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Panel Unit Root Tests: A Review

by

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Abstract: Over the last decade much research has been carried out on unit roots and cointegration in panel-data with integrated time series, due to the availability of new datasets where the time series dimension and the cross-section dimension are of the same order. The analysis of this peculiar panel data set requires new techniques. In the panel unit root test framework, two generations of tests have been developed: a first generation (Levin, Lin and Chu test (2002), Im, Pesaran and Shin test (2003) and *Fisher*-type tests) whose main limit is the assumption of cross-sectional independence across units; a second generation of tests that rejects the cross-sectional independence hypothesis. Within this second generation of tests, two main approaches can be distinguished: the covariance restrictions approach, adopted notably by Chang (2002, 2004), and the factor structure approach, including contributions by Bai and Ng (2004a), Phillips and Sul (2003), Moon and Perron (2004a), Choi (2002) and Pesaran (2003), among others.

JEL classification: C12, C22, C23

Key words: Non-stationary panel data, Panel unit root tests, Cross-section dependence

Introduction

The econometric theory for dealing with panel data[1] was largely developed for data sets where the number of time series observations (*T*) was small (often only four or five observations) but the number of groups or individuals (*N*) was large[2]. In this case, the asymptotic statistical theory was derived by letting $N \to \infty$, for fixed *T*, in contrast to time-series analysis which was done letting $T \to \infty$, for fixed *N*.

Nevertheless, during the past two decades a variety of new data sets have been constructed and are now available in electronic form (e.g. the Penn World Tables by Summer and Heston, 1991). One of the main features of these data sets is that sometimes both *T* and *N* are large and their orders of magnitude are similar. This feature has different implications for theoretical and empirical analysis, and understanding them is very important for economists intending to work on this kind of data. This is why recent years have seen an explosion in the number of papers on the subject of unit roots and cointegration in panels of data with integrated time series.

The attempt of the present work is to provide an updated overview of the recent developments in panel unit root tests literature and to underline the main issues which remain to be solved. This is fundamental for the econometric researcher who wants to apply existing tests or to develop new and better tests.

The advantages of panel data methods include the use of data from countries (when combined in a panel) for which the span of time series data is insufficient and would thus preclude the study of many hypotheses of interest. Other benefits include better power properties of the testing procedures (when compared to more standard time-series methods) and the fact that many of the issues studied, such as convergence or purchasing power parity, lend themselves naturally to being studied in a panel context.

A preliminary problem which has to be solved in order to develop appropriate test statistics in the large panel framework is the question of how to carry out an asymptotic analysis, as both *N* and *T* can go to infinity. Several approaches have been developed, considering how the two indexes go to infinity[3]. The main contribution in this framework has

[1] For surveys on panel data see Arellano (2003), Baltagi (2001), Hsiao (2003) and Wooldridge (2002).
[2] The cross-sectional units may be households, firms, regions, countries and so forth.
[3] The way this is done is crucial for determining the asymptotic properties of estimators and tests for nonstationary panels.

been made by Phillips and Moon (1999), who identified three main ways to approach asymptotic theory in this case:

1) *Sequential limits*: this procedure consists in letting one argument, say *T*, go to infinity first, and the other, say *N*, go to infinity second ($(T, N \to \infty)_{\text{seq}}$ hereafter). These sequential limits are easy to derive and helpful in extracting quick asymptotics, but they can sometimes give misleading asymptotic results (Phillips and Moon, 1999).

2) *Diagonal-path limits*: this consists in imposing restrictions on the relative rates at which *N* and *T* go to infinity[4]. The limit theory obtained by this approach depends on the specific functional relation $T(\cdot)$, and the assumed expansion path may not provide an appropriate approximation for a given (T, N) situation.

3) *Joint limits*: this allows both *N* and *T* to pass to infinity simultaneously without placing specific diagonal path restrictions on the divergence. In general, this procedure gives a more robust result than the other approaches, but there are also some disadvantages: a) the joint limit is usually more difficult to derive; b) stronger conditions (i.e. existence of higher moments) are required to allow for uniformity in the convergence arguments; c) it is not generally true that a sequential limit is equal to a joint limit[5].

All previous approaches are theoretically interesting; nevertheless, the limiting results are essentially the same as those of the sequential asymptotics, and, from a practical point of view, sequential asymptotic results will be adequate in most cases.

Initial theoretical work on the non-stationary panel data focuses on testing for unit roots in univariate panels, and since the work of Quah (1994) and Breitung and Meyer (1994), interest in this topic has been increasing significantly.

In general, the commonly-used unit root tests, such as the Dickey-Fuller (*DF*) and the Augmented *DF* (*ADF*) test (Dickey and Fuller, 1981), have non-standard limiting distributions[6] which depend on whether deterministic components are included in the regression equation.

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[4] For instance, Quah (1994) and Levin and Lin (1993) assumed that the two indexes pass to infinity along a specific diagonal path which can be determined by $T = T(N)$. This means that $T(N) \to \infty$ as $N \to \infty$ (being $T(\cdot)$ a monotonically increasing function).

[5] Phillips and Moon (1999) applied the multi-index asymptotic theory to joint limits in situations where T is large relative to N, that is $T, N \to \infty$ with $(T/N) \to \infty$ (examples of this type of data configuration are industries, regions and countries observed over a long span of time. A well-known example of this kind of data is provided by the Penn World Tables). However, the general approach given in Phillips and Moon (1999) is also applicable to situations in which $(T/N) \to 0$, although different limit results will generally obtain in that case.

[6] For example, the *ADF* test statistic converges to a function of Brownian motion (White, 1958) under very general conditions (Said and Dickey, 1984)

Moreover, in finite samples such tests show little power in distinguishing the unit root from stationary alternatives as well as unit root tests based on a single time series with highly persistent deviations from equilibrium.

The reason for the poor performance of standard unit root tests in the panel framework may be the different null hypothesis tested in this case. For instance, if we consider the simplified model:

$$\Delta y_{it} = \rho_i y_{it-1} + u_{it} \qquad i = 1,2,...,N \qquad t = 1,2,...,T\ .$$[7]

in the single equation case we are interested in testing $\rho_1 = 0$ against the alternative hypothesis $\rho_1 < 0$ and we apply a unit root for the first time series. Instead, in the panel data case, the hypothesis we are interested in is:

$$H_0 : \rho_i = 0 \text{ against } H_a : \rho_i < 0 \text{ for } i = 1,2,...,N\ .$$

In the large panel framework "*The hope ... is to combine the best of both worlds: the method of dealing with non-stationary data from the time series and the increased data and power from the cross-section*" (Baltagi and Kao, 2000).

Two generations of panel unit root tests have been developed as we can note in Table 1.

Table 1. Panel Unit Root Tests

First Generation	**Cross-sectional independence**
1. *Nonstationarity tests*	Levin and Lin (1992, 1993) and Levin, Lin and Chu (2002)
	Im, Pesaran and Shin (1997, 2003)
	Maddala and Wu (1999) and Choi (1999, 2001)
2. *Stationarity tests*	Choi's (2001) extension
	Hadri (2000)
Second Generation	**Cross-sectional dependence**
1. *Factor Structure*	Pesaran (2003)
	Moon and Perron (2004a)
	Bai and Ng (2002, 2004)
	Choi (2002)
2. *Other Approaches*	O'Connell (1998)
	Chang (2002, 2004)

The first generation of tests includes Levin, Lin and Chu's test (2002), Im, Pesaran and Shin (2003) and the *Fisher*-type test proposed first by Maddala and Wu (1999), then

[7] In the remainder of this paper, i and t are always assumed $i = 1,2,...,N$, $t = 1,2,...,T$ where not differently specified.

developed by Choi (2001). The main limit of these tests is that they are all constructed under the assumption that the individual time series in the panel are cross-sectionally independently distributed, when on the contrary a large amount of literature provides evidence of the co-movements between economic variables.

To overcome this difficulty, a second generation of tests rejecting the cross-sectional independence hypothesis has been proposed. Within this second generation of tests, two main approaches are distinguished. The first one consists in imposing few or no restrictions on the residual covariance matrix and has been adopted notably by Chang (2002, 2004), who proposed the use of nonlinear instrumental variable methods or the use of bootstrap approaches to solve the nuisance parameter problem due to cross-sectional dependency. The second approach relies on the factor structure approach and includes contributions by Bai and Ng (2004a), Phillips and Sul (2003), Moon and Perron (2004a), Choi (2002) and Pesaran (2003) among others.

This paper is organized as follows. The first section reviews first generation of panel unit root tests, section 2 discusses the second generation of panel unit root tests while my conclusions close the paper.

1. ***Panel unit root tests in the presence of cross-sectional independence***[8]

A first generation of models has analyzed the properties of panel-based unit root tests under the assumption that the data is independent and identically distributed (*i.i.d.*) across individuals. The firsts unit root tests are those of Quah (1992, 1994), Breitung and Mayer (1994) and Levin and Lin (1992, 1993).

In general, this type of panel unit root tests is based on the following univariate regression:

$$\Delta y_{it} = \rho_i y_{it-1} + z_{it}'\gamma + u_{it} \tag{1.1}$$

where $i = 1,2,...,N$ is the individual, for each individual $t = 1,2,...,T$ time series observations are available, z_{it} is the deterministic component and u_{it} is a stationary process. z_{it} could be zero, one, the fixed effects (μ_i), or fixed effect as well as a time trend (t).

[8] See Table A.1. in the Appendix for a summary of the main characteristics of this first generation of tests.

The null hypothesis is

$$\rho_i = 0 \ \forall i \,. \qquad (1.2)$$

The main difference between the proposed tests is the degree of heterogeneity considered under the alternative hypothesis.

Quah (1990, 1994) - using random field methods - derives the asymptotic standard normality of the *DF* unit root *t*-statistic for a model with *i.i.d.* disturbances and no heterogeneity across groups as both N and T grow arbitrarily large. Specifically, N and T are assumed to go to infinity at the same rate, such that N/T is constant[9]. Unfortunately, the random approach cannot be used to analyse more general model specifications (i.e. accommodating individual-specific fixed effects or serial correlation in the disturbances) or to multivariate analysis (i.e. testing for cointegration).

Breitung and Mayer (1994) derive the asymptotic normality of the *DF* test statistic for panel data with an arbitrarily large N and a small fixed T (which corresponds to the most common microeconomic panel dataset). In this framework, it is possible to incorporate arbitrary patterns of serial correlation for each individual (since this only involves a finite number of parameters), and time-specific random effects (which can be consistently estimated as N grows arbitrarily large).

The asymptotic dimensional assumptions of Breitung and Mayer are not appropriate for panel datasets in which T and N have the same or larger order of magnitude. Furthermore, their approach cannot be extended to allow for heterogeneous residual distributions and the influence of individual-specific effects can have large effects on the appropriate critical values at which to evaluate the unit root *t*-statistic.

Since the tests proposed by Quah and Breitung and Meyer have been by-passed by the papers by Levin and Lin (1992, 1993), they are not discussed here.

1.1. Levin, Lin and Chu (2002) test

Levin and Lin (1992, 1993) and Levin, Lin and Chu (2002)[10] (*LLC* thereafter) provide some new results on panel unit root tests. They generalize the Quah's model to allow for

[9] Via Monte Carlo simulations, Quah shows that the standard normal distribution is a good approximation for specific panel sizes (i.e. Summer-Heston dataset containing 25 annual observations for each 100 countries).

[10] Levin and Lin proposed their test in first time in 1992. In 1993 they generalised the analysis allowing for autocorrelation and heteroscedasticity. Their paper in 2002 (Levin, Lin and Chu, 2002) collect major results of their researches.

heterogeneity of individual deterministic effects (constant and/or linear time trend) and heterogeneous serial correlation structure of the error terms assuming homogeneous first order autoregressive parameters. They assume that both N and T tend to infinity but T increase at a faster rate, such that $N/T \to 0$.

They develop a procedure using pooled *t*-statistic of the estimator to evaluate hypothesis that each individual time series contains a unit root against the alternative hypothesis that each time series is stationary.

Thus, referring to the model (1.1), *LLC* assume homogeneous autoregressive coefficients between individual, i.e. $\rho_i = \rho$ for all i, and test the null hypothesis $H_0 : \rho_i = \rho = 0$ against the alternative $H_a : \rho_i = \rho < 0$ for all i.

Imposing a cross-equation restriction on the first-order partial autocorrelation coefficients under the null, this procedure leads to a test of much higher power than performing a separate unit root test for each individual.

The structure of the *LLC* analysis may be specified as follows:

$$\Delta y_{it} = \rho y_{it-1} + \alpha_{0i} + \alpha_{1i} t + u_{it}\,,\ i = 1,2,...,N\,,\ t = 1,2,...,T\,. \tag{1.1.1}$$

where a time trend ($\alpha_{1i} t$) as well as individual effects (α_i) are incorporated. Note that the deterministic components are an important source of heterogeneity in this model since the coefficient of the lagged dependent variable is restricted to be homogeneous across all units in the panel.

u_{it} is assumed to be independently distributed across individuals and follow a stationary invertible ARMA process for each individual:

$$u_{it} = \sum_{j=1}^{\infty} \theta_{ij} u_{it-j} + \varepsilon_{it} \tag{1.1.2}$$

and the finite-moment conditions are assumed to assure the weak convergence in Phillips (1987) and Phillips-Perron's (Phillips and Perron, 1988) unit root tests.

LLC consider several subcases of model (1.1.1) which are all estimated by *OLS* as pooled regression models.

Limiting distributions are derived by sequential limit theory $(T, N \to \infty)_{\text{seq}}$.

LLC show that the asymptotic properties of the regressions estimators and test statistics are a mixture of properties derived for stationary panel data, and properties derived in the time series literature on unit root tests: in contrast to the non-standard distributions of unit root test statistic for single time series (cf. Phillips, 1987; Phillips and Perron, 1988; Phillips and

Ouiliaris, 1990), the panel regression estimators and test statistics have limiting normal distributions, as in the case of stationary panel data (cf. Hsiao, 2003).

However, the presence of a unit root causes the convergence rate of the estimators and *t*-statistics is higher when $T \to \infty$ than when $N \to \infty$ (referred to as "super-consistency" in the time series literature).

In the case of *i.i.d.* disturbances and no individual-specific fixed effects, under the null, the panel regression unit root t-statistic t_ρ based on the pooled estimator $\hat{\rho}$ converges to the standard $\mathcal{N}(0,1)$ distribution when N and T tend to infinity and $\sqrt{N}/T \to 0$.

In contrast, if there are individual-specific fixed effects, time trends or serial correlation in the disturbances, the resulting test statistic is not centred at zero, with substantial impact on the size of unit root test; in this case, Levin and Lin suggest using an adjusted *t*-statistic:

$$t_\rho^* = \frac{t_{\rho=0} - N\tilde{T}\hat{S}_N \hat{\sigma}_{\tilde{\varepsilon}}^{-2} RSE(\hat{\rho}) \mu_{m\tilde{T}}^*}{\sigma_{m\tilde{T}}^*} \tag{1.1.3}$$

being $\mu_{\tilde{T}}^*$ and $\sigma_{\tilde{T}}^*$ the mean and the standard deviation adjustment terms which are obtained from Monte Carlo simulation and tabulated in Levin and Lin's paper (1992), $\hat{S}_{NT} = \frac{1}{N}\sum_{i=1}^{N}\frac{\hat{\sigma}_{yi}}{\hat{\sigma}_{ei}}$, where $\hat{\sigma}_{yi}^2$ denotes a kernel estimator of the long-run variance for the individual i.

Applying sequential limit theory, i.e. $(T, N \to \infty)_{\text{seq}}$, the following limiting distributions of $T\sqrt{N}\hat{\rho}$ and t_ρ are obtained:

$$T\sqrt{N}\hat{\rho} \Rightarrow \mathcal{N}(0,2) \tag{1.1.4}$$

$$t_{\rho=0} \Rightarrow \mathcal{N}(0,1). \tag{1.1.5}$$

Levin and Lin (1993)'s Monte Carlo simulation results indicate that when there are not individual-specific fixed effects, the standard normal distribution may provide a good approximation of the empirical distribution of the test statistic in relatively small samples, and that the panel framework can provide dramatic improvements in power compared to performing a separate unit root test for each individual time series.

As Levin *et al*. (2002) noted, their panel based unit root tests are more relevant for panels of moderate size (i.e., $10 < N < 250$ and $25 < T < 250$).

In fact, existing unit root test procedures are appropriate if T is very large, or T is very small but N is very large.[11]

However, for panels of moderate size standard multivariate procedures may not be computationally feasible or sufficiently powerful and the *LLC* test seems to be more appropriate.

Unfortunately, the *LLC* test has some limitations. First of all, the test depends crucially upon the independence assumption across individuals, and hence not applicable if cross sectional correlation is present.

But the major limitation is that the autoregressive parameters are considered being identical across the panel:

$$\begin{aligned} H_0 &: \rho_1 = \rho_2 = ... = \rho_N = \rho = 0 \\ H_a &: \rho_1 = \rho_2 = ... = \rho_N = \rho < 0 \end{aligned} \tag{1.1.6}$$

The null makes sense under some circumstances, but as Maddala and Wu (1999) pointed out, the alternative is too strong to be held in any interesting empirical cases.

This limitation has been overcame by *IPS* (Im, Pesaran and Shin, 1997, 2003) which proposed a panel unit root test without the assumption of identical first order correlation under the alternative.

1.2. Im, Pesaran and Shin (2003) tests

Im, Pesaran and Shin (2003) -*IPS* thereafter-, using the likelihood framework, suggest a new more flexible and computationally simple unit root testing procedure for panels (which is referred as *t*-bar statistic), that allows for simultaneous stationary and non-stationary series (i.e. ρ_i can differ between individuals).

Moreover, this test allows for residual serial correlation and heterogeneity of the dynamics and error variances across groups.

Instead of pooling the data, *IPS* consider the mean of (*A*)*DF* statistics computed for each cross-section unit in the panel when the error term u_{it} of the model (1.1) is serially correlated,

[11] If T is very large, then existing unit root test procedures will generally be sufficiently powerful to be applied separately to each individual in the panel, though pooling a small group of individual time series can be advantageous in handling more general patterns of correlation across individuals (Park, 1990; Johansen, 1991).
If T is very small, and N is very large, then existing panel data procedure will be appropriate allowing for very general temporal correlation patterns (MaCurdy, 1982; Hsiao, 2003; Holtz-Eakin *et al.*, 1988; Breitung and Mayer, 1994).

possibly with different serial correlation patterns across cross-sectional units (i.e. $u_{it} = \sum_{j=1}^{p_i} \varphi_{ij} u_{it-j} + \varepsilon_{it}$), and T and N are sufficiently large. Substituting this u_{it} in (1.1), and considering a linear trend for each of the N cross-section units, we get:

$$\Delta y_{it} = \alpha_{0i} + \rho_i y_{it-1} + \sum_{j=1}^{p_i} \varphi_{ij} \Delta y_{it-j} + \varepsilon_{it} \tag{1.2.1}$$

where, as usual, $i = 1,2,...,N$, $t = 1,2,...,T$.

The null hypothesis[12] is:

$$H_0 : \rho_i = 0 \quad \text{for all } i$$

against the alternative:

$$H_a : \begin{cases} \rho_i < 0 & \text{for } i = 1,...,N_1 \\ \rho_i = 0 & \text{for } i = N_1 + 1,...,N \end{cases} \quad \text{with } 0 < N_1 \le N$$

that allows for some (but not all) of individual series to have unit roots.

IPS compute separate unit root tests for the N cross-section units and define their t-bar statistic as a simple average of the individual *ADF* statistics, t_{iT} , for the null as:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_{iT} \ . \tag{1.2.2}$$

IPS assume that t_{iT} are *i.i.d.* and have finite mean and variance.

Therefore, by Lindeberg-Levy central limit theorem, the standardized t-bar statistic converges to a standard normal variate as $N \to \infty$ under the null hypothesis.

In order to propose a standardization of the t-bar statistic, the values of the mean and the variance have been computed via Monte Carlo methods for different values of T and $p_i's$ and tabulated by *IPS* (2003).

It is important to note that in this procedure only balanced panel data are considered. If unbalanced data are used, more simulations have to be carried out to get critical values. In the case of serial correlation, *IPS* propose using *ADF* t-test for individual series. However $E[t_{iT} | \rho_i = 0]$ and $Var[t_{iT} | \rho_i = 0]$ will vary as the lag length included in the *ADF* regression varies. They tabulate $E[t_{iT} | \rho_i = 0]$ and $Var[t_{iT} | \rho_i = 0]$ for different lag lengths. In practice, however, to use their tables, it is necessary to restrict all the *ADF* regressions for individual series having the same lag length.

[12] The null hypothesis in *IPS* also implies auxiliary assumptions about the individual effects as in *LLC* and in particular $\alpha_{0i} = 0$ for all i.

IPS's simulations show that, if there is not serial correlation, the *t*-bar test has the correct size and is very powerful, even for small values of T ($T = 10$): its power rises monotonically with N and T.[13]

Simulations show the importance of a correct choice of the underlying *ADF* regressions order especially when the panel contains deterministic time trends.

When the disturbances in the dynamic panel are serial correlated, size and power of the *t*-bar test are reasonably satisfactory, but T and N have to be sufficiently large. In this case it is also critically important not to under-estimate the order of the underlying *ADF* regressions.

Another important feature lies in the fact that the power of the *t*-bar test is much more favourably affected by a rise in T than by an equivalent rise in N.

Special care needs to be exercised when interpreting the results of this panel unit root tests. Due to the heterogeneous nature of the alternative hypothesis, rejection of the null hypothesis does not necessarily imply that the unit root null is rejected for all i, but only that the null hypothesis is rejected for $N_1 < N$ members of the group such that as $N \to \infty$, $N_1/N \to \delta > 0$. The test does not provide any guidance as to the magnitude of δ, or the identity of the particular panel members which the null hypothesis is rejected.

1.3. The Fisher's type test: Maddala and Wu (1999) and Choi (2001) test

Maddala and Wu (1999) and Choi (2001) consider the shortcomings of both the *LLC* and *IPS* frameworks and offer an alternative testing strategy.

Then, to test for unit root in panel data, they suggest to use a non parametric *Fisher*-type test which is based on a combination of the *p*-values of the test-statistics for a unit root in each cross-sectional unit (the *ADF* test or other non stationarity tests)[14]. Both *IPS* and *Fisher* tests combine information based on individual unit root tests and relax the restrictive assumption of the *LLC* test that ρ_i is the same under the alternative. However, the *Fisher* test is built under

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[13] This also confirms that better results arise when cross-section dimension is added to time-series dimension.

[14] Maddala and Wu (1999) note that the *IPS* test is good for testing the significance of the results from N independent tests of a hypothesis and propose a test combining the *p*-values. Pooling on the basis of *p*-value is a common practice in meta-analysis (see Tippett, 1931, Fisher, 1932, Becker ,1977, and Hedges and Olkin, 1985). It has the advantage of allowing for as much heterogeneity across units as possible. If the test statistics are continuous, the significance levels p_i ($i = 1,2,...,N$) are independent uniform $(0,1)$ variables, and $-2\log_e p_i$ has a χ^2 distribution with two degrees of freedom. Using the additive property of the χ^2 variables, we get $\lambda = -2\sum_{i=1}^{N} \log_e p_i$ has a χ^2 distribution with 2*N* degrees of freedom. This is the *Fisher* test (1932).

more general assumptions than the previously proposed ones (Quah's, *LLC* and *IPS* tests). In fact, as Choi (2001) noted, previous tests suffer from some common inflexibilities which can restrict their use in applications:

1) they all require an infinite number of groups.

2) all the groups are assumed to have the same type of nonstochastic component.

3) T is assumed to be the same for all the cross-section units and to consider the case of unbalanced panels further simulations are required[15].

4) as Levin and Lin, the critical values are sensitive to the choice of lag lengths in the *ADF* regressions.

5) finally, all the previous tests hypothesize that none of the groups have a unit root under the alternative hypothesis: they do not allow that some groups have a unit root and others do not.

Choi (2001) tries to overcome these limitations and proposes a very simple test based on the combination of p-values from a unit root test applied to each group in the panel data. There exists a number of possible p-value combinations[16] to this aim, but the *Fisher*'s one turns out to be the better choice[17].

Choi (2001) considers the model:

$$y_{it} = d_{it} + x_{it}\,, \tag{1.3.1}$$

with $i = 1,2,...,N$, $t = 1,2,...,T$ and:

$$\begin{aligned} d_{it} &= \alpha_{i0} + \alpha_{i1}t + ... + \alpha_{im_i} t^{m_i} \\ x_{it} &= \rho_i x_{i(t-1)} + u_{it} \end{aligned} \tag{1.3.2}$$

and u_{it} is integrated of order zero. Note that the observed data y_{it} are composed of a nonstochastic process d_{it} and a stochastic process x_{it}. Each time series y_{it} can have different sample size and different specification of nonstochastic and stochastic component depending on i. Notably u_{it} may be heteroskedastic.

The null hypothesis is:

$$H_0 : \rho_i = 1 \quad \text{for all}\, i \tag{1.3.3}$$

which implies that all the time series are unit root nonstationary. The alternative hypothesis may be:

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[15] *IPS* consider only briefly such case and the required moment calculations make it difficult to use.
[16] See note 15.
[17] As we will see later, the *Fisher* test is an exact and non-parametric test, and may be computed for any arbitrary choice of a test for unit root in a cross-sectional unit.

$$H_a : |\rho_i| < 1 \quad \text{for at least one } i \text{ for finite } N \tag{1.3.4}$$

that is some time series are nonstationary while the others are not, or

$$H_a : |\rho_i| < 1 \quad \text{for some } i\text{'s for infinite } N \tag{1.3.5}$$

which includes as a special case the alternative that all the time series are stationary, as it is considered in *LLC*.

Let G_{iT_i} be an one-sided unit root test statistic (e.g. *DF* tests) for the *i*-th group in model (1.3.1) and assume that:

a) under the null hypothesis, as $T_i \to \infty$, $G_{iT_i} \Rightarrow G_i$ (where G_i is a non degenerate random variable);

b) u_{it} is independent of u_{js} for all t and s when $i \neq j$;

c) $N_k / N \to k$ (a fixed constant) as $N \to \infty$.

Let p_i be the *p*-value of a unit root test for cross-section *i*, i.e., $p_i = F(G_{iT_i})$, where $F(\cdot)$ is the distribution function of G_i. The proposed *Fisher* type test is:

$$P = -2\sum_{i=1}^{N} ln\, p_i \tag{1.3.6}$$

which combines the *p*-value from unit root tests for each cross-section i to test for unit root in panel data. Under null hypothesis of unit root, *P* is distributed as $\chi^2(2N)$ as $T_i \to \infty$ for all *N*.

Fisher test holds some important advantages: 1) it does not require a balanced panel as in the case of *IPS* test; 2) it can be carried out for any unit root test derived; 3) it is possible to use different lag lengths in the individual *ADF* regression.

The main disadvantage of this test is that the *p*-values have to be derived by Monte Carlo simulation.[18]

When *N* is large, it is necessary to modify the *P* test since in the limit it has a degenerate distribution. Having for the *P* test $E[-2\ln p_i] = 2$ and $Var[-2\ln p_i] = 4$, Choi (2001) proposes a *Z* test:

$$Z = \frac{1}{2\sqrt{N}} \sum_{i=1}^{N} \left(-2\, ln\, p_i - 2\right) \tag{1.3.7}$$

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[18] The *IPS* test is easy to use because there are tables available in the paper for $E(t_{iT})$ and $V(t_{iT})$. However, these are valid only for the *ADF* test.

This statistic corresponds to the standardized cross-sectional average of individual *p*-values. Under the cross-sectional independence assumption of the p_i's, the Lindeberg-Levy central limit theorem is sufficient to show that under the unit root hypothesis *Z* converges to a standard normal distribution as $(T_i, N \to \infty)_{\text{seq}}$.

Choi (2001) also studies the effects of serial correlation in u_{it} on the size for the panel unit root tests and concludes that this is an important source of size distortions.

1.4. Comparison between the previous tests

Extensive simulations have been conducted to explore the finite sample performance of previous panel unit root tests, e.g., Im *et al.* (1997), Karlsson and Lothgren (1999), Maddala and Wu (1999), Choi (2001) and Levin *et al.* (2002).

Monte Carlo simulations show that *t*-bar test is more powerful than *LLC* and Quah test.

Breitung (2000) studies the local power of *LLC* and *IPS* tests statistics versus a sequence of local alternatives. He finds that both tests suffer for a dramatic loss of power if individual specific trends are included. This is due to the bias correction that also removes the mean under the sequence of local alternatives.

It is necessary to note (Maddala and Wu, 1999, and Levin *et al*., 2002) that a direct comparison between *LLC* and *t*-bar is not possible because they present some relevant differences. Even if both tests have the same null hypothesis, the alternatives are quite different: in *LLC* test the alternative provides for individual stationary series with identical first order autoregressive coefficient (although there is heterogeneity in the error variances and the serial correlation structure of the errors), while in *IPS* test it provides for different individual first order autoregressive coefficients.

As a consequence *LLC* test is based on pooled regressions (which is more advantageous if the stationary alternative with identical AR coefficients across individuals is appropriate) meanwhile *IPS* test amounts to a combination of different independent tests and does not pool the data as *LLC* test does. Then, in the power comparisons, it should be kept in mind that the worse performances of *LLC* test may be due to the fact that this test has to use the panel estimation method which is not valid if there is no need for pooling.

Im *et al*. (2003) simulation results are:

- in the case of models without serially correlated errors, *LLC* test tends to over-reject null hypothesis as *N* is allowed to increase; for small *T* the *t*-bar test has a slightly higher power though the *LLC* test has a larger size.

- in the case of models with serially correlated errors, *LLC* test tends to over-reject null hypothesis as *N* is allowed to increase and *t*-bar test is more powerful;

- in general, if a large enough lag order is selected for the underlying *ADF* regressions, then the finite sample properties of the *t*-bar test is reasonably satisfactory and generally better than that of the *LLC* test.

While *LLC* and *t*-bar test are not directly comparable, *Fisher* and *IPS* *t*-bar test are.

Both tests are combination of different independent tests and they verify the same hypothesis[19].

The main difference between the two tests is that the *Fisher* test is based on the combination of the significance levels of the different tests, and the *IPS* test is based on combining the test statistics.

Furthermore, the *Fisher* test is a non-parametric test, whereas the *IPS* test is a parametric test. The distribution of the *t*-bar statistic involves the mean and variance of the *t*-statistics used. *IPS* compute this for the *ADF* test statistic (using different numbers of lags and sample sizes), but their tables are valid only if the *ADF* test is used for the unit root tests and the length of the time series is the same for all samples. Otherwise *Fisher* test can be used with any unit root test and even if the *ADF* test is used, the choice of the lag length for each sample can be separately determined. Also, there is no restriction of the sample sizes for different samples.

The fact that *Fisher* test is an exact test whereas the *IPS* test is an asymptotic test does not lead to a large difference in finite sample results: the adjustment terms in the *IPS* test and the *p*-values in the *Fisher* test are all derived from simulations. However, the asymptotic validity of the *IPS* test depends on $N \to \infty$ while for the *Fisher* test it depends on $T \to \infty$.

Maddala and Wu (1999) conduct simulations not size-corrected to compare their *Fisher* test, *LLC* test and *t*-bar test and show that their test performs better than the other two tests. In comparing the performances of the tests, the main results are:

- when there is no cross-sectional correlation in the errors, the *IPS* test is more powerful than the *Fisher* test (the *IPS* test has higher power when the two have the same size). Both tests are more powerful than the *LLC* test;

[19] Note that if there is contemporaneous correlation (i.e. there is correlation among the individual test statistics), both tests will need modifications.

- all the tests can take care of heteroscedasticity and serial correlation in the errors, but when there is cross-sectional correlation, none of the tests can handle this problem well. The Monte Carlo evidence suggests that this problem is less severe with the *Fisher* test than with the *LLC* or the *IPS* test[20].

- when a mixture of stationary and nonstationary series in the group is included as an alternative hypothesis, the *Fisher* test is the best because it has the highest power in distinguishing the null and the alternative.

Choi's simulations (2001) compare *t*-bar and *Fisher* tests performances and show that the size of both tests is reasonably close to their nominal size 0.05 when N is small and *t*-bar test has the most stable size to the different values of N and T. The power of both test rises as N increases (which justifies the use of panel data), but it decreases considerably when a linear trend is included in the model. However in terms of size-adjusted power, *Fisher* test is superior to the *t*-bar. Considering the trade-off between size and power, the Z test (1.3.7) seems to outperform the other tests. Furthermore, we remember that this test can be used for both finite and infinite N.

Also Banerjee *et al.* (2005) point out the assumption of cross-unit independence in panel framework.

They analyze the PPP hypothesis and suggest an alternative explanation for the mismatch existing between the results of the panel data analysis and the results of the univariate analysis[21]. They observe that empirically the no cross-sectional independence hypothesis of panel unit root tests is violated. As a consequence the empirical size of these tests is substantially higher than the nominal level, and usual tests would over-reject the non-stationarity null when there are common sources of non-stationarity (see also Lyhagen, 2000, and Pedroni and Urbain, 2001).

Banerjee *et al.* (2005) carry out Monte Carlo simulations to compare performances of *LLC*, *t*-bar, *LM-IPS* and *Fisher* tests. They show that when there are cross-country cointegrating relationships, the *LLC* test suffers the least from size distortion. This occurs because *LLC* is the only pooled test which takes some account of the relationships linking the units. In addition, pooling is not an unreasonable restriction when the autoregressive parameter ρ to be estimated is homogeneous under both the null and alternative hypotheses.

[20] More specifically, when T is large but N is not very large, the size distortion with the *Fisher* test is small. But for medium values of T and large N, the size distortion of the *Fisher* test is of the same level as that of the *IPS* test.

[21] The PPP hypothesis is often accepted in the former case and rejected in the latter.

However, when the DGP allows for heterogeneity, the power of the *LLC* test is lower than for the other panel tests considered.

Banerjee *et al*. (2005) show that in the presence of cross-unit cointegration the null hypothesis of unit root is rejected too often.

All this results show that it is very important to take into account the presence of cross-country cointegration relationships in testing for unit roots

1.5. __Stationarity tests__

All previous test procedures evaluate the null hypothesis of unit root but, as Hadri (2000) noted, it is a well known fact that the classical hypothesis testing accept the null hypothesis unless there is strong evidence to the contrary. This is confirmed in the time series literature by the fact that the standard unit root tests does not result very powerful against relevant alternatives and fails to reject the null hypothesis for many economic series. To decided whether economic data are stationary or integrated, DeJong and Whiteman (1991) suggest to perform test of the null hypothesis of stationarity as well as of a unit root.

Testing for stationarity in a panel data instead of single time series leads to the same advantage invoked for panel unit root tests: as N grows the power of the test increases and the distributions of the test statistics get asymptotically normal.

Nevertheless, it is also necessary to recall that the time series tests for the null of stationarity tend to have serious size distortions when the null is close to the alternative of a unit root. Panel tests for the null of stationarity are no different in this respect, and caution should be exercised when interpreting the results of panel stationarity tests.

These tests may be used in conjunction with panel data tests for the null of a unit root. Using both kinds of test it is possible to distinguish series that appear to be stationary, series that appear to have a unit root, and series for which it is not possible establish it they are stationary or integrated.

The previously presented Choi's test can also be used for the null of stationarity.

Hadri (2000) proposes a residual based Lagrange Multiplier test which is an extension of stationarity test for time series of Kwiatkowki *et al*. (1992).

1.5.1 Choi's test extension

Choi's testing procedures can be extended to verify the null of stationarity. In this case, the null hypothesis is formulated as:

$$H_0 : |\rho_i| < 1 \quad \text{for all } i \tag{1.5.1.1}$$

which implies that all the time series are stationary, and the alternative hypothesis may be:

$$H_a : |\rho_i| = 1 \quad \text{for at least one } i \text{, for finite } N \tag{1.5.1.2}$$

or:

$$H_a : |\rho_i| = 1 \quad \text{for some } i\text{'s , for infinite } N \tag{1.5.1.2'}$$

Now, if G_{iT_i} is a test for the null of stationarity (e.g. Kwiatkowski *et al.*, 1992; Tsay, 1993; Choi, 1994, ecc…) and $p_i = F(G_{iT_i})$ is the associated *p*-value, the tests and asymptotic theories that we have previously seen can be applied to the hypothesis system (1.5.1.1)-(1.5.1.2').

1.5.2. Hadri (2000) test

Hadri (2000) proposes a parametrization which provides an adequate representation of both stationary and nonstationary variables and permits an easy formulation for a residual-based *LM* test of stationarity. More specifically, Hadri adopts the following components representation:

$$y_{it} = z_{it}'\gamma + r_{it} + \varepsilon_{it} \tag{1.5.2.1}$$

where z_{it} is the deterministic component, r_{it} is a random walk:

$$r_{it} = r_{it-1} + u_{it} \tag{1.5.2.2}$$

$u_{it} \sim \text{iid}(0, \sigma_u^2)$ and ε_{it} is a stationary process. The null hypothesis of trend stationary corresponds to the hypothesis that the variance of the random walk equals zero.
(1.5.2.1) can be written as:

$$y_{it} = z_{it}'\gamma + e_{it} \tag{1.5.2.3}$$

where:

$$e_{it} = \sum_{j=1}^{t} u_{ij} + \varepsilon_{it} \tag{1.5.2.4}$$

Let $\hat{e}_{it}$ be the residuals from the regression in (1.5.2.3), $\hat{\sigma}_e^2$ be a consistent estimator of the error variance under H_0 [22] and let S_{it} be the partial sum process of the residuals:

$$S_{it} = \sum_{j=1}^{t} \hat{e}_{ij} \,. \tag{1.5.2.5}$$

The *LM* statistic can be defined as:

$$LM = \frac{1}{\hat{\sigma}_e^2} \frac{1}{NT^2} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} S_{it}^2 \right). \tag{1.5.2.6}$$

which is consistent and has an asymptotic normal distribution as $(T, N \to \infty)_{\mathrm{seq}}$.

The main advantage of Hadri (2000) test is that the moments of the asymptotic distribution of the Hadri test are exactly derived[23].

Besides, these tests allow the disturbance terms to be heteroscedastic across i. Let consider equation (1.5.2.6): it is sufficient to compute σ_e^2 for each individual time series i, say $\sigma_{e,i}^2$, and then apply the following formula:

$$LM = \frac{1}{\hat{\sigma}_{e,i}^2} \frac{1}{NT^2} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} S_{it}^2 \right) \tag{1.5.2.7}$$

Sometimes it is also possible allow for serial dependence substituting the assumption that the errors ε_{it} are *i.i.d.* $\mathcal{N}\left(0, \sigma_\varepsilon^2\right)$ over t with the assumption that they satisfy the strong mixing regularity conditions of Phillips and Perron (1988) or the linear process conditions of Phillips and Solo (1992). Then it is possible to replace σ_e^2 by the long-run variance σ^2 defined as:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} \lim_{T \to \infty} T^{-1} \left(S_{iT}^2 \right) \tag{1.5.2.8}$$

A consistent estimator of σ^2 can be obtained using one of the estimators found by Andrews and Monahan (1992), Lee and Phillips (1994), Newey and West (1994) and Den Haan and Levin (1996).

Monte Carlo simulations show that *T* and *N* dimensions are very important for the test size. Test size is close to the nominal 5% level if $T > 10$ and it is the correct size if $T > 25$.

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[22] A possible consistent estimator of σ_e^2 is given by $\hat{\sigma}_e^2 = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{e}_{it}^2$ which should be corrected for the degrees of freedom for finite samples.

[23] For all the tests cited above, the critical values or the moments of the asymptotic distributions are evaluated using Monte Carlo simulations or numerical integration.

2. ***Panel unit root tests in the presence of cross-sectional dependence***[24]

All previously presented tests were constructed under the assumption that the individual time series in the panel were cross-sectionally independently distributed.

This condition is needed in order to satisfy the Lindberg-Levy central limit theorem and to obtain asymptotically normal distributed test statistics.

Nevertheless more recently, a large amount of literature (i.e. Backus and Kehoe, 1992) provided evidence on the strong co-movements between economic variables and it was recognized that the assumption of independence across members of the panel is rather restrictive, particularly in the context of cross country (region) regressions. Moreover, this cross-sectional correlation may affect the finite sample properties of panel unit root test[25] (O'Connell, 1998). For instance, the limit distribution of the usual Wald type unit root test based upon *OLS* and *GLS* system estimators depend upon various nuisance parameters defining correlations across individual units. Various attempts to eliminate the nuisance parameters in such systems have been proposed[26]; unfortunately, even if this procedure could partly deal with the problem, it is not appropriate if pair-wise cross-section covariances of the error terms differed across the individual series. This is why new panel unit root tests have been proposed in the literature[27].

To build these tests, a preliminary issue is to specify the cross-sectional dependence. But since individual observations in a cross-section have no natural ordering this specification is not obvious.

Various methods have been developed and they can be organized in two main streams:

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[24] See Table A.2. in the Appendix for a summary of the main characteristics of this second generation of tests. The reader should note that this area of research is very recent and the linked literature is still under development, given the diversity of the potential cross-sectional correlations.

[25] Inappropriately assuming cross-sectional independence can lead to a severe distortion in the size of the test when this assumption is not valid.

[26] For example, the cross-sectionally de-meaning of the series before application of the panel unit root test. This is what has been done by Im, Pesaran and Shin (1997) which consider a simple form of cross-sectional correlation using time-specific effects.

[27] Other attempts to solve the cross-sectional correlation problem are those of Driscoll and Kraay (1998) and Conley (1999). The former presents a simple extension of common nonparametric covariance matrix estimation techniques which yields standard errors that are robust to very general forms of spatial and temporal dependence as the time dimension becomes large. Conley (1999) presents a spatial model of dependence among agents using a metric of economic distance that provides cross-sectional data with a structure similar to time-series data. In this context, a generalized method of moments (GMM) is proposed using the dependent data and a class of nonparametric covariance matrix estimators that allow for a general form of dependence characterized by economic distance.

1. a first more general approach consists in imposing few or none restrictions on the covariance matrix of residuals (O'Connel, 1998; Maddala and Wu, 1999; Taylor and Sarno, 1998; Chang, 2002, 2004).

2. in a second approach the cross sectional dependence is modelled in the form of a low dimensional common factor model, which is estimated and conditioned out prior to construction of the panel unit root test (Bai and Ng, 2004a; Moon and Perron, 2004a; Phillips and Sul, 2003; Pesaran, 2003). The main advantage of factor models is that they allow us to model the cross-sectional dependence using a (small) number of unobserved common factors.

2.1. The covariance restrictions approach

The first attempt to deal with the problem of cross-sectional correlation in panel data was done in O'Connell (1998). He refers to a *GLS*-based unit root test for homogeneous panels and considers a covariance matrix similar to the one that would arise in an error component model with mutually independent random time effects and random individual effects. However, this approach is theoretically valid only when N is fixed. When $N \to \infty$, consistent estimation of the *GLS* transformation matrix is not a well defined concept since the sample cross-section covariance matrix will have rank T when $N > T$ even when the population covariance matrix is rank N.

Another attempt was proposed in Maddala and Wu (1999). They bootstrap the critical values of the *LLC*, *IPS* or *Fisher*'s type test statistics in order to get the empirical distributions and make inferences. This approach results in a decrease of the size distortions due to the cross-sectional correlations, although it does not eliminate them. The main disadvantage of this methodology is that it is technically difficult to implement and Maddala and Wu do not provide the validity of using bootstrap methodology for panel data.

More recently, Chang (2004) proposed a second generation bootstrap unit root test that, contrary to the previous tests, successfully overcomes the nuisance parameters problem in panels with cross-sectional dependence.

2.1.1. Chang (2002) test

Chang (2004) applies bootstrap methods to Taylor and Sarno's (1998) multivariate *ADF* and other related tests and computes appropriate critical values by conditioning on the estimated cross sectional dependence. More specifically, in her general framework, each panel is driven by general linear processes which may differ across cross-sectional units; she approximates these processes by an autoregressive integrated process of finite order which increases as T grows. In order to take into account the dependence among the innovations generating the individual series, a unit root tests based on the estimation of the whole system of N equations is suggested. The limit distributions of the tests are derived as T goes to infinity and N is fixed. Therefore bootstrap methodology is applied to approximate autoregressions and obtain the critical values for the panel unit root tests based on the original sample. To overcome the inferential difficulty of standard panel unit root tests in the presence of cross-sectional dependence, that have non standard limit distributions (i.e. *LLC*, *IPS*), Chang proposes to use the bootstrap method and shows that bootstrap panel unit root tests are consistent and perform well in finite samples relative to the *IPS* t-bar statistic.

Chang (2002) also proposes an alternative non-linear instrumental variable (IV) approach. As previous, the goal is to solve the nuisance parameter problem and to do this, Chang (2002) tries to render the panel statistics asymptotically invariant to cross sectional dependence: for each cross-section unit, she estimates the AR coefficient from an usual *ADF* regression using the instruments generated by an integrable transformation of the lagged values of the endogenous variable. Then the author constructs N individual t-statistics for testing the unit root based on these N nonlinear IV estimators. These t-statistics have limiting standard normal distribution under the null hypothesis. Finally, a cross-sectional average of the individual IV t-ratio statistics is considered, as in the *IPS* approach.

Specifically, Chang considers a panel model generated by a first-order autoregressive regression:

$$y_{it} = \rho_i y_{it-1} + u_{it}, \qquad (2.1.1.1)$$

where as usual $i = 1,..., N$ denotes individual cross-sectional units and $t = 1,..., T_i$ denotes time series observations. Note that the total number T for each individual i may differ across cross-sectional units, i.e. unbalanced panels are allowed.

The initial values $(y_{10},..., y_{N0})$ are set at zero for simplicity.

The error term u_{it} is given by an $AR(p_i)$ invertible process:

$$\lambda^i(L) u_{it} = \varepsilon_{it} \qquad (2.1.1.2)$$

where $\lambda^i(L)=1-\sum_{j=1}^{p_i}\beta_{ij}L^j$ being L the usual lag operator. Cross-sectional dependence of the innovations $\varepsilon_{it} \sim i.i.d.(0,\sigma_{\varepsilon_i}^2)$ that generate the errors u_{it}'s is allowed.

The null hypothesis of interest is:

$$H_0 : \rho_i = 1 \text{ for all } y_{it}$$

against the alternative:

$$H_a : |\rho_i| < 1 \text{ for some } y_{it}.$$

Thus, the null implies that all y_{it}'s have unit roots, and it is rejected if any one of y_{it}'s is stationary with $|\rho_i| < 1$. The rejection of the null therefore does not imply that the whole panel is stationary.

Giving the (2.1.1.1) and (2.1.1.2) it is now possible re-write the model as:

$$y_{it} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \beta_{ij} u_{it-j} + \varepsilon_{it}.$$

and, since $\Delta y_{it} = u_{it}$ under the unit root null hypothesis, the above regression becomes:

$$y_{it} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \beta_{ij} \Delta y_{it-j} + \varepsilon_{it} \qquad (2.1.1.3)$$

To deal with the cross-sectional dependence, Chang uses the instrument generated by a nonlinear function $F(\cdot)$ of lagged values y_{it-1}, i.e. $F(y_{it-1})$ which is called the *Instrument Generating Function* (IGF).

$F(\cdot)$ is a regularly integrable function which satisfies $\int_{-\infty}^{\infty} xF(x)dx \neq 0$, i.e. the nonlinear instrument $F(\cdot)$ is correlated with the regressor y_{it-1}.

For the lagged demeaned differences $x'_{it} = (\Delta y_{it-1},..., \Delta y_{it-p_i})$, the variables themselves are used as the instruments.

Let $X_i = (x_{ip_i+1},..., x_{iT})'$ be the (T, p_i) matrix of the lagged differences, let $y_{li} = (y_{ip_i},..., y_{iT-1})$ be the vector of lagged values and $\varepsilon_i = (\varepsilon_{ip_i+1},..., \varepsilon_{iT})'$ be the vector of residuals.

The augmented regression (2.1.1.3) can be written in matrix form as

$$y_i = y_{li}\rho_i + X_i\beta_i + \varepsilon_{it} \qquad (2.1.1.4)$$

where $\beta_i = (\beta_{i1},..., \beta_{ip_i})'$. Under the null, the nonlinear IV estimator of the parameter ρ_i denoted $\hat{\rho}_i$, is:

$$\hat{\rho}_i = \left[F(y_{li})' y_{li} - F(y_{li})' X_i (X_i' X_i)^{-1} X_i' y_{li} \right]^{-1} \left[F(y_{li})' \varepsilon_i - F(y_{li})' X_i (X_i' X_i)^{-1} X_i' \varepsilon_i \right]$$

and its variance is:

$$\hat{\sigma}^2_{\hat{\rho}_i} = \hat{\sigma}^2_{\varepsilon_i} \left[F(y_{li})' y_{li} - F(y_{li})' X_i (X_i' X_i)^{-1} X_i' y_{li} \right]^{-2}$$
$$\left[F(y_{li})' F(y_{li}) - F(y_{li})' X_i (X_i' X_i)^{-1} X_i' F(y_{li}) \right]$$

where $\hat{\sigma}^2_{\varepsilon_i} = (1/T) \sum_{t=1}^{T_i} \hat{\varepsilon}^2_{it}$ and $\hat{\varepsilon}_{it}$ is the fitted residual from augmented regression (2.1.1.3).

For testing the unit root hypothesis $H_0 : \rho_i = 1$ for all y_{it}, Chang constructs a *t*-ratio statistic from the nonlinear IV estimator $\hat{\rho}_i$, denoted Z_i:

$$Z_i = \frac{\hat{\rho}_i - 1}{\hat{\sigma}_{\hat{\rho}_i}} \underset{T \to \infty}{\overset{d}{\to}} \mathcal{N}(0,1) \text{ for all } i = 1,...N$$

which asymptotically converges to a standard normal distribution if a regularly integrable function is used as an IGF.

This asymptotic Gaussian result is fundamentally different from the usual unit root limit theories and it is entirely due to the nonlinearity of the IV. More importantly, the limit distributions of individual Z_i statistics are cross-sectionally independent. So, these asymptotic orthogonalities lead to propose a panel unit root test based on the cross-sectional average of these individual independent statistics. Chang proposes an *average IV t-ratio statistic*, defined as:

$$S_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} Z_i \ .$$

The factor $N^{-1/2}$ is used just as a normalization factor. S_N results having a limit standard normal distribution[28]. Then by this result it is possible to do simple inference based on the standard normal distribution even for unbalanced panels with general cross-sectional dependence.

More specifically, Chang's limit theory does not require a large spatial dimension; consequently N may take any value, large or small.

Finally, the model with deterministic components can be analyzed similarly using demeaned or detrended data.

[28] It should be noted that the usual sequential asymptotic is not used here. The limit theory is derived for $T \to \infty$, which in not followed by $N \to \infty$.

Chang (2002) asserts that her approach is very general and has good finite sample properties. Her simulation results show that the finite sample sizes of S_N calculated from using the standard normal critical values quite closely approximate the nominal test sizes. Moreover, the test S_N has noticeably higher discriminatory power than the commonly used *IPS t*-bar tests. The panel nonlinear IV unit root test seems to improve significantly upon the *t*-bar test under cross-sectional dependency, especially for the panels with smaller time and spatial dimensions.

However, Im and Pesaran (2003) showed that Chang's test is valid only if N is fixed as $T \to \infty$. Their Monte Carlo simulations show that Chang's test is grossly over-sized for moderate degrees of cross section dependence, even for relatively small values of N.

2.2. The factor structure approach

Pesaran (2003), Bai and Ng (2002), Moon and Perron (2004a), and Phillips and Sul (2003) treated the cross-section dependence by allowing the common factors to have differential effects on different cross section units.

In the context of a residual one-factor model Phillips and Sul (2003) showed that when there is cross-sectional dependence the standard panel unit root tests are no longer asymptotically similar. Thus, they proposed an orthogonalization procedure to asymptotically eliminate the common factors before applying standard panel unit root tests and provide asymptotic results for $(T, N \to \infty)_{\text{seq}}$.

Moon and Perron (2004a) and Bai and Ng (2004a) provided similar orthogonalization procedures in a more general context.

Moon and Perron (2004a) proposed a pooled panel unit root test based on "de-factored" observations in which the factor loadings are estimated by the principal component method. Deriving asymptotic properties of the test under the unit root null hypothesis and local alternatives, as N and $T \to \infty$ with $N/T \to 0$, this test has good asymptotic power properties if the model does not contain deterministic trends[29].

Bai and Ng (2004a) specify the model allowing for the possibility of unit roots (and cointegration) in the common factors. They consider the first-differenced version of the model

[29] Moon, Perron and Phillips (2005) propose a point optimal invariant panel unit root test which is shown to have local power even in the presence of deterministic trends.

and apply the principle component procedure. Standard unit root tests are then applied to the factors loadings and the individual de-factored series, both computed as partial sums of the estimated first differences. Also in this case asymptotic properties are derived as N and $T \to \infty$ with $N/T \to 0$.

In order to describe these approaches to panel unit root testing in the presence of cross-sectional correlation, let assume a common factor representation in which an observed data series y_{it} can write as the weighted sum of (unobserved) common and idiosyncratic components:

$$y_{it} = \sum_{k=1}^{K} D_{ik}(L)\eta_{kt} + C_i(L)\varepsilon_{it}, \tag{2.2.1}$$

where $i = 1,...,N$ are the cross-sectional units, $t = 1,...,T$ the time series observations and $k = 1,...,K$ the common factors and $K << N$.

The common shock terms η_{kt} are assumed to be $i.i.d.(0, \sigma_{f_k}^2)$ variables, and the idiosyncratic errors ε_{it} are also $i.i.d.(0, \sigma_{e_i}^2)$ with η_{kt} and ε_{it} mutually independent for all i, k, t. The lag polynomials $D_{ik}(L) = \sum_{j=1}^{\infty} d_{ikj} L^j$, where L is the lag operator, describe the (dynamic) dependence of the observed data on the common factor, and $C_i(L) = \sum_{j=1}^{\infty} c_{ij} L^j$ generate individual specific dynamics. Pesaran's (2003), Moon and Perron's (2004a) and Bai and Ng's (2004a) models can be obtained from (2.2.1) by suitable restrictions on its lag polynomials.

2.2.1. Pesaran (2003) test

Pesaran (2003) presents a new and simple procedure for testing unit roots in dynamic panels subject to possibly cross sectionally dependent as well as serially correlated errors.

In this approach, the observations y_{it} are supposed to be generated according to a simple dynamic linear heterogeneous panel data model:

$$y_{it} = (1-\rho_i)\mu_i + \delta_i y_{it-1} + u_{it} \tag{2.2.1.1}$$

where μ_i is a deterministic component, the initial values y_{i0} are given and the disturbances follow a one-factor structure:

$$u_{it} = \lambda_i f_t + \varepsilon_{it}. \tag{2.2.1.2}$$

The idiosyncratic shocks, ε_{it} are assumed to be independently distributed both across i and t with zero mean, variance σ_i^2 and finite forth-order moments.

The unobserved common factor f_t is serially uncorrelated with zero mean, constant variance σ_f^2 and finite forth-order moment. Without loss of generality, σ_f^2 is set equal to one. The variables ε_{it}, λ_i and η_t are assumed to be mutually independent for all i.

The assumptions made about ε_{it} and f_t imply serial uncorrelation for the u_{it}[30].

(2.2.1.1) and (2.2.1.2) can more conveniently be written as:

$$\Delta y_{it} = \alpha_i - (1-\rho_i) y_{it-1} + \lambda_i f_t + \varepsilon_{it} \tag{2.2.1.3}$$

being $\alpha_i = (1-\rho_i)\mu_i$ and $\Delta y_{it} = y_{it} - y_{it-1}$. Pesaran (2003) considers the following unit root hypothesis:

$$H_0 : \rho_i = 1 \text{ for all } i$$

against the possibly heterogeneous alternatives:

$$H_1 : \begin{cases} \rho_i < 1 & \text{for } i = 1,\ldots,N_1 \\ \rho_i = 1 & \text{for } i = N_1,\ldots,N \end{cases}$$

where the fraction of the stationary individuals is such that $N_1/N \to \kappa$, as $N \to \infty$ with $0 < \kappa \le 1$.

Instead of basing the unit root tests on deviations from the estimated common factors, Pesaran (2003) proposes a test based on standard unit root statistics in a *Cross-sectionally Augmented DF* (*CADF*) regression - that is a *DF* (or *ADF*) regression which is augmented with the cross section averages of lagged levels and first-differences of the individual series[31]:

$$\Delta y_{it} = a_i + b_i y_{it-1} + c_i \bar{y}_{t-1} + d_i \Delta \bar{y}_t + e_{it} \tag{2.2.1.4}$$

where $\bar{y}_t = N^{-1}\sum_{j=1}^{N} y_{jt}$, $\Delta\bar{y}_t = N^{-1}\sum_{j=1}^{N}\Delta y_{jt}$ and e_{it} is the regression error[32].

[30] This assumption and the assumption that $K = 1$ (there is only one common factors) could be relaxed. Pesaran (2003) considers an example where a stationary *p*-th order autoregression for u_{it} is obtained including *p* lagged values of u_{it} in (2.2.1.2). Then it is possible to write $u_{it} = \lambda_i f_t + e_{it}$ where $f_t = \Phi(L)\eta_t$ and $e_{it} = \Phi(L)\varepsilon_{it}$ are stationary and invertible MA processes, and $\Phi(L)^{-1}$ is the AR polynomial of u_{it}. Note that in this setting any non-stationarity of the y_{it} is due to the presence of a unit root in the autoregressive part of (2.2.1.1): the common factor f_t is always assumed to be stationary.

[31] This is a natural extension of the DF approach in order to deal with residual serial correlation where lagged changes of the series are used to filter out the time series dependence when *T* is sufficiently large.

[32] Note that (2.2.1.4) is valid for serially uncorrelated u_{it}. For the more general case, lagged values of Δy_{it}, but also of $\Delta\bar{y}_t$ need to be included in the estimation.

The cross-sectional average $\bar{y}_t$ and its lagged values are included into (2.2.1.4) as a proxy for the unobserved common factor f_t[33].

Let $CADF_i$ be the *ADF* statistic for the *i*-th cross-sectional unit given by the *t*-ratio of the *OLS* estimate $\hat{b}_i$ of b_i in the *CADF* regression (2.2.1.4).

Individual *CADF* statistics are used to develop a modified version of the *IPS t*-bar test (denoted *CIPS* for *Cross-sectionally Augmented IPS*) that simultaneously take account of cross-section dependence and residual serial correlation:

$$CIPS = \frac{1}{N}\sum_{i=1}^{N} CADF_i \,. \qquad (2.2.1.5)$$

The asymptotic null distributions of the single *CADF* statistics are similar and do not depend on the factor loadings. Unfortunately, *CADF* statistics are correlated due to their dependence on the common factor. Then, even if *CIPS* statistics can be built, it is not possible to apply standard central limit theorems to them. Moreover, in contrast to the results obtained by Im *et al*. (2003) under cross-sectional independence[34], the distribution of the *CIPS* statistic is shown to be non-standard even for large *N*.

Pesaran also considers a truncated version of *CADF* (*CADF**) to avoid excessive influence of extreme outcomes that could arise for small *T* samples. The results for *CADF* and *CIPS* are valid also for *CADF** and the related *CIPS** and, even if it is not normal, the null asymptotic distribution of *CIPS** statistic exists and is free of nuisance parameter.

He proposes simulated critical values of *CIPS* for various samples sizes and three specification of deterministic components (i.e. models without intercept or trend, models with individual-specific intercepts and models with incidental linear trends).

Following Maddala and Wu (1999) or Choi (2001), Pesaran also proposes *Fisher* type tests based on the significant levels of individual *CADF* statistics. In this case as well the statistics do not have standard distributions because of the previous reasons.

Finally, Pesaran (2003) extends his approach to serially correlated residuals.

For an AR(p) error specification, the relevant individual *CADF* statistics can be computed from a p^{th} order cross-section/time series augmented regression:

$$\Delta y_{it} = \alpha_i + \rho_i y_{it-1} + c_i \bar{y}_{t-1} + \sum_{j=0}^{p} d_{ij}\Delta\bar{y}_{t-j} + \sum_{j=0}^{p}\beta_{ij}\Delta y_{t-j} + \mu_{it}$$

[33] This approximation is applicable if $\bar{\lambda} = N^{-1}\sum_{j=1}^{N}\lambda_j$ and $\bar{\lambda} \neq 0$ for a fixed *N* and as $N \to \infty$ (Pesaran, 2003).

[34] In this case, a standardized average of individual ADF statistics was normally distributed for large *N*.

Finally, note that Pesaran's (2003) *CADF* and *CIPS* tests are designed for testing for unit roots when cross-sectional dependence is due to a single common factor but the *CIPS* test has better power properties than the individual *CADF* tests and should therefore be preferred.

2.2.2. The Phillips and Sul (2003) and Moon and Perron (2004a) tests

Phillips and Sul (2003) and Moon and Perron (2004a) propose tests for the null of a unit root in the observable series y_{it}. These approaches are not based on separate tests on the individual and common components and use the factor model in a similar way: this is why these tests are both illustrated in this section.

First Moon and Perron procedure is presented allowing for a more general specification of the common components respect to the Phillips and Sul test.

Moon and Perron (2004a) test

Moon and Perron (2004a) represent the observation series y_{it} as AR(1) processes with fixed effects and assume, as Pesaran (2003), that common factors are present in the error term. They consider the following dynamic panel model:

$$\begin{aligned} y_{it} &= \mu_i + x_{it} \\ x_{it} &= \rho_i x_{it-1} + u_{it} \\ u_{it} &= \lambda_i' f_t + e_{it} \end{aligned} \qquad (2.2.2.1)$$

where the observed y_{it} ($i = 1,2,...,N$; $t = 1,2,...,T$) are generated by a deterministic component μ_i and an autoregressive process x_{it} and $x_{i0} = 0$ for all i.

To model the correlation among the cross-sectional units, the error component u_{it} is assumed to follow an approximate factor model where f_t is a $(K \times 1)$ vector of unobservable random factors, λ_i is the corresponding vector of non-random factor loading for cross-section i and e_{it} is an idiosyncratic shock. The number of factors K is possibly unknown.

As it is easy to note panel data are assumed to be generated by idiosyncratic shocks and unobservable dynamic factors that are common to all the individual units but to which each individual reacts heterogeneously.

Model (2.2.2.1) can also be re-written as:

$$\Delta y_{it} = (1-\rho_i)\mu_i + \rho_i y_{it-1} + u_{it}. \tag{2.2.2.2}$$

Comparing (2.2.2.2) and (2.2.1.1) it is now straightforward to note that Pesaran (2003) and Moon and Perron (2004a) models are identical in the case where a single common factor is present in the composite error term.

For the error term u_{it} in (2.2.2.1) similar assumptions are made as by Pesaran (2003):

- the idiosyncratic part e_{it} follows a stationary and invertible infinite MA process, and is cross-sectionally uncorrelated: $e_{it} = \Gamma_i(L)\varepsilon_{it}$, where $\Gamma_i(L) = \sum_{j=0}^{\infty}\gamma_{ij}L^j$ and $\varepsilon_{it} \sim i.i.d.(0,1)$ across i and t with finite eighth moment;

- also the common factor f_t follows a stationary, invertible MA(∞) representation: $f_t = \Phi(L)\eta_t$, where $\Phi_i(L) = \sum_{j=0}^{\infty}\phi_{ij}L^j$ is a K-dimensional lag polynomial and $\eta_t \sim i.i.d.(0, I_K)$. Furthermore, the covariance matrix of f_t is (asymptotically) positive definite: this implies that under the null hypothesis it is possible to have cointegrating relations among the nonstationary factors;

- there exists at least one common factor in the data but their maximum number $\overline{K}$ $\left(1 \le K \le \overline{K} < \infty\right)$ is supposed to be known *a priori*. Also, the contribution from each factor to at least one of the y_{it} is assumed to be significant by imposing $\frac{1}{N}\sum_{i=1}^{N}\lambda_i\lambda_i' \xrightarrow{p} \Sigma_\lambda > 0$; however this assumption does not impose that all cross-sections respond to all factors so that some of the factor loadings could be zero;

- short-run variance $\sigma_{e_i}^2$ $(=\sum_{j=0}^{\infty}\gamma_{ij}^2)$, long-run variance $\omega_{e_i}^2$ $(=\left(\sum_{j=0}^{\infty}\gamma_{ij}\right)^2)$ as well as the one-sided long-run covariance φ_{e_i} $(=\left(\sum_{l=1}^{\infty}\sum_{j=0}^{\infty}\gamma_{ij}\gamma_{ij+l}\right))$ are supposed to exist for all idiosyncratic disturbances e_{it}; additionally, these parameters are assumed to have non-zero cross-sectional averages $\sigma_e^2 = \frac{1}{N}\sum_{i=1}^{N}\sigma_{e_i}^2$, $\omega_e^2 = \frac{1}{N}\sum_{i=1}^{N}\omega_{e_i}^2$ and $\varphi_e^2 = \frac{1}{N}\sum_{i=1}^{N}\varphi_{e_i}^2$.

In referring to the model (2.2.2.1), the null hypothesis of interest is:

$$H_0 : \rho_i = 1 \text{ for all } i = 1,...,N \tag{2.2.2.3}$$

against the heterogeneous alternative:

$$H_a : |\rho_i| < 1 \text{ for some } i$$

It is simple to note that under the null, y_{it} in (2.2.2.1) results to be influenced by two components: the integrated factors $\sum_{s=1}^{T} f_s$ and the integrated idiosyncratic errors $\sum_{s=1}^{T} e_{is}$.

The model allows for both integrated or cointegrated factors (in this case the rank of $\Phi(1)$ is $r < K$) but excludes the possibility of cointegrating relations of the integrated idiosyncratic errors.

When common factors are present in the panel, tests based on the assumption of cross-sectional independence among units suffer from size distortions. To pass this difficulty, Moon and Perron (2004a) transform the model in order to eliminate the common components of the y_{it} series and apply the unit root test on de-factored series. Resulting test statistics have a normal asymptotic distributions as those of Im *et al*. (2003) or Levin and Lin (1992, 1993); moreover being computed from de-factored data, they are also independent in the individual dimension.

More specifically, to remove cross-sectional dependence in (2.2.2.1), Moon and Perron use a projection onto the space orthogonal to the factor loadings (i.e. the space generated by the columns of the matrix of factor loading $\Lambda = (\lambda_1, ..., \lambda_N)'$). Then, Λ is estimated[35] to construct a projection matrix $Q_\Lambda = I_N - \Lambda(\Lambda'\Lambda)^{-1}\Lambda'$.

Let $\hat{\Lambda}$ and $Q_{\hat{\Lambda}_k}$ be the estimator of the matrix Λ and the corresponding estimator of the projection matrix.

Consistent estimates of the above defined nuisance parameters can be obtained non-parametrically from the de-factored residuals $\hat{e} = \hat{u}Q_{\hat{\Lambda}_k}$, $\hat{u} = (\hat{u}_1, ..., \hat{u}_N)$ with $\hat{u}_i = (\hat{u}_{i1}, ..., \hat{u}_{iT})'$. Denote the estimates of short-run and long-run variances, $\sigma^2_{e_i}$ and $\omega^2_{e_i}$, as $\hat{\varphi}_{e_i}$ and $\hat{\omega}^2_{e_i}$ respectively, and let $\hat{\varphi}_e$ and $\hat{\omega}^2_e$ be their cross-sectional averages[36].

The unit root test is implemented from the de-factored data obtained as YQ_Λ (being Y the matrix of observed data). Specifically, the modified pooled estimator of δ suggested by Moon and Perron (2004a) is:

$$\rho^*_{\text{pool}} = \frac{tr\left(Y_{-1}Q_{\hat{\Lambda}_k}Y'\right) - NT\hat{\varphi}_e}{tr\left(Y_{-1}Q_{\hat{\Lambda}_k}Y'_{-1}\right)} \tag{2.2.2.4}$$

where Y_{-1} is the matrix of lagged observed data and $tr(\cdot)$ the trace operator.

In order to obtain feasible statistics, Moon and Perron procedure requires estimating the number K of factors in (2.2.2.1), apart from the projection matrix Q_{Λ_k}.

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[35] For this purpose, Moon and Perron suggest to use the principal component method used in Stock and Watson (1998) and Bai and Ng (2004a).

[36] For these estimates, see Moon and Perron (2004a).

From the estimator ρ^*_{pool}, two modified t-statistics based on pooled estimation of the first-order serial correlation coefficient of the data are suggested for the null hypothesis (2.2.2.2):

$$t^*_\alpha = \frac{T\sqrt{N}\left(\hat{\rho}^+_{\text{pool}} - 1\right)}{\sqrt{\dfrac{2\hat{\phi}^4_e}{\hat{\omega}^4_e}}} \tag{2.2.2.5.a}$$

$$t^*_b = T\sqrt{N}\left(\hat{\rho}^+_{\text{pool}} - 1\right)\sqrt{\frac{1}{NT^2} tr\left(Y_{-1} Q_{\hat{\Lambda}_k} Y'_{-1}\right)\frac{\hat{\omega}^2_e}{\hat{\phi}^4_e}} \tag{2.2.2.5.b}$$

being $\hat{\rho}^+_{\text{pool}}$ the bias-corrected pooled autoregressive estimate of (2.2.2.4) and $\hat{\phi}^4_e$ the estimate of and the cross sectional average of $\omega^4_{\hat{e},i}$.

Moon and Perron show that as N and $T \to \infty$, with $N/T \to 0$, the statistics (2.2.2.5.a) and (2.2.2.5.b) have a limiting standard normal distribution under the null hypothesis, while diverge under the stationarity alternative.

It is possible to note that the convergence rate of the pooled estimator (corrected or no) of the autoregressive root is the same as the one obtained in the *LLC* model: in fact, removing the cross unit dependence in the Moon and Perron model, the model which is obtained on transformed data is similar to the *LLC* model with common autoregressive root, under the cross-unit independence hypothesis.

Finally, Moon and Perron simulations show that the tests are very powerful and have good size when no estimation of deterministic components is necessary (i.e. only a deterministic constant is included in the model) for different specifications and different values of *T* and *N*. When such estimation is necessary, the tests have no power beyond their size.

Note that the Moon and Perron (2004a) tests using defactored data allow for multiple common factors. Therefore, their use has to be recommended when cross-section dependence is expected to be due to several common factors.

Phillips and Sul (2003) test

Phillips and Sul (2003) consider a rather more restrictive model than Moon and Perron (2004a) which contains only one factor f_t independently distributed as $\mathcal{N}(0,1)$ across time. The main difference existing between the two tests stays in the approach used to remove the common factors: Moon and Perron suggested to estimate the projection matrix Q_Λ using principal component analysis; Phillips and Sul propose to use a moment-based method.

The obtained de-factored data are used to compute a series of panel unit root tests. The first one is defined as:

$$G_{OLS}^{+} = \frac{1}{\sqrt{N}\sigma_{\xi}} \sum_{i=1}^{N-1} \left[\frac{\hat{\rho}_i^{+} - 1}{\hat{\sigma}_{\hat{\rho}_i^{+}}} - \mu_{\xi} \right] \xrightarrow{d} \mathcal{N}(0,1) \tag{2.2.2.6}$$

where $\hat{\rho}_i^{+}$ and $\hat{\sigma}_{\hat{\rho}_i^{+}}$ are the cross-sectional autoregressive estimates and their standard errors computed for each i from the de-factored data[37] and μ_{ξ} and σ_{ξ} are the mean and standard error of the statistic. Phillips and Sul show that G_{OLS}^{+} converges to a standard normal distribution as $(T, N \to \infty)_{\mathrm{seq}}$.

As in Bai and Ng (2004a), Phillips and Sul also propose tests based on the meta-analysis. Specifically the better test seems to be the *inverse normal test* given by:

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N-1} \Phi^{-1}\left(p_{\hat{e}_i}^{c}\right), \tag{2.2.2.7}$$

where $p_{\hat{e}_i}^{c}$ is the *p*-value of the *ADF* test associated with cross section element i, and $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function for a standard normal variable. Expression (2.2.2.7) converges to a standard normal distribution.

Note finally that both tests (2.2.2.6) and (2.2.2.7) require summing up only $N-1$ elements because the Phillips and Sul procedure reduces the cross sectional dimension by 1.

2.2.3. Bai and Ng test (2004a)

Bai and Ng (2004a) propose a different procedure to test for panel unit root allowing for cross-section correlation or cointegration. It does not treat cross-section dependence as a disturbance as the previously tests did: the nature of the comovements of economic variable are themselves an object of interest of the analysis.

In the context of model (2.2.2.1), assuming that the null hypothesis (2.2.2.2) holds and that (2.2.2.1) admits only one factor, this approach is based on the decomposition of each panel series y_{it} in the sum of a deterministic component, a common –stochastic- component (all the common factors) and an individual component (the idiosyncratic error term):

[37] Phillips and Sul (2003) also propose the G_{EMS}^{+} test based on the median estimates of ρ_i^{+} which seems to have marginal better property than the G_{OLS}^{+} test.

$$y_{it} = D_{it} + \lambda_i' F_t + E_{it} \qquad (2.2.3.1)$$

where the deterministic component D_{it} is a polynomial function containing either a constant α_i or a linear trend $\alpha_i + \beta_i t$, λ_i is a $(K \times 1)$ vector of factor loadings, F_t is a $(K \times 1)$ vector of common factors[38], and E_{it} is an error term.

It is worth noting that the presence of the common factor F_t, with his specific elasticity λ_i, is at the origin of the cross-sectional dependence.

As Pesaran (2003) and Moon and Perron (2004b), Bai and Ng (2004a) consider a balanced panel with N cross-sectional units and T time series observations.

A series with this factor structure is nonstationary if at least one common factor of the vector F_t is nonstationary and/or the idiosyncratic error is nonstationary[39].

The possibility that one or more common factors are integrated allows Bai and Ng test for considering possible presence of cross-section cointegration relationships.

Rather than directly testing the nonstationarity of y_{it} ($i = 1,...,N$)[40], this approach analyzes the common and individual components separately. This is why, it is referred as PANIC (*Panel Analysis of Nonstationarity in the Idiosyncratic and Common components*). Therefore the aim of *PANIC* is to determine if nonstationarity comes from a pervasive (F_t) or an idiosyncratic source (E_{it}) and to construct valid pooled tests for panel data when the units are correlated.

This procedure potentially solves three econometric problems: 1) firstly, it solves the problem of the size distortion[41]; 2) since in this approach y_{it} will be strongly correlated across units if the data follow a factor structure (in previous factor model the idiosyncratic components can only be weakly correlated across *i* by design), pooled tests based upon e_{it} are more likely to satisfy the cross-section independence assumption required for pooling; 3) pooled tests exploit cross-section information and are more powerful than univariate unit root tests.

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[38] K is assumed to be known.

[39] Under the null hypothesis of unit root the data in the Pesaran's (2003) or Moon and Perron's (2004b) model contains a common, as well as an idiosyncratic stochastic trend.

[40] It is well known that, if a series is defined as the sum of two components with different dynamic properties, has itself dynamic properties which are very different from its entities. Since common factor and idiosyncratic term can have different dynamic properties, it may difficult to check the stationarity of y_{it} if this series contains a large stationary component. This is why, Bai and Ng suggest to separately test the presence of unit root in the common and individual components.

[41] For example, existing tests tend to over-reject the null hypothesis of nonstationarity when the series being tested is the sum of a weak I(1) component and a strong stationary component.

First of all, to analyze the factors F_t and the idiosyncratic components e_{it} that are both unobserved, Bai and Ng try to find consistent estimates of these series preserving their integration features[42].

The complete model considered by Bai and Ng can be written as follow:

$$y_{it} = c_i + \beta_i t + \lambda_i' F_t + e_{it} \tag{2.2.3.2.a}$$

$$F_t = F_{t-1} + f_t \tag{2.2.3.2.b}$$

$$e_{it} = \rho_i e_{it-1} + \varepsilon_{it} \tag{2.2.3.2.c}$$

where $f_t = \Phi(L)\eta_t$ and $\Phi(L) = \sum_{j=0}^{\infty} \phi_j L^j$ is a K–dimensional lag polynomial and $rank(\Phi(1)) = k_1$; $\eta_t \sim i.i.d.(0, \Sigma_\eta)$ with finite fourth-order moment. Then, the F_t are assumed to follow an AR(1) process that contains $k_1 \leq K$ independent stochastic trends and consequently $K - k_1$ stationary components.

The idiosyncratic terms e_{it} are also modelled as AR(1) processes and are allowed to be either I(0) or I(1); ε_{it} follows a mean zero, stationary, invertible MA process, such that $\varepsilon_{it} = \Gamma_i(L) \in_{it}$ with $\in \sim i.i.d.(0, \sigma^2_{\in_i})$.

Bai and Ng (2004a) impose the cross-sectional independence of the idiosyncratic term[43] only to validate pooled testing. It is obvious that the assumption that Σ_η is not (necessarily) a diagonal matrix is more general than the assumption of uncorrelation for the innovations of the common factors made by Moon and Perron (2004a). ΔF_t has a short-run covariance matrix of full rank while, as $rank(\Phi(1)) = k_1$, the long-run covariance matrix has reduced rank and permits cointegration among the common factors. As in Moon and Perron (2004a), (asymptotically) redundant factors are ruled out.

So, the objective of PANIC is to determine the number of non-stationary factors k_1 and to test whether $\rho_i = 1$ for each $i = 1,...,N$.

Bai and Ng accomplish this goal by utilizing a suitable transformation of y_{it}. Specifically, if the intercept only is included in the model (i.e. $y_{it} = c_i + \lambda_i' F_t + e_{it}$), the first differences are

[42] In other words, the common variations must be extracted without appealing to stationarity assumptions and/or cointegration restrictions.

[43] Bai and Ng(2004a) permit some weak cross-sectional dependence of the shock terms driving the ε_{it}. The full set of assumptions can be found in their paper.

employed, $\Delta y_{it} = y_{it} - y_{it-1}$, while in the presence of a linear trend ($y_{it} = c_i + \beta_i t + \lambda_i' F_t + e_{it}$) y_{it} is de-trended, i.e. $y_{it}^d = \Delta y_{it} - \Delta \bar{y}$, where $\Delta \bar{y}_{it} = \frac{1}{T-1} \sum_{t=2}^{T} \Delta y_{it}$.

The Bai and Ng (2004a) procedure implies, in a first step, to estimate the common factors and idiosyncratic errors in Δy_{it} or y_{it}^d by a simple principal component method; in a second step these estimators, denoted as $\hat{f}_t$ and $\hat{e}_{it}$ respectively, have to be re-cumulate to remove the effect of possible over-differencing. This yields:

$$\hat{F}_t = \sum_{s=2}^{T} \hat{f}_s \qquad \hat{E}_{it} = \sum_{s=2}^{T} \hat{e}_{is} \tag{2.2.3.3}$$

Now it is simple to test for the null hypothesis of a unit root the common factor $\hat{F}_t$ and each idiosyncratic components $\hat{E}_{it}$ separately:

- for the first component, when only a factor is detected[44], Bai and Ng use the *ADF* test; when more than one factor is detected, they employ a modified version of Stock and Watson (1988) common trend test;
- for the idiosyncratic components, a method based on meta-analysis is used[45].

- Common factors stationarity analysis

In order to test the nonstationarity of the common factors, Bai and Ng (2004a) suggest to use either an *ADF* test, or a rank test, depending on whether there is only one, or several common factors[46].

In the former case (i.e. $K = 1$), consider:

$$\Delta \hat{F}_{it} = D_{it} + \theta_0 \hat{F}_{it-1} + \sum_{j=1}^{p} \theta_{ij} \Delta \hat{F}_{it-j} + \xi_{it} \tag{2.2.3.4}$$

where ξ_{it} is the regression error and D_{it} is defined as in equation (2.2.3.1).

Now, denote the *t*-statistic for $\theta_0 = 0$ as $ADF_{\hat{F}}^c$ (for the intercept only case) or $ADF_{\hat{F}}^{\tau}$ (for the linear trend case). Limiting distributions of these statistics are *DF* type distributions[47].

[44] The number of factors is estimated by using Bai and Ng's (2002) procedure.
[45] This procedure was been originally presented in Maddala and Wu (1999) and in Choi (2001).
[46] It is straightforward to verify that $k_1 = 0$ corresponds to the case where there are *N* cointegrating vectors for *N* common factors, i.e. all factors are *I*(0).
[47] The asymptotic 5% critical values are -2.86 and -3.41, respectively.

In the latter case ($K > 1$), individually testing each of the factors for the presence of a unit root generally overstates the number of common trends. So, to select k_1 (i.e. the number of common independent stochastic trends in the common factors) Bai and Ng implement an iterative procedure, similar to the Johansen (1988) trace test for cointegration.

They use demeaned or de-trended factor estimates, depending on whether the model (2.2.3.1) contain the intercept only, or also a linear trend.

Then, they define $\tilde{F}_t = \hat{F}_t - \overline{\hat{F}_t}$ where $\overline{\hat{F}_t} = (T-1)^{-1}\sum_{t=2}^{T}\hat{F}_t$ in the intercept only case; in the linear trend case, $\tilde{F}_t$ represents the residuals from a regression of $\hat{F}_t$ on a constant and linear trend.

Using this defined $\tilde{F}_t$, the proposed test procedure can be described as follow.

Initially, we consider $m = K$.

1. If $\hat{\beta}_\perp$ are the *m* eigenvectors associated to the *m* largest eigenvalues of $T^{-2}\sum_{t=2}^{T}\tilde{F}_t\tilde{F}_t'$ with $\hat{X}_t = \hat{\beta}_\perp'\tilde{F}_t$, it is possible to consider two different statistics:

   (a) Let $K(j) = 1 - j/(J-1)$, $j = 1,...,J$; in this case the considered statistic $MQ_c^c(m)$ -in the constant only case- or $MQ_c^\tau(m)$ -in the linear trend case- is defined as:

   $$T[\hat{v}_c(m) - 1]$$

   where $\hat{v}_c(m)$ is the smallest eigenvalue of:

   $$\hat{\Phi}_c(m) = \frac{1}{2}\left[\sum_{t=2}^{T}\left(\hat{X}_t\hat{X}_{t-1}' + \hat{X}_{t-1}\hat{X}_t'\right) - T\left(\hat{\Sigma}_1 + \hat{\Sigma}_1'\right)\right]\left(\sum_{t=2}^{T}\hat{X}_{t-1}\hat{X}_{t-1}'\right)^{-1}$$

   with $\hat{\Sigma}_1 = \sum_{j=1}^{J}K(j)\left(\frac{1}{T}\sum_{t=2}^{T}\hat{\xi}_{t-j}\hat{\xi}_t'\right)$, being $\hat{\xi}_t$ the residuals from estimating a VAR(1) in $\hat{X}_t$

   (b) For *p*, fixed that does not depend on *N* or *T*, the considered statistic $MQ_f^c(m)$ -in the constant only case- or $MQ_f^\tau(m)$ -in the linear trend case- is defined as:

   $$T[\hat{v}_f(m) - 1]$$

   where $\hat{v}_f(m)$ is the smallest eigenvalue of:

   $$\hat{\Phi}_f(m) = \frac{1}{2}\left[\sum_{t=2}^{T}\left(\hat{x}_t\hat{x}_{t-1}' + \hat{x}_{t-1}\hat{x}_t'\right)\right]\left(\sum_{t=2}^{T}\hat{x}_{t-1}\hat{x}_{t-1}'\right)^{-1}$$

$\hat{x}_t = \hat{\Pi}(L)\hat{X}_t$ is obtained by filtering $\hat{X}_t$ by $\hat{\Pi}_L$, the polynomial coefficients of an estimate Var(*p*) in $\Delta\hat{X}_t$, i.e. $\hat{\Pi}(L) = I_m - \hat{\Pi}_1 L - ... - \hat{\Pi}_p L^p$.

2. If the null $H_0 : k_1 = m$ is rejected, it is necessary to set $m = m-1$ and return to Step 1. If the null is not rejected, we set $\hat{k}_1 = m$ and we can stop.

Then, if there are $K > 1$ common factors, Bai and Ng consider two tests: the first *filters* the factors under the assumption that they have finite order VAR representations. The second *corrects* for serial correlation of arbitrary form by non-parametrically estimating the relevant nuisance parameters. This is why they have been called MQ_c and MQ_f respectively[48]. As it is obvious, the MQ_f test is valid only when the common trends can be represented as finite order $\mathrm{AR}(p)$ processes and MQ_c is more general.

The limiting distributions of these tests are non-standard; Bai and Ng provide 1%, 5%, and 10% critical values for all four statistics and various *m*.

- Idiosyncratic components stationarity analysis

To test the non-stationarity of the idiosyncratic components, Bai and Ng implement a methodology that consists in pooling individual *ADF* *t*-statistics computed for each de-factored $\hat{E}_{it}$ in a model with no deterministic term:

$$\Delta\hat{E}_{it} = d_{i0}\hat{E}_{it-1} + \sum_{i=1}^{p} d_{ij}\Delta\hat{E}_{it-j} + \upsilon_{it}\,; \tag{2.2.3.5}$$

υ_{it} is a regression error.

Let $ADF_{\hat{E}}^{c}(i)$ (if a constant is included in the DGP) and $ADF_{\hat{E}}^{\tau}(i)$ (if a constant and a linear trend are included in the DGP) be the *t*-statistics to test the hypothesis $d_{i0} = 0$.

The limiting distribution of $ADF_{\hat{E}}^{c}(i)$ coincides with the usual *DF* distribution for the case of no constant and the 5% critical value is -1.95. Instead, the asymptotic distribution of $ADF_{\hat{E}}^{\tau}(i)$ is proportional to the reciprocal of a Brownian bridge. Critical values for this distribution are not tabulated yet and have to be simulated.

Thus, contrary to the other panel unit root tests previously described, these statistics do not have the advantages of a standard normal limiting distribution. This happens because the

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[48] Note that $MQ_c^{c,\tau}(m)$ and $MQ_f^{c,\tau}(m)$ statistics are modified version of the Q_c and Q_f tests developed in Stock and Watson (1988).

panel information has been used to consistently estimate e_{it}, but not to analyze its dynamic properties.

As Bai and Ng noted, PANIC procedure is characterized by some significant features: first, the tests on the factors do not depend on whether e_{it} is I(1) or I(0), as well as the tests on the idiosyncratic errors do not depend on whether F_t is I(1) or I(0); second, the unit root tests for e_{it} is valid whether e_{jt}, $j \neq i$, is I(1) or I(0), and in any event, such knowledge is not necessary.

The independence of the limiting distribution of $ADF_{\hat{E}}^{c}(i)$ and $ADF_{\hat{E}}^{\tau}(i)$ on the common factors makes possible for Bai and Ng (2004a) to propose a pooled *Fisher*-type test[49] as suggested in Maddala and Wu (1999) or Choi (2001).

The test statistic is given by:

$$P_{\hat{E}}^{\circ} = \frac{-2\sum_{i=1}^{N} \log p_{\hat{E}}^{\circ}(i) - 2N}{\sqrt{4N}} \xrightarrow{d} \mathcal{N}(0,1) \tag{2.2.3.6}$$

where $P_{\hat{E}}^{\circ}$ denotes $P_{\hat{E}}^{c}$ or $P_{\hat{E}}^{\tau}$, depending on the deterministic specification, and $p_{\hat{E}}^{\circ}(i)$ is the associated *p*-value of the *ADF* test on the estimated residual $\hat{e}_{it}$.

For N and $T \to \infty$, this statistic converges to a standard normal distribution, but only if independence among the error terms is assumed: in this case, pooled testing is valid and it is possible to derive the statistic distribution. This seems a contradiction: the aim of Bai and Ng (2004a) test was precisely to take into account these individual dependence. Nevertheless, it is straightforward to note that Bai and Ng do not assume the cross-sectional independence hypothesis on the whole series y_{it} as Im *et al*. (2003) or Maddala and Wu (1999) do, but they only hypothesize the asymptotic independence between the individual components e_{it}. Under this hypothesis, the test statistics based on the estimate components $\hat{e}_{it}$ are asymptotically independent and the *p*-values $p_{\hat{e}i}$ are also independently distributed as uniform laws on $[0,1]$. Then, the hypothesis that all individual components e_{it} for $i = 1,\dots,N$ are I(1) is sufficient to assure that the test statistic $P_{\hat{E}}^{c}$ or $P_{\hat{E}}^{\tau}$ is standard normally distributed, for all panel sizes *N*.[50]

[49] In principal, also an *IPS*-type test using a standardized average of the above described *t*-statistics should be possible.
[50] It is possible to adopt the Choi (2001)'s standardization for panels with large sizes.

The *PANIC* procedure has the advantage that the estimated common factors and idiosyncratic components are consistent whether they are stationarity or non-stationarity. This is due to the fact that the unobserved components are estimated from the first-differenced (or de-trended) data, then the estimates are re-accumulated to remove the effect of possible overdifferencing if the factors or errors are stationary. Hence, the obtained estimates could also be used for stationarity tests, which is discussed in Bai and Ng (2004b).

As Banerjee and Zanghieri (2003) noted the implementation of Bai and Ng test shows very well the role of cross-section cointegration relationship. In fact, the Bai and Ng tests considering the factors common of different series, accept null hypothesis of unit roots for the factors, leading to the conclusion that the series is nonstationarity.

Simulations show that Bai and Ng test has good finite sample properties with satisfying size and a power even for small panel ($N = 40$).

2.2.4. Choi (2002) test

Choi (2002)'s approach is very similar to the one of Moon and Perron (2004a). Both approaches test the unit root hypothesis using the modified observed series y_{it} that allows the elimination of the cross-sectional correlations and the potential deterministic trend components. However, Choi (2002) procedure differs from the one of Moon and Perron in some main points.

Choi considers a two-way error-component model:

$$y_{it} = \alpha_0 + x_{it} \tag{2.2.4.1.a}$$

$$x_{it} = \mu_i + \lambda_t + v_{it} \tag{2.2.4.1.b}$$

$$v_{it} = \sum_{j=1}^{p_i} d_{ij} v_{it-j} + \varepsilon_{it} \tag{2.2.4.1.c}$$

where α_0 is the common mean for all i, μ_i is the unobservable individual effect, λ_t is the unobservable time effect represented as a weakly stationary process, v_{it} is the remaining random component which follows the autoregressive process of order p_i and ε_{it} is $i.i.d.\left(0, \sigma_{\varepsilon i}^2\right)$ for fixed i and independently distributed across individuals.

The number of the time series T may differ across cross-sectional units even if, for simplicity, in the model specification T is retained the same for all time series.

Differently from Bai and Ng (2002) and Moon and Perron (2004a) approaches, here the cross sectional units y_{it} are considered to be influenced homogeneously by a single common factor ($K = 1$), i.e. the time effect λ_t. Other authors also consider only one common factor, but they assume an heterogeneous specification of the sensitivity to this factor, like $\theta_i \lambda_t$ (i.e. Phillips and Sul, 2003). Choi justifies his choice by the fact that the logarithmic transformation of the model (2.2.4.1) allows to introduce such a sensitivity. Furthermore in this model, it is straightforward to check the stationarity hypothesis of the λ_t process, while it is not the case when an heterogeneous sensitivity is assumed. This represents an important advantage since, from a macroeconomic point of view, these time effects are supposed to capture the breaks in the international conjuncture, and nothing guarantees that they are stationary,

Referring to the model (2.2.4.1), the null hypothesis of interest is:

$$H_0 : \sum_{j=1}^{p_i} d_{ij} = 1 \;\; \forall i = 1,...N ,$$

that suppose the presence of a unit root in the idiosyncratic component v_{it} for all individuals, against the alternative hypothesis that:

$$\sum_{j=1}^{p_i} d_{ij} < 1 \text{ for some } i$$

Another difference with the Moon and Perron (2004a) approach concerns the orthogonalization of the individual series y_{it}. To get ride of the cross-sectional correlations, Choi isolates v_{it} by eliminating the individual effect α_0 and the common error term λ_t. To do that, he use a two-step procedure: first he demeans the data (i.e. removes the intercept) following the method suggested in Elliott *et al.* (1996); then he subtracts from the demeaned data the cross-sectional means to suppress the time effect. So, the new variables are independent across the units i for large *N* and *T*.

When the v_{it} component is stationary, *OLS* provides a fully efficient estimator of the constant term. However, when v_{it} is *I*(1) or presents a near unit root, Elliott *et al.* (1996) show that using *GLS* to estimate the constant term on quasi-differenced data provides unit root tests with better finite sample properties. Choi (2002) extende this approach in a panel context.

More formally, Choi’s methodology can be presented as follows.

To introduce the individual series orthogonalization process, let assume that the largest root of the v_{it} process is $1+c/T$ (near unit root process), for all $i=1,...N$. It is then possible to construct two quasi-differenced series $\tilde{y}_{it}$ and $\tilde{c}_{it}$ such that, for $t \geq 2$:

$$\tilde{y}_{it} = y_{it} - \left(1+\frac{c}{T}\right) y_{it-1}, \quad \tilde{c}_{it} = 1 - \left(1+\frac{c}{T}\right) \tag{2.2.4.2}$$

Now, let consider the case of a model without time trend[51] and set $c=-7$ (the value given by Elliott *et al.*, 1996, for this case). Regressing $\tilde{y}_{it}$ on the deterministic variable $\tilde{c}_{it}$, the demeaned series can be written for large *T* as:

$$y_{it} - \hat{\alpha}_{0i} \approx \lambda_t - \lambda_1 + v_{it} - v_{i1} \tag{2.2.4.3}$$

where $\hat{\alpha}_{0i}$ is the *GLS* estimate obtained for each individual *i* on quasi-differenced data. Relation (2.2.4.3) holds whatever the process v_{it} is *I*(1) or near-integrated.

Now the aim is to eliminate from the (2.2.4.3) the common component λ_t which can induce correlation across individuals.

Choi suggests demeaning $y_{it} - \hat{\alpha}_{0i}$ cross-sectionally to obtain:

$$z_{it} = (y_{it} - \hat{\alpha}_{0i}) - \frac{1}{N}\sum_{i=1}^{N}(y_{it} - \hat{\alpha}_{0i}) \approx (v_{it} - \bar{v}_t) - (v_{i1} - \bar{v}_1) \tag{2.2.4.4}$$

whit $\bar{v}_t = (1/N)\sum_{i=1}^{N} v_{it}$. It is possible to note that the deterministic components α_0, μ_i and λ_t were removed from z_{it} by the time series and cross-sectional demeanings. It is straightforward also to note that z_{it} are independent in the individual dimension for large *T* and *N* since the means $\bar{v}_1$ and $\bar{v}_1$ converge in probability to 0 when *N* goes to infinity for any *T*.[52]

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[51] It is possible to extend this procedure to the case of a model with an individual time trend.

[52] Choi shows that a similar approach can be used when an individual time trend is present in the model (2.2.4.1). In fact, many macroeconomic and financial variables contain a linear time trend in addition to a stochastic component (see Nelson and Plosser, 1982) and also some economic theories provide for a linear time trend in economic variables (i.e. Solow, 1956). In this case, the extended model becomes:

$$y_{it} = \alpha_0 + \alpha_1 t + x_{it}, \quad x_{it} = \mu_i + \lambda_t + \gamma_i t + v_{it}$$

where v_{it} is specified as previously seen and γ_i is the individual trend effect. In this case, to obtain the estimates $\hat{\alpha}_i$ and $\hat{\gamma}_i$, it is necessary to regress the series $\tilde{y}_{it}$ on $\tilde{c}_{it}$ and $\tilde{d}_{it} = 1 - c/T$ using *GLS*. Then c is set to $c=-13.5$ (i.e. the value given by Elliott *et al.*, 1996, for the case of a model with a time trend). It is now possible to define w_{it} as:

$$w_{it} = (y_{it} - \hat{\alpha}_{0i} - \hat{\gamma}_i t) - \frac{1}{N}\sum_{i=1}^{N}(y_{it} - \hat{\alpha}_{0i} - \hat{\gamma}_i t) \tag{a}$$

As in the previous case, the unit root tests using w_{it} are independent across *i* for large *T* and *N* and this relation holds either when v_{it} is stationary or not.

Next step is running a unit root test on the transformed series z_{it}.

The main idea of the Choi (2002) test is to combine *p*-values from independent unit root tests applied to each time series as Maddala and Wu (1999) and Choi (2001) already did[53].

To do this, the *ADF* test is applied to each detrended time series and the *p*-values are calculated.

In instance, the *ADF* test using the series $\{z_{it}\}_{t=2}^{T}$, with z_{it} defined as in equation (2.2.4.4), is the *t*-ratio for coefficient estimate $\hat{\rho}_0$ from the regression:

$$\Delta z_{it} = \hat{\rho}_0 z_{it-1} + \sum_{j=1}^{p_i - 1} \rho_j \Delta z_{it-j} + \hat{u}_{it} \tag{2.2.4.5}$$

This test has the *DF t*-distribution without time trends as $T \to \infty$ and $N \to \infty$.[54]

Denoted by p_i the asymptotic *p*-value of one of the *DF-GLS* tests for each unit *i*, Choi suggests three panel test statistics based on the individual t_{ρ_i} statistics which are independent the ones on the others:

$$P_m = -\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left[\log(p_i) + 1\right] \tag{2.2.4.6.a}$$

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(p_i) \tag{2.2.4.6.b}$$

$$L^* = \frac{1}{\sqrt{\pi^2 N/3}} \sum_{i=1}^{N} \log\left(\frac{p_i}{1 - p_i}\right) \tag{2.2.4.6.c}$$

where $\Phi(\cdot)$ is the standard cumulative normal distribution function.

It is straightforward to note that the P_m test is a modification of Fisher's (1932) inverse chi-square tests and rejects the null hypothesis for positive large value of the statistics. The test statistic Z is called the *inverse normal test* (see Stouffer *et al.*, 1949) and the L^* is a modified logit test. These last two tests reject the null for large negative values of the statistics.

Finally under the null all the tests converge to a standard normal distribution as N and $T \to \infty$.

As for Choi (2001) or Maddala and Wu (1999), the main difficulty in this approach is in the fact that the *p*-values (p_i) used to build the test statistics have to be simulated using bootstrap methods[55].

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[53] This procedure makes the same test more powerful in the panel framework than in the time series framework.

[54] When there is a time trend, this statistic applied to w_{it} follows a distribution tabulated by Elliot *et al.* (1996).

It is important to note that the obtained *p*-values are somewhat sensitive to the normality assumption for the u_t. When *T* is large, wrongly suppose the normality of the residuals u_t does not affect the results. However, when *T* is small, the *p*-values obtained under the normality hypothesis may be not accurate, if the underlying distribution is quite different from the normal one.

Choi's simulation shows that all the tests keep nominal size quite well and this property tends to improve as *T* grows; moreover, the power of all the tests increases as *N* grows, which justifies the use of panel data; unfortunately, the size and the power of all the tests decrease when a linear time trend term is included in the model.

In terms of size-adjusted power, the *Z* and P_m tests seem to be superior to the L^* test; in fact, the *Z* and P_m tests seem to outperform the L^* test both in terms of size and power.

The empirical size of the tests is not quite sensitive to the cross-sectional correlation of innovation terms[56]. Nevertheless, when the innovation terms independence assumption is violated, the empirical size of the tests deteriorates as *T* increases and the tests tend to reject more often under the alternative; instead, the empirical power properties seem to be the same.

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[55] Data are generated on *T* periods according to the process $x_t = x_{t-1} + u_t$, where $x_0 = 0$, $u_t \sim i.i.d.\mathcal{N}(0,1)$ and $t = 1,...,T+30$. It is recommended to consider realizations of x_t on a longer period and to retain only the last *T* observations to avoid sensitivity to initial conditions.
Choi followed the MacKinnon (1994) methodology which can be summarized as follow:

- for different values of *T* ($T = 30,50,75,100,250,500,1000$), he generated $\{x_{it}\}_{t=1}^{T}$ *i* times. Then, for each series he computed the t_{ρ_i} statistic in the *ADF* model (a) -see note 52-, and, given the *i* realizations $\hat{t}_{\rho_i}$, he constructed 399 equally spaced percentiles;
- he repeated 50 times the first step and recorded the percentiles: for the seven considered values of *T*, he obtained 50 values for each of the 399 percentiles. Let $q_j^p(T_k)$ be the value of the percentile obtained at the *j*-th simulation ($j = 1,...,350$) for $p = 0.0025,0.0050,...,0.9975$ and size *T*;
- for each level p and each size *T*, the following equation:

$$q_j^p(T_k) = \eta_\infty^p + \eta_1^p T_k^{-1} + \eta_2^p T_k^{-2} + \varepsilon_j \quad j = 1,...,350$$

is estimated, using *GLS* and the 50 disposable observations;

For a given size *T*, 399 realizations of the *GLS* estimates of the parameters η_∞^p, η_∞^p and η_2^p are obtained. For example, for the 20th Choi obtained $\hat{\eta}_\infty^{0.05} = -1.948$, $\hat{\eta}_1^{0.05} = -19.36$ and $\hat{\eta}_2^{0.05} = 143.4$. Therefore, using these estimates, the 5% percentile of the t_{ρ_i} statistic at $T = 100$ is:

$$-1.948 - \frac{19.36}{100} - \frac{143.4}{100^2} = -2.1273$$

Then, for each size *T*, 399 percentiles values are obtained and using linear interpolation it is possible to derive the *p*-values p_i of the *DF-GLS* tests statistic that will be used in the construction of the P_m, *Z* and L^* test statistics.

[56] By contrast, O'Connell (1998) reports quite severe size distortions of Levin and Lin's (1992) test under a similar circumstance.

2.2.5. Comparison between the previous tests

From previous discussions it is straightforward to note that Bai and Ng (2004a) approach is more general than the ones of Pesaran (2003) and Moon and Perron (2004a). All these tests assume the same dynamic structure of the data and are computationally simple to implement (they simply require some tabulated critical values and the selection of the number of common factor *K*). The use of factor models by all three approaches is a convenient way to model cross-correlation or cointegration between panel members and the assumption of independence between the common factors and error terms (necessary for pooled testing) is far less restrictive than the assumption of independent cross-sections (*IPS* and *LLC* test).

However these tests differ in several ways:

- Bai and Ng (2004a) allow the non-stationarity of the data to come from common or idiosyncratic sources, while the Pesaran (2003) and Moon and Perron (2004a) approaches assume common and idiosyncratic stochastic trends under the null hypothesis.

- Pesaran (2003) and Moon and Perron (2004a) assume the same order of integration for the idiosyncratic and the common component of the data, while Bai and Ng (2004a) allow them to differ.[57]

- Pesaran (2003) and Moon and Perron (2004a) exclude the possibility of cointegration among the y_{it} as well as between the observed data and the common factors; Bai and Ng (2004a) models include both possibilities.

- Pesaran's (2003) and Bai and Ng's (2004a) models include either a constant or a linear trend; Moon and Perron (2004a) test is proposed for the case in which only a restricted constant is present.

Due to these features, in the case where the observed nonstationarity depends only on a nonstationary common factor (i.e. the individual series are pairwise cointegrated along the cross sectional dimension) only the Bai and Ng (2004a) tests enable us to detect this situation. On the contrary, both Moon and Perron (2004a) and Pesaran (2003) tend to systematically reject the non-stationarity of the series.

Gengenbach *et al.* (2004) analyze the small sample behaviour of the proposed tests and show that:

[57] Note that on the basis of the these observations, Pesaran's (2003) Moon and Perron's (2004a) null hypothesis can be viewed as a special case of Bai and Ng's (2004a) setup, namely where $K \geq 1$ and all idiosyncratic errors are I(1).

- Moon and Perron (2004a) tests are more powerful than the Pesaran (2003) tests, but the latter is simpler to compute;

- the $P_{\hat{E}}^{c}$ tests is more powerful than the $ADF_{\hat{E}}^{c}$ in detecting unit roots in the idiosyncratic components;

- the $ADF_{\hat{F}}^{c}$ for testing for the presence of unit roots in the common factor is found to have good small sample properties when *N* and *T* are sufficiently large (≥ 50) and the serial correlation in the common factor is not too persistent;

- in a multi-factor setting, the MQ_{c}^{c} test shows better performance than the MQ_{f}^{c} test but both tests fail to distinguish high but stationary serial correlation from non-stationarity in the common factors.

Due to this observations, Gengenbach *et al*. (2004) propose a procedure to unit root testing in panels with dynamic factors: in step one, use Pesaran's (2003) *CIPS* test to test for the presence of unit roots in the data when on expects that only a single common factor generates cross-sectional dependence. Use one of the tests proposed by Moon and Perron (2004a) to test for unit roots when cross-section dependence is likely to be due to multi common factors. In a second step, use $P_{\hat{E}}^{c}$ and the $ADF_{\hat{F}}^{c}$ tests proposed by Bai and Ng (2004a) to test for the presence of unit roots in the idiosyncratic components and the common factors respectively.

The case where the Bai and Ng (2004a) tests reject the unit roots for the idiosyncratic components but not for the factor while the Pesaran (2003) or Moon and Perron (2004a) tests both reject the unit root null, is an indication of cross-member cointegration.

Further, Choi's (2002) tests are largely oversized, except when the cross-section units respond homogeneously to the common factor. In addiction, unlike Phillips and Sul (2004a) results, the G_{OLS}^{+} test seems to show better properties than the pooled *Z* test of Choi, but large size distortions are detected for both tests when more than one factor is included in the simulation study. Finally, all tests lack power when a deterministic trend is included in the process.

Thus, one must be careful when using the previously reviewed panel unit root tests with variables, such as real GNP or industrial production, that are probably influenced by a deterministic trend.

Conclusions

This work gives a review of the main results presented in the panel unit root test literature. Much research has been carried out recently on the topic of econometric nonstationary panel data, especially because of the availability of new data sets (e.g. the Penn World Tables by Summer and Heston, 1991) in which the time series dimension and the cross-section dimension are of the same order.

Giving a larger quantity of information, new data sets ask for new tools of analysis. In order to work with this new kind of data, it is necessary that new tools including advantages and limits are well-known by researchers.

The aim of this paper is to provide a survey of the topic, making it easy to see the directions in which the research has developed, sorting out what appears worthwhile from the dead ends, and determining future areas in which it would be productive to undertake research.

In particular, in the panel unit root test framework, two directions have been developed since the seminal work by Levin and Lin (1992), leading to two generations of panel unit root tests. The first one concerns heterogeneous modellings with contributions by Im, Pesaran and Shin (2003), Maddala and Wu (1999), Choi (2001) and Hadri (2000). A second and more recent area of research aims at taking cross-sectional dependence into account. This latter category of tests is still under development, given the diversity of the potential cross-sectional correlations.

Researchers should bear in mind that all tests for the null of a unit root must be used with panel data tests for the null of stationarity. This procedure allows us to distinguish series that seem stationary, series that appear to have a unit root, and series for which it is not possible to establish whether they are stationary or integrated.

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APPENDIX

Table A.1. The first generation of panel unit root tests.

Test	Hypotesis test	Model specification	Advantages (+) /disadvantages (-)	Properties
LLC	• nonstationarity for all individual • homogeneous alternative	• individual effects • time trends • heterogeneous serial correlation structure of the errors	+ unbalanced panels are allowed, but further simulations are required - it requires an infinite number of groups - all the groups are assumed to have the same type of nonstochastic components - the critical values are sensitive to the choice of lag lengths in the individual *ADF* regressions - it does not allow that some groups have a unit root and others do not	• it is a pooled test • more relevant for panel of moderate size ($10 < N < 250$ and $25 < T < 250$) • superconsistency of the estimators • there is a loss of power when time trends are included
IPS	• nonstationarity for all individual • heterogeneous alternative	• individual linear trend • heterogeneous serial correlation structure of the errors	+ unbalanced panels are allowed, but further simulations are required - it requires an infinite number of groups - all the groups are assumed to have the same type of nonstochastic components - the critical values are sensitive to the choice of lag lengths in the individual *ADF* regressions - it does not allow that some groups have a unit root and others do not	• it is an averaged *t*-test • there is a loss of power when time trends are included • generally, it is more powerful than *LLC* and *Fisher* tests
Fisher	• nonstationarity for all individual • heterogeneous alternative	• individual fixed effects and time trend • heterogeneous serial correlation structure of the errors	+ unbalanced panels are allowed + it can be carried out for any unit root test derived + it is possible to use different lag lengths in the individual *ADF* regressions - the *p*-value have to be derived by Monte Carlo simulations - problems of size distortion with serial correlated errors	• it is a combination test • there is a loss of power when time trends are included • with cross-sectional correlated errors it is more powerful than *LLC*
Hadri	• stationarity for all individual • homogeneous alternative	• individual specific variances and correlation patterns	+ it avoids oversized tests due to treating not only *N* but also *T* asymptotic + the moments of the asymptotic distribution of the test are exactly derived	• it is a residual based LM test

Table A.2. The second generation of panel unit root tests: the problem of cross-sectional dependence.

Test	Hypotesis test	Model specification	Advantages (+) /disadvantages (-)	Properties
Chang (2002)	• nonstationarity for all individual • heterogeneous alternative	• fixed effects • time trends	+ unbalanced panels are allowed + N can take any value, large or small - for moderate cross-section dependence, it is valid only for N fixed $T \to \infty$	• it is a nonlinear instrumental variable approach • good finite sample properties • more powerful than *IPS* especially for little panels
Pesaran (2003)	• nonstationarity for all individual • heterogeneous alternative	• fixed effects • time trends • cross-section dependence and/or serial correlation	+ unbalanced panels are allowed	
Moon and Perron (2004a)	• nonstationarity for all individual • heterogeneous alternative	• one-way error component model • identical panel composition • heterogeneous restrictions	+ unbalanced panels are allowed	• the test suffers from size distortion when common factors are present on cross-sectional independence • the test is very powerful and has good size when only a deterministic constant is included in the model • recommended when cross-section dependence is expected to be due to several common factors
Bai and Ng (2004a)	• for all individual • heterogeneous alternative	• fixed individual effects • time trends	+ unbalanced panels are allowed + it is possible to determine if nonstationarity comes from a pervasive or an idiosyncratic source + this procedure solves the problem of the size distortion + it is a pooled test, then it is more powerful than univariate unit root test	• good finite sample properties (even for little N)
Choi (2002)	• nonstationarity for all individual • heterogeneous alternative	• two-way error component model • time trend • one single common factor is considered	+ unbalanced panels are allowed	• the tests keep nominal size quite well • the power increases as N grows • the size and the power decrease when a linear time trend is included in the model