m,\ \delta m$	$a - \beta c,\ \delta c - d$

Table 6.1: Strategic form of the game

Appendix B

Payoffs

Student payoffs:

$$\begin{aligned} E[\text{On Time}] &= a - c \\ E[\text{Late}] &= p(a - \beta c - m) + (1 - p)(a - \beta c) \\ &= -mp + a - \beta c \end{aligned}$$

Student plays on time if:

$$\begin{aligned} a - c &> a - mp - \beta c \\ \beta c &> c - mp \\ \beta &> \frac{c - mp}{c} \end{aligned}$$

Student plays late if:

$$\beta < \frac{c - mp}{c}$$

Lecturer Payoffs:

$$\begin{aligned} E[\text{Penalty}] &= q(-\delta m) + (1 - q)(0) \\ &= q(-\delta m) \\ E[\text{No Penalty}] &= q(\delta c - d) + (1 - a)(0) \\ &= q(\delta c - d) \end{aligned}$$

Lecturer gives a penalty if:

$$
\begin{aligned}
q(-\delta m) &> q(\delta c - d) \\
-\delta m &> \delta c - d \\
d &> \delta(c+m) \\
\delta &< \frac{d}{c+m} \\
\delta &< \bar{\delta}
\end{aligned}
$$

Lecturer gives no penalty if:

$$
\begin{aligned}
\delta &\geq \frac{d}{c+m} \\
\delta &\geq \bar{\delta}
\end{aligned}
$$

Best Responses

Solving for the best responses:

$$p = \text{Probability that the lecturer gives a penalty} = \bar{\delta} = \text{Prob}(\delta < \bar{\delta})$$

Substitute into the student's best response function - student hands in on time if:

$$\beta > \frac{c - m(\bar{\delta})}{c}$$

Since $0 \leq \beta \leq 1$, β cannot be greater than 1. This implies

$$
\begin{aligned}
\frac{c - m(\bar{\delta})}{c} &\leq 1 \\
c - m\bar{\delta} &\leq c \\
-m\bar{\delta} &\leq 0 \\
0 &\leq \bar{\delta}
\end{aligned}
$$

Since $0 \leq \bar{\delta} \leq 1$, this condition will always hold.

β cannot be less than 0:

$$\begin{aligned}
\frac{c - m\bar{\delta}}{c} &< 0 \\
c - m\bar{\delta} &< 0 \\
-m\bar{\delta} &< -c \\
\bar{\delta} &> \frac{c}{m}
\end{aligned}$$

if $\bar{\delta} > \frac{c}{m} \Rightarrow \beta = 0$, otherwise:

$$\beta = \frac{c - m\bar{\delta}}{c}$$

Best response function for the student:

$$BR_{\beta}(\bar{\delta}) = \begin{cases} \frac{c - m\bar{\delta}}{c} & \text{if} \quad \bar{\delta} \leq \frac{c}{m} \\ 0 & \text{if} \quad \bar{\delta} > \frac{c}{m} \end{cases}$$

Best response function for the lecturer:

$$BR_{\delta}(\delta) = \begin{cases} \text{Penalty} & \text{if} \quad \delta < \frac{d}{c+m} \\ \text{No penalty} & \text{if} \quad \delta \geq \frac{d}{c+m} \end{cases}$$

Bayesian Nash Equilibrium

The Bayesian Nash equilibrium occurs at the point where the best response functions intersect. For the BRFs to cross:

$$\begin{aligned}
\text{Substitute } \bar{\delta} = \frac{d}{c+m} &\text{ into } \beta = \frac{c - m\bar{\delta}}{c} \\
\text{Then: } \beta = \frac{c}{c} - \frac{m}{c}&\left(\frac{d}{c+m}\right) \\
\beta = 1 - &\frac{md}{c^2 + mc}
\end{aligned}$$

BNE strategy for the student

$$s^*_{\beta}(\beta) = \begin{cases} \text{On time} & \text{if} \quad \beta > 1 - \frac{md}{c^2+mc} \\ \text{Late} & \text{if} \quad \beta \leq 1 - \frac{md}{c^2+mc} \end{cases}$$

BNE strategy for the lecturer

$$s_\delta^*(\delta) = \begin{cases} \text{Penalty} & \text{if} \quad \delta < \frac{d}{c+m} \\ \text{No penalty} & \text{if} \quad \delta \geq \frac{d}{c+m} \end{cases}$$