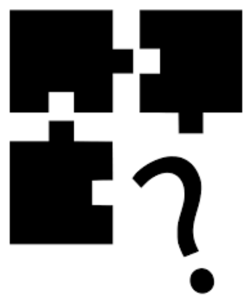
A GAME THEORETIC APPROACH TO DEADLINE ADHERENCE

Microeconomics 871 Essay



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18 October 2021 | Word Count: 1980

Table of Contents

1. Introduction

The relationship between a supervisor and a subordinate have been studied extensively (Conrad, 1983),(Conrad, 1991), (Liu & Shi, 2017). A subordinate's action is often influenced by the behaviour of the supervisor. Identity-based motivation theory describes that a positive relationship of trust and leniency between a subordinate and a supervisor will ultimately lead to both parties being better off in terms of reaching their goals, performance, and mental health (Liu & Shi, 2017). Game theory models have been used to analyze the interactions between a supervisor and a subordinate (Osborne, 2004: 1). A common situation that arises is a subordinate being assigned a task that should be complete before a deadline. Game theory can offer insights as to when a subordinate should submit a task on time or miss the deadline if there is incomplete information.

This article contributes to the existing literature by analyzing the dynamic relationship between a student and a lecturer. We investigate a case where a student is required to submit an assignment with a deadline restriction but experiences a crisis before the deadline and must choose whether to submit her assignment on time or miss the deadline. We impose a structure of continuous types for both players, with a discrete set of actions. This essay is structured as follows: section 2 briefly discusses the literature on games of incomplete information and some applications. Section 3 presents the deadline adherence game theory model; and section 4 analyses the results of the game. Section 5 provides a brief extension of the game and the final section concludes 6.

2. Games of Incomplete Information

Real-life situations often lack full information Trabelsi (2020). Von Neumann and Morgenstern (1944: 30 cited in Myerson, 2004) first used the term *incomplete information* to a game theory model in which parts of the normal form structure is unspecified. However, Von Neumann and Morgenstern (1944: 30 cited in Myerson, 2004) deemed any further research into this model as unimportant. Disagreeing Luce & Adams (1956) extended on the literature on incomplete information by assuming that each player has a perception, which could be correct or incorrect, of the payoff function of the other player. However, Luce & Adams (1956) did not consider uncertainty about strategies. Harsanyi (1995) addressed the concerns of Luce & Adams (1956) by developing a general analytical framework to analyze games of incomplete information.

We follow the approach of Harsanyi (1995) in applying a game-theoretic model of incomplete information, where players have less than full information about each other's payoff functions. Based on the Bayesian methodology, both players have expectations in the form of subjective probability distributions. Players have different types, which are randomly assigned and represents their belief about the game being played. However, players do not know the type of the other player Trabelsi (2020). Both players attempt to estimate the probability of each other's types, subject to the available information. To solve the model, the game of incomplete information will be reinterpreted as a game with complete and imperfect information, by transforming the basic mathematical structure.

3. A Model of Deadline Adherence

A student receives an assignment, which is due by a certain date set by the lecturer. While the student is working on the assignment, she undergoes a crisis and therefore spends less time on the assignment. She has two options: she can hand in the assignment on time or she can hand in late. If she hands in on time, she will get a payoff of $a - c$, where a is the potential pre-crisis mark, and c is the negative impact the crisis has on her mark. However, if she submits her project late, she has some time to recover after the crisis and reduce its academic impact. Her payoff is $a - \beta c - m$ if the lecturer gives her a penalty, where m is the size of the penalty. She gets a payoff of $a - \beta c$ if there is no penalty. β represents the type of the student, where a high β suggests a low resiliency to crises and a low β suggests a high resiliency and a better academic recovery. The student observes her own type but does not know the lecturer's type.

On the other hand, the lecturer is faced with the decision either to give a penalty (m) if a student submits late or not to give a penalty. If the lecturer gives a penalty, he feels bad since the student has gone through a crisis. The size of his disutility depends on the size of the penalty (m) and how empathetic the lecturer is, where the level of empathy describes the lecturer's type (δ). The more empathetic the lecturer is, the higher δ is. The lecturer observes his own type but not that of the student. The lecturer's and student's types are both continuous types, which are independently and randomly chosen by nature at the start of the game from a uniform distribution[1]: $\delta \sim Uniform(0, 1)$ and $\beta \sim Uniform(0, 1)$. If the lecturer decides not to impose a penalty, he feels good that he did not impose on a student experiencing a crisis, and gets a positive payoff of δc. However, the lecturer knows that by waving the penalty, he may be encouraging this student, and other students to hand in late in the future. The lecturer would rather deter late hand-ins, and receives a negative payoff $-d$ for not deterring late hand-ins. This deterrent parameter relates to the literature on games of repeated interaction and reputations.

The parameters a, c, m & d are all common knowledge. This is a game of incomplete information because the players' types are not common knowledge. The type spaces are continuous and the action spaces are discrete. Each player needs to choose his/her action based on his/her own type, what each believes the other player's type is, and the values of a, c, m & d. Figure 3.1 shows the game in extensive form[2]. And a summary of the game's parameters and restrictions are given in figure 3.1

[1] A uniform distribution puts equal chance on any of the outcomes between 0 and 1 happening.
[2] The simultaneous form game can be found in the appendix, 6.1

below.

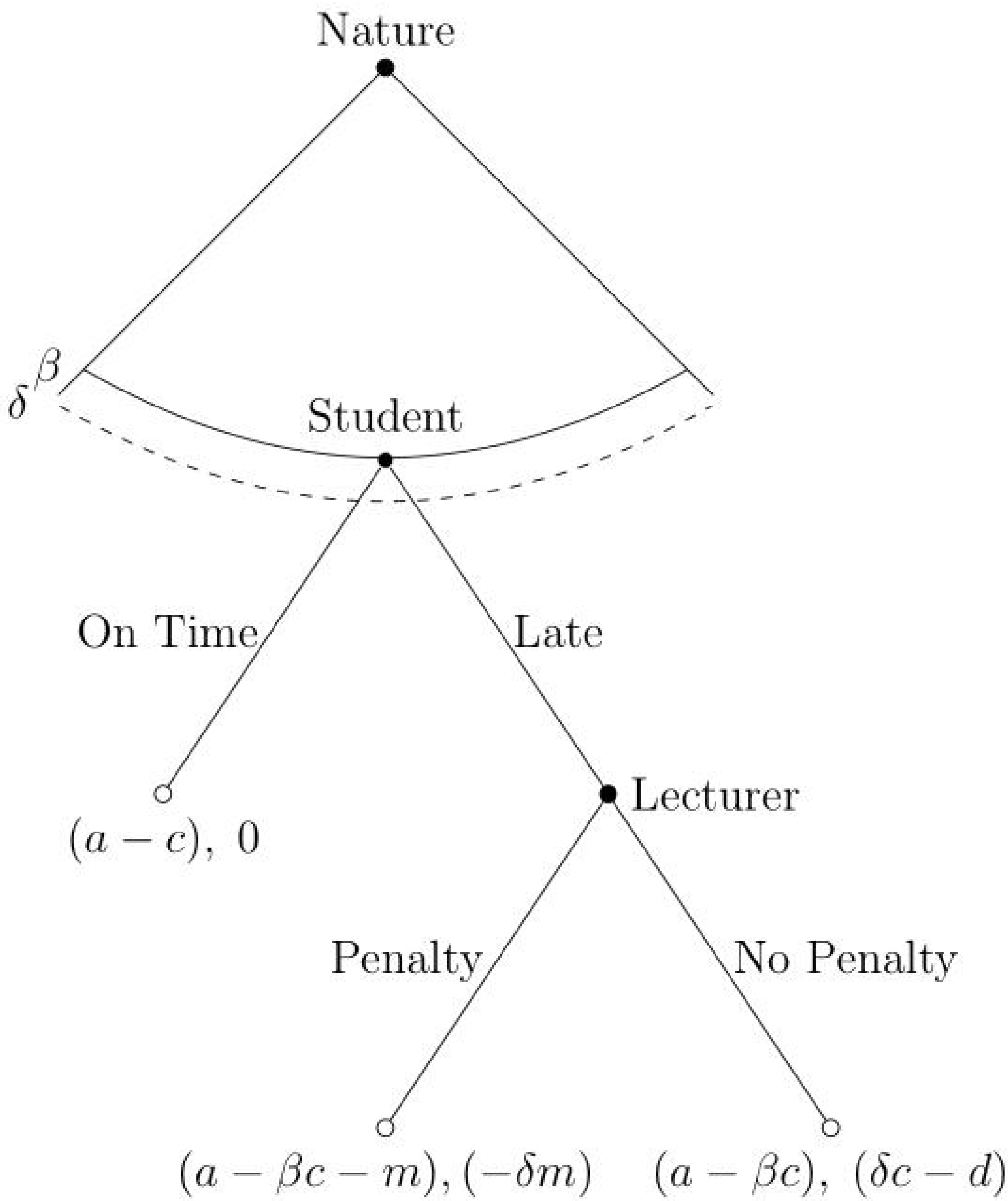

Figure 3.1: This game is dynamic, where nature first chooses the student's and lecturer's types. The dashed line represents information only known by the lecturer and the solid line represents information only known by the student. Then the student moves, deciding to hand in on time or late after experiencing a crisis. If the student hands in late, the lecturer decides to impose a penalty or not.

Parameter	Explanation	Restriction
a	Potential assignment mark	$0 \leq a \leq 1$
c	Cost of crisis to assignment mark	$0 < c \leq 1$
β	Student's type: level of resiliency	$\beta \sim Uniform(0,1)$
m	Mark penalty	$0 < m \leq 1$
δ	Lecturer's type: level of empathy	$\delta \sim Uniform(0,1)$
d	Detterent	$0 < d \leq 1$

Table 3.1: Game Parameters

4. Results and Discussion

In order to understand how the lecturer and student will make their decisions given their beliefs, we need to solve for their best responses. A best response for the student would be to hand in on time if the expected payoff from handing in on time is higher than the expected payoff of submitting late. Defining p as the probability that the lecturer will give a penalty, a student should hand in on time where:

$$\beta > \frac{c - mp}{c}$$

The right hand side is a constant[3]. This implies that there is some threshold value of β, for which a student should hand in on time. If the student believes that the lecturer will give no penalty (i.e. $p = 0$), then she should only hand in on time if $\beta > \frac{c}{c} \Rightarrow \beta > 1$. Since β lies between 0 and 1, the inequality will never hold and she should always hand in late. From an intuitive stand point, this makes sense: a student can never do worse by handing in late if there is no penalty[4] but she will do better to hand in late if she is resilient in any way ($\beta < 1$) and can partially recover from the crisis. However, if the student believes that the lecturer will give a penalty with some positive probability ($p > 0$) then her decision to hand in on time depends on her level of resiliency, the magnitude of the crisis and the size of the penalty. As the cost of the crisis increases, the threshold value increases (ceteris paribus), and the student becomes more likely to hand in late (unless her she has a very low resiliency). If the mark penalty is high, the threshold value is smaller, and the student is more likely to hand in on time (unless is very resilient: β is very low). We can analyse the lecturer's best response rule similarly. The lecturer should impose a penalty where:

$$\delta < \frac{d}{c + m}$$

[3]Since c and m are known, and p is a belief the student holds.

[4]Handing in late is weakly dominant.

The threshold value for the lecturer is the ratio between the deterrent factor and the sum of the crisis cost and the penalty mark. If the deterrent factor is high relative to the cost of crisis and the penalty mark, then the lecturer is more likely to give a penalty (unless he is highly empathetic). If the cost of the crisis large and the mark penalty is high relative to the deterrent factor, the lecturer is more likely to waive the penalty (unless he has very low empathy levels).

While the best response analysis is useful, if we want to understand the outcome of the game and the player's strategies, we need to solve for the Bayesian Nash Equilibrium (BNE). The full derivations of the solution concepts are given in the appendix (6).

After observing her private type, the student chooses the following hand-in pattern:

$$s_{\beta}^{*}(\beta) = \begin{cases} \text{On time} & \text{if} \quad \beta > 1 - \frac{md}{c^2+mc} \\ \text{Late} & \text{if} \quad \beta \leq 1 - \frac{md}{c^2+mc} \end{cases}$$

And the lecturer's BNE strategy is given by:

$$s_{\delta}^{*}(\delta) = \begin{cases} \text{Penalty} & \text{if} \quad \delta < \frac{d}{c+m} \\ \text{No penalty} & \text{if} \quad \delta \geq \frac{d}{c+m} \end{cases}$$

When the lecturer and the student play their equilibrium strategies, neither has an incentive to deviate and we get a Bayesian Nash equilibrium. We would interpret the student's BNE strategy similarly to her best response; however, her strategy profile only depends on her own type and no longer on her beliefs about the lecturer's type. The lecturer's best response function and equilibrium strategy profile are the same because the lecturer observes whether the student hands in late or not and therefore has no need to hold beliefs about when the student will hand in.

5. Extension

We extend the game explained in section 3 to account for grade inflation. The lecturer has an incentive to inflate the marks of the student since a higher grade is associated with the student giving a positive evaluation of the lecturer which could lead to an increase in salary and promotions. However, inflation the mark if ethically wrong therefore the lecturer incurs cost γ if he decides to inflate. If he decides to leave the mark unchanged, then he occurs a sympathy cost δ. A student experiences a cost of ω for asking the lecturer and the lecturer experiences a cost of ϕ of being bothered by the lecturer. If the lecturer decides not to penalize the student, then he will never choose to inflate a students mark. Knowing this, a student that hands in late and receives no penalty will always accept his mark. If a student hands-on time, a lecture will always leave the mark unchanged at the request of the student, therefore the student will always accept the mark. If the lecturer decides to penalize a late student, the lecturer will choose to inflate a student's mark if the moral cost is smaller than the empathy cost the lecturer experiences.

$$\gamma < \delta m$$

If the moral cost that the lecturer experiences is bigger than the empathy cost the lecturer experiences for giving a penalty, then the lecturer is strict, otherwise the lecturer is lenient. Therefore, a strict lecturer will not inflate a student mark but a lenient lecture will.
A summary of the game's parameter's and restrictions are shown in table 5.1 and the game is represented in Figure 5.1

Table 5.1: Extended Game Parameters

Parameter	Explanation	Restrictions
ω	Exogenous cost of asking for the lecturer for higher mark	$0 < \omega \leq 1$
δ	Ethical cost of inflating students mark	$0 < \delta \leq 1$
ϕ	Cost of annoyance lecture experience for student bothering him	$0 <\phi \leq 1$
x	Additional marks that student get when lecturer decides to inflates	$1 < \mathrm{x} \leq 2$

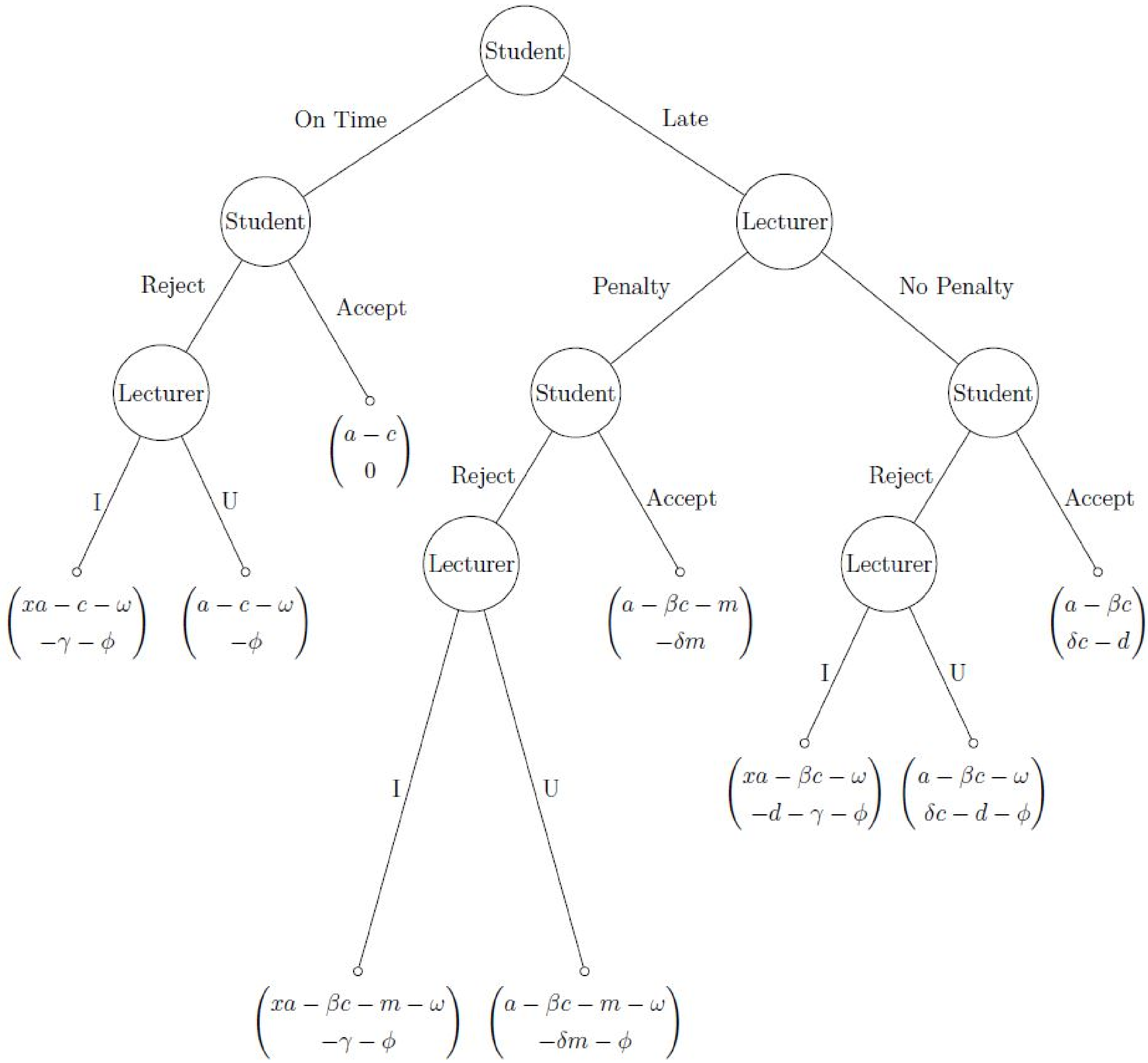

Figure 5.1: This is an extended version of the game presented in Figure 3.1. To account for grade inflation, the student gets to accept the grade the lecturer game him, or reject the grade and ask the lecturer for a better mark. The lectuerer can then decided to inflate (I) the students mark or leave the mark unchanged (U).

6. Conclusion

The results of the model are intuitive and provide a useful insight into how student's think about handing in assignments, and how lecturers respond to late submissions. One shortcoming of the model is its assumption that c is common knowledge. Although some lecturers are in touch with their students and would know whether they are experiencing a crisis and how impactful the crisis would be, most lecturers have too many students to know that information.

Extensions, generality Shortcomings

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Appendix A

	Penalty	No Penalty
On Time	$a-c,\ 0$	$a-c,\ 0$
Late	$a-\beta c-m,\ \delta m$	$a-\beta c,\ \delta c-d$

Table 6.1: Strategic form of the game

Appendix B

Payoffs

Student payoffs:

$$\begin{aligned} E[\text{On Time}] &= a-c \\ E[\text{Late}] &= p(a-\beta c-m)+(1-p)(a-\beta c) \\ &= -mp+a-\beta c \end{aligned}$$

Student plays on time if:

$$\begin{aligned} a-c &> a-mp-\beta c \\ \beta c &> c-mp \\ \beta &> \frac{c-mp}{c} \end{aligned}$$

Student plays late if:

$$\beta < \frac{c-mp}{c}$$

Lecturer Payoffs:

$$\begin{aligned} E[\text{Penalty}] &= q(-\delta m)+(1-q)(0) \\ &= q(-\delta m) \\ E[\text{No Penalty}] &= q(\delta c-d)+(1-a)(0) \\ &= q(\delta c-d) \end{aligned}$$

Lecturer gives a penalty if:

$$\begin{aligned}
q(-\delta m) &> q(\delta c - d) \\
-\delta m &> \delta c - d \\
d &> \delta(c + m) \\
\delta &< \frac{d}{c + m} \\
\delta &< \bar{\delta}
\end{aligned}$$

Lecturer gives no penalty if:

$$\begin{aligned}
\delta &\geq \frac{d}{c + m} \\
\delta &\geq \bar{\delta}
\end{aligned}$$

Best Responses

Solving for the best responses:

$$p = \text{Probability that the lecturer gives a penalty} = \bar{\delta} = \text{Prob}(\delta < \bar{\delta})$$

Substitute into the student's best response function - student hands in on time if:

$$\beta > \frac{c - m(\bar{\delta})}{c}$$

Since $0 \leq \beta \leq 1$, β cannot be greater than 1. This implies

$$\begin{aligned}
\frac{c - m(\bar{\delta})}{c} &\leq 1 \\
c - m\bar{\delta} &\leq c \\
-m\bar{\delta} &\leq 0 \\
0 &\leq \bar{\delta}
\end{aligned}$$

Since $0 \leq \bar{\delta} \leq 1$, this condition will always hold.

β cannot be less than 0:

$$\begin{aligned}
\frac{c - m\bar{\delta}}{c} &< 0 \\
c - m\bar{\delta} &< 0 \\
-m\bar{\delta} &< -c \\
\bar{\delta} &> \frac{c}{m}
\end{aligned}$$

if $\bar{\delta} > \frac{c}{m} \Rightarrow \beta = 0$, otherwise:

$$\beta = \frac{c - m\bar{\delta}}{c}$$

Best response function for the student:

$$BR_\beta(\bar{\delta}) = \begin{cases} \frac{c - m\bar{\delta}}{c} & \text{if} \quad \bar{\delta} \leq \frac{c}{m} \\ 0 & \text{if} \quad \bar{\delta} > \frac{c}{m} \end{cases}$$

Best response function for the lecturer:

$$BR_\delta(\delta) = \begin{cases} \text{Penalty} & \text{if} \quad \delta < \frac{d}{c+m} \\ \text{No penalty} & \text{if} \quad \delta \geq \frac{d}{c+m} \end{cases}$$

Bayesian Nash Equilibrium

The Bayesian Nash equilibrium occurs at the point where the best response functions intersect. For the BRFs to cross:

$$\begin{aligned}
\text{Substitute } \bar{\delta} = \frac{d}{c+m} &\text{ into } \beta = \frac{c - m\bar{\delta}}{c} \\
\text{Then: } \beta = \frac{c}{c} - \frac{m}{c}&\left(\frac{d}{c+m}\right) \\
\beta = 1 - \frac{md}{c^2 + mc}&
\end{aligned}$$

BNE strategy for the student

$$s^*_\beta(\beta) = \begin{cases} \text{On time} & \text{if} \quad \beta > 1 - \frac{md}{c^2+mc} \\ \text{Late} & \text{if} \quad \beta \leq 1 - \frac{md}{c^2+mc} \end{cases}$$

BNE strategy for the lecturer

$$s_{\delta}^{*}(\delta) = \begin{cases} \text{Penalty} & \text{if} \quad \delta < \frac{d}{c+m} \\ \text{No penalty} & \text{if} \quad \delta \geq \frac{d}{c+m} \end{cases}$$