MATH 200: TEST 2

11.7 QUADRATIC SURFACES SKETCHES

- **Quadric surfaces** are the graphs of any equation that can be put into the general form:
  $Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$
- A ***surface of revolution*** is a ***surface*** generated by rotating a two-dimensional curve about an axis.

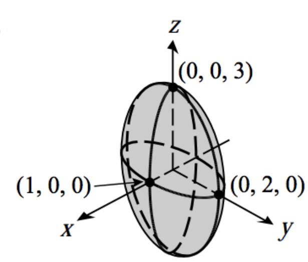
Table 11.7.2

IDENTIFYING A QUADRIC SURFACE FROM THE FORM OF ITS EQUATION

EQUATION	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$z^2-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$z-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$z-\frac{y^2}{b^2}+\frac{x^2}{a^2}=0$
CHARACTERISTIC	No minus signs	One minus sign	Two minus signs	No linear terms	One linear term; two quadratic terms with the same sign	One linear term; two quadratic terms with opposite signs
CLASSIFICATION	Ellipsoid	Hyperboloid of one sheet	Hyperboloid of two sheets	Elliptic cone	Elliptic paraboloid	Hyperbolic paraboloid

11. 7 HOMEWORK (15 - 25 odd, 33 - 35)

15.



Ellipsoid

- **Problem 15 (ellipsoid):**
  Sketch the graph $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$
  - Find x coordinates by set $y = 0, z = 0$
    $P_1 = (1, 0, 0)$
  - Find y coordinates by set $x = 0, z = 0$
    $P_2 = (0, 2, 0)$
  - Find z coordinates by set $x = 0, y = 0$
    $P_3 = (0, 0, 3)$

- **Problem 17 (hyperboloid of 1 sheets):** $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$
  - Find x coordinates = 2
  - Find y coordinates = 3
  - Set $z = \pm 4$:

    $x^2 + \frac{y^2}{9} - \frac{4^2}{16} = 1$

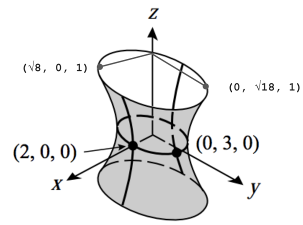
    $\frac{x^2}{4} + \frac{y^2}{9} = 2$ (this is an ellipses with x = $\sqrt{8}$, and y = $\sqrt{18}$)

  So we have 2 ellipses / circle:

    $z = 0$: $(1, 3, 0)$

    $z = 1 : (\sqrt{8}, \sqrt{18}, 1)$

17.



Hyperboloid of one sheet

- **Problem 19 (elliptical cone):** $4z^2 = x^2 + 4y^2$

  Set $z = 1$:

    $4 = x^2 + 4y^2$

    $4 = x^2$ if $y = 0$

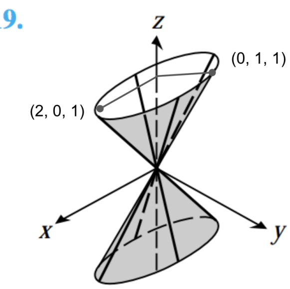
    $x = 2$

    $4 = 4y^2$ if $x = 0$

    $y = 1$

  There is a circle $(2, 1, 1)$ at z = 0

19.



Elliptic cone

- **Problem 21 (hyperboloid of two sheets):**

  $9z^2 - 4y^2 - 9x^2 = 36$
  - Get the z coordinates ($x = 0, y = 0$): $z = 2$

    $P_1 = (0, 0, 2)$ and $P_2 = (0, 0, -2)$
  - Get a random value for z (larger than 2):

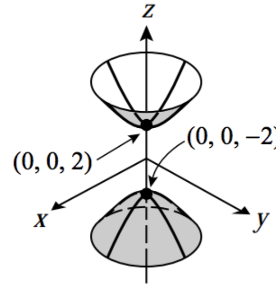
    $z = 2\sqrt{2}$

    $9 * (2\sqrt{2})^2 - 4y^2 - 9x^2 = 36$

    $9 * 8 - 4y^2 - 9x^2 = 36$

    $72 - (4y^2 + 9x^2) = 36$

21.

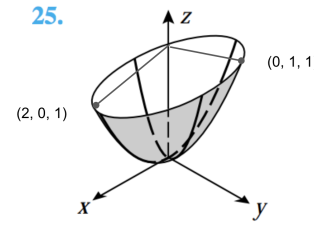


$4y^2 + 9x^2 = 36$
$y = 3$ and $x = 2$

- So another 4 points are:
  $P_1(2, 0, 2\sqrt{2})\ \ - P_2(-2, 0, 2\sqrt{2})$
  $P_3(0, 3, 2\sqrt{2}) - P_4(0, -3, 2\sqrt{2})$

- **Problem 23:** $z = y^2 - x^2$ **(**let's skip this, since it's a hyperbolic paraboloid)

25.

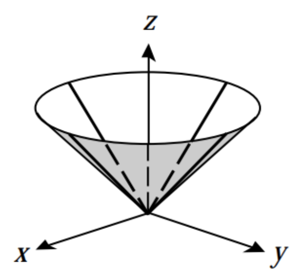


Elliptic paraboloid

- **Problem 25 (elliptic paraboloid) :** $4z = x^2 + 2y^2$
  Set $z = 1$, we have $(x = 2, y = 1)$

- **Problem 33: (circular cone)**
  $z = \sqrt{x^2 + y^2}$
  $z^2 = x^2 + y^2$
  These are just circles stack on each other with radius (0, 1, 2, 3....etc.) so it's a cone

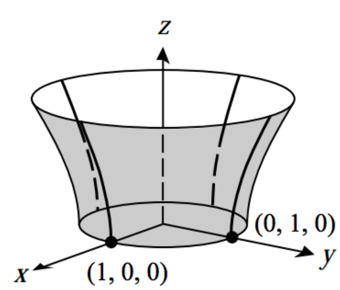
33.



- **Problem 35:** $z = \sqrt{x^2 + y^2 - 1}$
  $z^2 = x^2 + y^2 - 1$
  $z^2 - x^2 - y^2 = -1$
  $z^2 - (x^2 + y^2) = -1$
  $(x^2 + y^2) - z^2 = 1$

  Set $z = 0$, $(x^2 + y^2) = 1$ (a circle of radius 1)
  Set $z = 1$, $(x^2 + y^2 = 2$ (A circle of radius 2)
  This appear to be a hyperboloid of two sheets

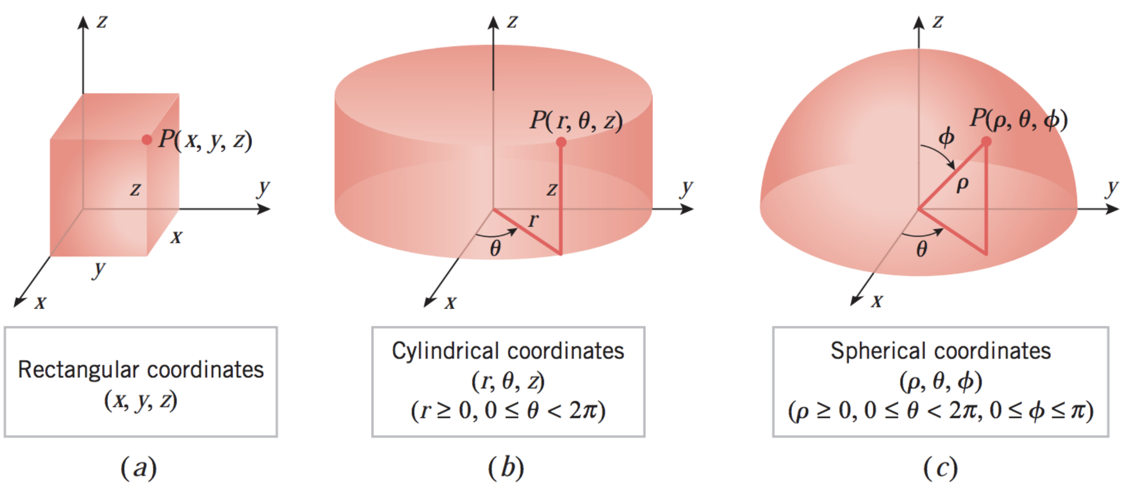
35.



11.8 QUADRATIC SURFACES SKETCHES

POINTS ON A COORDINATE

- **Theory:**
  - part (a) of the figure shows the rectangular coordinates (x, y, z) of a point P
  - part (b) shows the cylindrical coordinates (r, θ , z) of P
  - part (c) shows the spherical coordinates (ρ , θ , ϕ) of P



- **Rectangular coordinates:**
  $x = 0 \qquad y = 0 \qquad z = 0$

- **The cylindrical coordinates:**
  $r = 0 \qquad \theta = \theta_0 \qquad z = 0$

- **The spherical coordinates:**
  $p = 0 \qquad \theta = \theta_0 \qquad \phi = \phi_0$

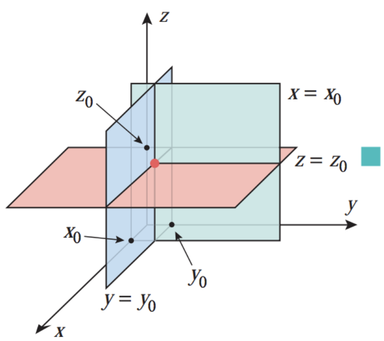


Figure 11.8.2

11.1 - rectangular coordinates

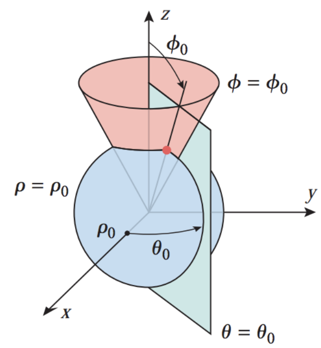


Figure 11.8.4

11.2 - cylindrical coordinates

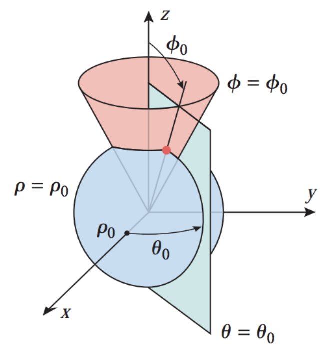


Figure 11.8.4

11.3 - Spherical coordinates

POINTS ON A COORDINATE

Table 11.8.1
CONVERSION FORMULAS FOR COORDINATE SYSTEMS

CONVERSION		FORMULAS	RESTRICTIONS
Cylindrical to rectangular Rectangular to cylindrical	$(r, \theta, z) \to (x, y, z)$ $(x, y, z) \to (r, \theta, z)$	$x = r\cos\theta,\quad y = r\sin\theta,\quad z = z$ $r = \sqrt{x^2 + y^2},\quad \tan\theta = y/x,\quad z = z$	
Spherical to cylindrical Cylindrical to spherical	$(\rho, \theta, \phi) \to (r, \theta, z)$ $(r, \theta, z) \to (\rho, \theta, \phi)$	$r = \rho\sin\phi,\quad \theta = \theta,\quad z = \rho\cos\phi$ $\rho = \sqrt{r^2 + z^2},\quad \theta = \theta,\quad \tan\phi = r/z$	$r \ge 0, \rho \ge 0$ $0 \le \theta < 2\pi$ $0 \le \phi \le \pi$
Spherical to rectangular Rectangular to spherical	$(\rho, \theta, \phi) \to (x, y, z)$ $(x, y, z) \to (\rho, \theta, \phi)$	$x = \rho\sin\phi\cos\theta,\quad y = \rho\sin\phi\sin\theta,\quad z = \rho\cos\phi$ $\rho = \sqrt{x^2 + y^2 + z^2},\quad \tan\theta = y/x,\quad \cos\phi = z/\sqrt{x^2 + y^2 + z^2}$	

11.8 HOMEWORK (11, 19, 21, 23, 27, 29, 35, 37)

- **Problem 11:** Convert from spherical to cylindrical coordinates:
  $(5, \pi/4, 2\pi/3)$

  Plug in the formula:
  $r = 5sin(2\pi/3)$
  $\theta = \pi/4$
  $z = 5cos(2\pi/3)$

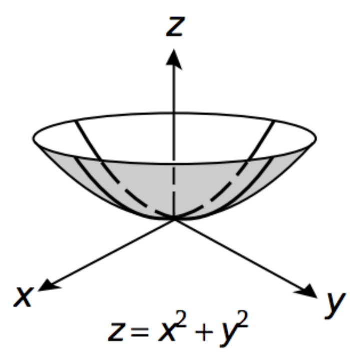
- **Problem 19:** Convert from cylindrical to rectangular coordinates:

$r = 3$

Since $x^2 + y^2 = r^2$ therefore $x^2 + y^2 = 9$
This is a cylinder of radius 3 (since z coordinates is not specified)"

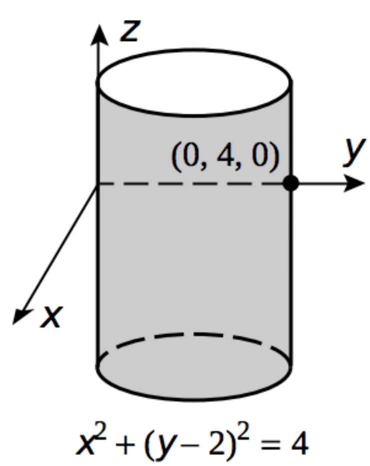
- **Problem 21:** $z = r^2$

- **Problem 21:** the equation is $z = r^2$
  Therefore $z = x^2 + y^2$ or $z - x^2 - y^2 = 0$
  This is the equation for an elliptic paraboloid



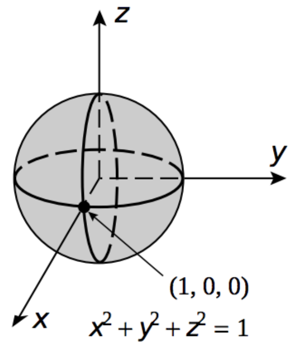
Problem 21

- **Problem 23:** the equation is $r = 4sin(\theta)$
  - Therefore
    $r^2 = r * 4sin(\theta) = 4r * sin(theta)$
  - $x^2 + y^2 = 4r * sin(\theta) = 4y$
  - $x^2 + y^2 - 4y = 0$
  - Complete the square: $x^2 + y^2 - 4y + 4 = 4$
    $x^2 + (y-2)^2 = 2^2$
  - This is the shape of a circle with radius 2 and center at $(0, 2)$



Problem 23

- **Problem 25:** $r^2 + z^2 = 1$
  - Therefore $x^2 + y^2 + z^2 = 1$
  - This is the equation for an ellipsoid in which it is a perfect sphere, center at (0, 0, 0) with a radius of 1



Problem 25

SPHERICAL → RECTANGULAR

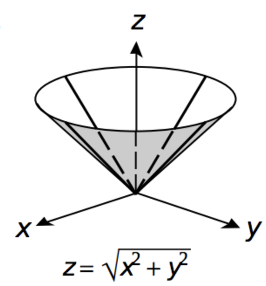
Problem 27: $p = 3$

- Rectangular to spherical: $\sqrt{x^2 + y^2 + z^2} = p = 3$
  $x^2 + y^2 + z^2 = 9$
  Therefore this is the shape of a sphere center at $(0, 0, 0)$ with a radius of 3

Problem 29: $\phi = \pi/4$

- $cos(\pi/4) = z/\sqrt{x^2 + y^2 + z^2}$
- $cos^2(\pi/4) = \frac{z^2}{x^2+y^2+z^2}$
- $0.5 = \frac{z^2}{x^2+y^2+z^2}$

  $0.5(x^2 + y^2 + z^2) = z^2$
  $x^2 + y^2 + z^2 = 2z^2$
  $x^2 + y^2 = 2z^2 - z^2 = z^2$
  $x^2 + y^2 = z^2$
  $z^2 - x^2 - y^2 = 0$
  This is the shape of an **elliptical cone.**



Cone

Problem 31: $p = 4 * cos\phi$
Therefore $p^2 = 4p * cos\phi = 4z$
Therefore $x^2 + y^2 + z^2 = 4z$

Problem 33: $p * sin\phi = 2 * cos\theta$
(ask later)!!!!

RECTANGULAR → CYLINDRICAL / SPHERICAL

Problem 35: $z = 3$

- a) Cylindrical: $z = 3$
- b) Spherical: $p = \sqrt{x^2 + y^2 + 9}$
  $p^2 = x^2 + y^2 + 9$
  $p^2 = r^2 + 9$
  $p^2 = p^2 sin\phi + 9$
  $p^2 - p^2 sin\phi = 9$
  $p^2(1 - sin\phi) = 9$
  Not sure what to do after this ?

Problem 37: $z = 3x^2 + 3y^2$

a) Cylindrical:

- $z = 3(x^2 + y^2)$
- $z = 3r^2$

b) Spherical:
Not sure what to do here

12.1 VECTOR VALUE FUNCTIONS

ORIENTATION OF A LINE

- The parametric equations:
  $x = 1 - t$
  $y = 3t$
  $z = 2t$

  - If $t = 1 :$ $(0, 3, 2)$
  - If $t = 0 :$ $(1, 0, 0)$

- Since x decreases as t increases, the orientation of the line is as Figure 12.1

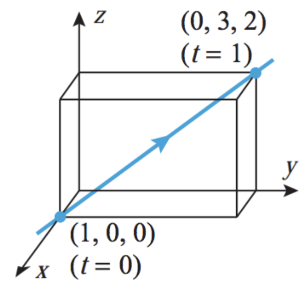


Figure 12.1

DESCRIBE PARAMETRIC CURVE

- **Describe the parametric curve represented by the equations:**
  $x = a\,cos(t)$
  $y = a\,sin(t)$
  $z = ct$
  in which a and c is a positive constant
  - As t increases, the parametric curve moves upwards

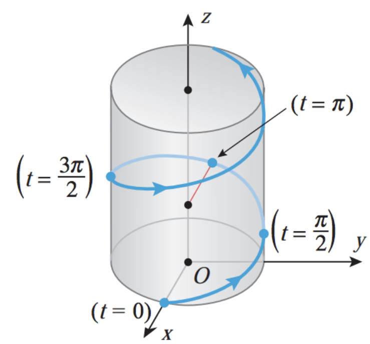


Figure 12.2

VECTOR VALUE FUNCTIONS

- Given $r = (x, y, z) = (t, t^2, t^3) = ti + t^2j + t^3k$
  The component functions of r :
  $x(t) = t$
  $y(t) = t^2$
  $z(t) = t^3$

- **Find the natural domain of**
  $x(t) = ln|t-1|i$ domain: $(-\infty, 1)$ and $(1, \infty)$
  $y(t) = e^t j$ domain: $(-\infty, +\infty)$
  $z(t) = \sqrt{t}$ domain: $(0, \infty)$

HOMEWORK

Chapter 12.1: 1-13 odd, 21, 23, 27, 39, 47

- **Problem 1:** Find the domain of r(t) and the value of r(t_0):
  $r(t) = cos(t) * i - 3tj$ $t_0 = \pi$
  Domain: $(\infty, +\infty)$
  Value = $cos(\pi) - 3(\pi) = -1 - 3\pi$

- **Problem 3:** Find the domain of r(t) and the value of r(t_0):
  r(t) = $cos(\pi t)i - ln(t) * j + \sqrt{t-2}k$ $\quad\quad t_0 = 3$
  For i, the domain is $(\infty, +\infty)$
  For j, the domain of natural log is $(0, +\infty)$
  For k, the domain is $(2, \infty)$
  Therefore the domain is $(2, \infty)$
  Value = $cos(3\pi)i - ln(3)j + k$

- **Problem 5:** Express the parametric equations as a single vector equation of the form
  $r = x(t)i + y(t)j...$
  - Given: $x = 3cos(t)$
    $y = t + sin(t)$
  - Therefore $r = (3cos\, t)i + (t + sin(t))j$

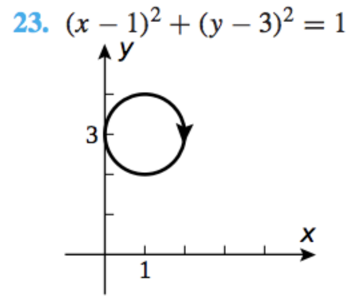
- **Problem 7:** Find the parametric equations that correspond to the given vector equation.
  $r = 3t^2 i - 2j$
  Therefore:
  $x = 3t^2$
  $y = -2$

- **Problem 9:** Describe the graph with the equation $(3 - 2t) + 5tj$
  Set $t = 0$, $(3, 0)$
  Set $t = 1$, $(1, 5)$
  Set $t = 2$, $(-1, 10)$
  So it appears as t increases, it moves towards the negative direction in x and positive direction in y. This is a vector in 2 space (x, y) through the point $(3, 0)$ with direction $a = (-2i + 5j)$

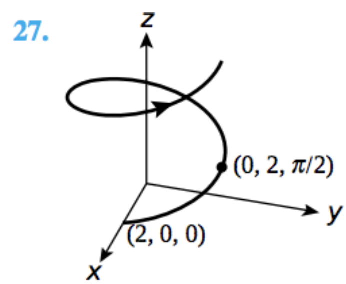
- **Problem 11**: Describe the graph with the equation $r = (2t)i - 3j + (1 + 3t)k$
  This is a graph in 3 space through the point $(0, -3, 1)$ with a direction of $a = (2i + 3k)$

- **Problem 13:** Describe the graph with the equation $r = (2cost)i + 3(sint)j + k$
  This is a graph of an elliptic centered at (0, 0, 1)

- **Problem 23:** Sketch r(t) = (1 + cost)i + (3 − sin t)j
  $t = 0, (2, 3)$
  $t = \pi/2, (1, 2)$
  $t = \pi(0, 3)$
  $t = 2\pi(2, 3)$
  Sketching this result in a circle with the coordinates (0, 2) for x and (2, 4) for y
  So it is a circle centered at (1, 3)
  $(x-1)^2 + (y-3)^2 = 1$



- **Problem 27:** Sketch $(2cost)i + (2sint)j + tk$
  This is a 3D graph of a circular motion wraps around a cylinder object
  - $(2, 0, 0)$
  - $(0, 2, \pi, 2)$
  - $(-2, 0, \pi)$
  - $(0, -2, 3\pi/2)$
  - $(-2, 0, \pi)$



- **Problem 39:**
  Show that the graph of
  r = $(tsint)i$ + $(tcost)j$ + $(t^2)k$ lies on the paraboloid $z = x^2 + y^2$

  $z = t^2(sin^2t + cos^2t) = t^2$
  And for the third term in the equation r, we see $(t^2)k$ so this match with the alue of z we have earlier

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12.2 CALCULUS OF VECTOR FUNCTION

- **Theory:** Let r0 = r(t0) and v0 = r′ (t0). It follows from Formula (9) of Section 11.5 that the tangent line to the graph of r(t) at r0 is given by the vector equation
  $r = r0 + tv_0$

- **Find parametric equations of the tangent line to the circular helix**
  $x = cost, y = sint, z = t$
  The vector equation is:

$$r(t) = (cost)i + (sint)j + tk$$
$$r_0 = r(t_0) = (cost_0)i + (sint_0)j + (t_0)k$$
$$v_0 = r'(t_0) = (-sint_0)i + (cost_0)j + k$$

- Tangnet line: $r_0 + tv_0$

$= (cost_0)i + (sint_0)j + (t_0)k + t[(-sint_0)i + (cost_0)j + k]$

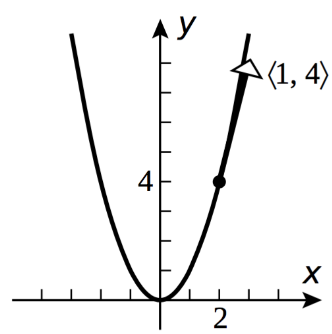
- Combine the terms
  $(cost_0 - t * sint_0)i + (sint_0 + t * cost_0)j + (t_0 + t)k$
- Parametric equation, plug in $t_0 = \pi$
  $x = cos(\pi) - t * sin\pi = -1 - 0 = -1$
  $y = -t$
  $z = \pi + t$

HOMEWORK

Problem 12.2: 11 19 21 23 25 31 33 35 37 39 45 49 53

- **Problem 11:** Find the vector r′ (t0); then sketch the graph of

then sketch the graph of r(t) in 3-space and draw the tangent vector r′(t0)



Given $r(t) = (t, t^2)$ and $t_0 = 2$
So $r'(t_0) = (1, 2t) = (1, 4)$

- **Problem 19:** Find parametric equations of the line tangent to the graph of r(t) at the point where $t = t_0$

$$r(t) = t^2 i + (2 - ln(t)) \qquad t_0 = 1$$

$$r'(t) = (2t, -1/t)$$
$$r'(t_0) = (2, -1)$$
$$r(t_0) = (1, 2)$$

$$x = r(t_0) + tr'(t_0) = 1 + 2t$$

$y = r(t_0) + tr'(t_0) = 2 - t$

- **Problem 21:** Find parametric equations of the line tangent to the graph of r(t) at the point where $t = t_0$:
  $2cos(\pi * t) + 2sin(\pi * t) + 3tk$ at $t_0 = \frac{1}{3}$
  $r'(t) = -2\pi sin(\pi t)i + 2\pi cos(\pi t)j + 3k$
  $r'(t_0) = (-\sqrt{3}\pi, \pi, 3)$
  $r(t_0) = (1, \sqrt{3}, 1)$
  $x = r(t_0) + tr'(t_0) = 1 - \sqrt{3}\pi t$
  $y = r(t_0) + tr'(t_0) = \sqrt{3} + \pi t$
  $z = 1 + 3t$

- **Problem 23:** Find a vector equation of the line tangent to the graph of r(t) at the point P_0 on the curve
  $r(t) = (2t - 1)i + \sqrt{3t + 4}$ at $P_0(-1, 2)$
  - Find t: $2t - 1 = -1$ therefore $t = 0$
  - Confirm by plugging in $t = 0$ for second point $\sqrt{3 * 0 + 4} = 2$

  $r'(t) = (2, \frac{3}{2\sqrt{3t+4}})$
  $r'(0) = (2, 3/4)$
  $r(0) = (-1, 2)$

  $R = r(t_0) + t * r'(t_0) = (-1 + 2t)i + (2 + \frac{3}{4}t)j$

- **Problem 31: Evaluate the indefinite integral**
  $\int (3i + 4t)dt = (3t)i + (t^2)j$

- **Problem 33: Evaluate the indefinite integral** using integration by parts
  $\int udv = uv - \int vdu$
  $\int (te^t, ln\, t)dt = (e^t, t * ln(t))$

  $u = t \qquad du = 1$
  $dv = e^t \qquad v = e^t$

  $\int te^t = te^t - \int e^t = te^t - e^t + C$

$\int ln\, t = tlnt - t + C$

Result = $(te^t - e^t, tlnt - t) + C$
in which C is a vector

- **Problem 35: Evaluate the definite integral**

$\int cos(2t), sin(2t)dt$ with $a = 0, b = \pi/2$

$\int cos(2t)sin(2t)dt = \frac{1}{2}sin(2t) - \frac{1}{2}cos(2t)$

$(\frac{1}{2}sin(2\pi/2) - \frac{1}{2}sin(0), -\frac{1}{2}cos(2\pi/2) + -\frac{1}{2}cos(0))$

$= (0, 1)$

- **Problem 37 (Integration by Substitution):**

$\int_0^2 ||ti + t^2j||$

The magnitude of a vector is $\sqrt{t^2 + t^4}$

$\int \sqrt{t^2 + t^4} = \int \sqrt{t^2(1 + t^2)} = \int t\sqrt{1 + t^2} = \int (1 + t^2)^{1/2}tdt$

Let set $u = (1 + t^2)$
Therefore $du = 2tdt$
But since we have tdt in the original one, we have to multiply $1/2$ outside

$a = (1 + 0^2) = 1$
$b = (1 + 2^2) = 5$

$\frac{1}{2}\int_1^5 u^{1/2}du = \frac{1}{2} * \frac{2}{3}u^{3/2} = \frac{1}{3}(u)^{3/2}$

$= \frac{1}{3}(5)^{3/2} - \frac{1}{3}(1)^{3/2}$

$= \frac{1}{3}(5^{3/2} - 1)$

- **Problem 45: Solve the vector initial-value problem for y(t) by integrating and using the initial conditions to find the constants of integration.**
  $y'(t) = (2t)i + 3t^2 j$ and $y(0) = i - j$
  $y(t) = \int y'(t) = (t^2, t^3) + C$
  $y(0) = (1, -1)$
  $y(t) = (1 + t^2, -1 + t^3)$

- **Problem 49:**
  a) **Find the points where the curve** $r = ti + t^2 j - 3tk$ **intersects the plane** $2x - y + z = -2$

  Substitute into the plane the value of (x, y, z) in which:
  $2t - t^2 - 3t = -2$
  $-t^2 - t = -2$
  $t^2 + t - 2 = 0$
  Using a quadratic solver,
  $t^2 + 2t - t - 2 = 0$
  $t(t + 2) - (t + 2) = 0$
  So $t = 1$ or $t = -2$

  The three points (x, y, z) with $t = 1$: $(1, 1, -3)$
  The three points (x, y, z) with $t = -2$: $(-2, 4, 6)$

  b) For the curve and plane in part (a), find, to the nearest degree, the acute angle that the tangent line to the curve makes with a line normal to the plane at each point of intersection
  - Normal line to the plane: $(2, -1, 1)$
  - The tangent line to the curve at point $P(1, 1, -3)$ or $t = 1$:
    $R = r(t_0) + tr'(t_0)$
    $r(t_0) = (1, 1, -3)$
    $r'(t_0) = (1, 2t, -3) = (1, 2, -3)$
  - **Acute angle formula:**
    $cos(\theta) = \frac{n_1 \cdot n_2}{||n_1||||n_2||}$

$(2, -1, 1) \cdot (1, 2, -3) = -3$
$||(2, -1, 1)|| * ||(1, 2, -3)|| = \sqrt{6} * \sqrt{14} = \sqrt{84}$

$cos(\theta) = \frac{-3}{\sqrt{84}}$
$\theta = 71^o$

- The tangent line to the curve at point $P(-2, 4, 6)$ or $t = -2$
  $r'(t_0) = (1, -4, 3)$
  Do the same process as above and the result is $\theta = 76^o$

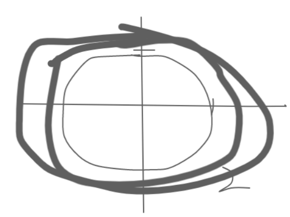
13.1 MULTIVARIABLE FUNCTION

PROBLEM 1:

- $z = f(x, y)$
- $f(x, y) = \sqrt{x^2 + y^2 - 4}$

Domain: $x^2 + y^2 - 4 \geq 0$
$x^2 + y^2 \geq 4$

This is a graph that include a circle and everything that is outside the circle

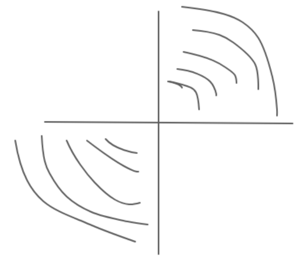


Graph include the circle + everything outside it

PROBLEM 2:

- $f(x, y) = ln(xy)$
  $xy > 0$

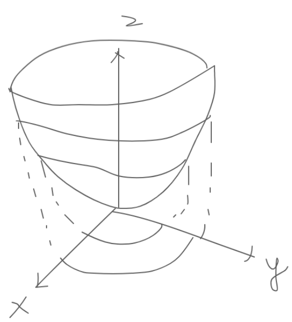
This is the graph that include everything in the first quadrant and third quadrant since the product of 2 positive number and 2 negative number is always > 0



Graph include everything in 1st and 3rd quadrant

PROBLEM 3:

- $z = x^2 + y^2$

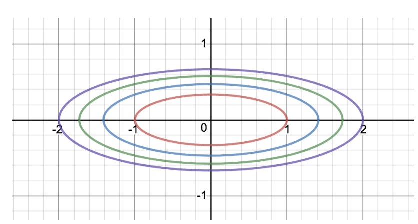


Example of a multi-level curve?

PROBLEM 4

- Given $z = x^2 + 9y^2$ for $k = 0, 1, 2, 3, 4$
  Graph $z = k$
  $z = 0$ therefore $0 = x^2 + 9y^2$
  $(0, 0)$

  $1 = x^2 + 9y^2$ therefore $(1, \sqrt{1/9})$
  $2 = x^2 + 9y^2$ therefore $(\sqrt{2}, \sqrt{2/9})$



Graph

13.1 HOMEWORK (5, 15, 17, 19, 23, 25, 27, 31, 35, 41, 49, 51, and 55)

- **Problem 5:** Find F if $F(x, y) = xe^{xy}, g(x) = x^3$ = xe^{xy} and $h(y) = 3y + 1$

- **Problem 23**: Sketch the domain of f. Use solid lines for portions of the boundary included in the domain and dashed lines for portions not included.
  $f(x, y) = ln(1 - x^2 - y^2)$

13.3 PARTIAL DERIVATIVES

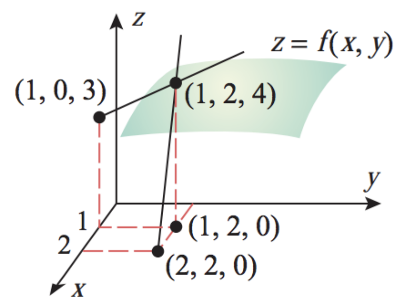
Problem 13.3: 13 15 25 35 37 47 55 57 59 61 65 71 73 81 89

- **Problem 13:** Let $z = sin(y^2 - 4x)$
  (a) Find the rate of change of z with respect to x at the point (2, 1) with y held fixed.
  (b) Find the rate of change of z with respect to y at the point (2, 1) with x held fixed.

  - Plug in y (fixed) as 1:
    $\frac{dx}{dz} sin(y^2 - 4x) = \frac{dx}{dz} sin(1^2 - 4x)$
    $= -4cos(1 - 4 * 2) = -4cos(7)$
  - Plug in x (fixed) at 2:
    $\frac{dy}{dz} sin(y^2 - 4x) = 2cos(y^2 - 4 * 1) = 2cos(1 - 8) = 2cos(7)$

- **Problem 15:** Use the information in the accompanying figure to find the values of the first-order partial derivatives of f at the point (1, 2)

  - The slope of the first line (parallel to the xz plane):



$(1,2,4)-(2,2,0)$ (take only x and z) coordinates:
$(4-0)/(1-2)=-4$

- The slope of the second line (parallel to the yz plane):
  $(1,2,4)-(1,0,3)$ (take only y and z. coordinates)
  $(4-3)/(2-0)=1/2$

- **Problem 25:** Find $\frac{dz}{dy}$ and $\frac{dz}{dx}$

  $z=4e^{x^2y^3}$

  $\frac{dz}{dx}=4e^{x^2y^3}*(2x*y^3)$

  $\frac{dz}{dy}=4e^{x^2y^3}*(3y^2*x^2)$

- **Problem 35:** Find fx (x, y) and fy (x, y)
  - fx (x, y) = $(y^2*tanx)^{-4/3}$
  - fy (x, y) = $(y^2*tanx)^{-4/3}$

  $f(x)=\frac{-4}{3}(y^2*tanx)^{-7/3}*(y^2*sec^2*4)$
  $f(y)=\frac{-4}{3}(y^2*tanx)^{-7/3}*(2y*tanx)$

- **Problem 55:** A point moves along the intersection of the elliptic paraboloid $z=x^2+3y^2$ and the plane $y=1$. At what rate is z changing with respect to x when the point is at $(2,1,7)$

- **Problem 61:**
  $P=\frac{KT}{V}$

  $V=\frac{KT}{P}$

- Find V_p when $V = 50, T = 80$

  $P = \frac{10*80}{50} = 16$

  $V_P = \frac{-kT}{P^2} = \frac{-10*80}{16^2} = \frac{-800}{256} = -3.125$

- **Problem 71:** Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$using implicit differentiation
  $x^2 + zsin(xyz) = 0$

  - Find $\frac{dz}{dx}$:

  $2x + \frac{dz}{dx}sin(xyz) + zcos(xyz) * (yz + xy\frac{dz}{dx}) = 0$
  $2x + \frac{dz}{dx}sin(xyz) + yz^2cos(xyz) + xyz * cos(xyz)\frac{dz}{dx} = 0$
  $\frac{dz}{dx}(sin(xyz) + xyz * cos(xyz)) = -2x - yz^2cos(xyz)$

  $\frac{dz}{dx} = \frac{-2x-yz^2cos(xyz)}{sin(xyz)+xyz*cos(xyz)}$

  - Find $\frac{dz}{dy}$:

  $\frac{dy}{dz} * sin(xyz) + z * cos(xyz)(xz + xy * \frac{dz}{dy}) = 0$
  $\frac{dz}{dy} * sin(xyz) + xz^2 * cos(xyz) + xyz * cos(xyz) * \frac{dz}{dy} = 0$
  $\frac{dy}{dz} * (sin(xyz) + xyz * cos(xyz)) = -xz^2 * cos(xyz)$

  $\frac{dy}{dz} = \frac{-xz^2*cos(xyz)}{(sin(xyz)+xyz*cos(xyz)}$

- **Problem 73:**

  $(x^2 + y^2 + z^2 + w^2)^{3/2} = 4$

  $\frac{dw}{dx} = \frac{3}{2}(x^2 + y^2 + z^2 + w^2)^{1/2}(2x + 2w\frac{dw}{dx}) = 0$
  $\frac{dw}{dx} = -2x/-2w = \frac{-x}{w}$

Similarly, $\frac{dw}{dy} = -\frac{y}{w}$

Similarly, $\frac{dw}{dz} = -\frac{z}{w}$

- **Problem 81:**
  $z(x, y) = \sqrt{x} * cos(y)$

  $\frac{d^2z}{dx^2} = \frac{d}{dx} * \frac{dz}{dx}$

  So $\frac{dz}{dx} = \frac{1}{2\sqrt{x}} * cos(y) = \frac{cos(y)}{2\sqrt{x}}$

  Therefore $\frac{dx}{dz} = \frac{d}{dx}\frac{cos(y)}{2\sqrt{x}}$

- $\frac{d^z}{dxdy} = \frac{dz}{dx} * \frac{dz}{dy}$
  - First differentiating z with respect to y, keeping x as a constant
    $\frac{dz}{dy} = -\sqrt{x} * sin(y)$

    $\frac{dz}{dx} = -sin(y) * \frac{1}{2}$

    $\frac{dz}{dy} =$

Problem 89: Confirm that the mixed second-order partial derivatives of f are the same

$f(x, y) = ln(4x - 5y)$

- Prove that $\frac{d}{dy} * \frac{d}{dx} = \frac{d}{dy} * \frac{d}{dx}$
- So $\frac{d}{dy} * \frac{d}{dx} = ln(4x - 5y) = \frac{d}{dy}\frac{4}{4x-5y} \quad = 4 * (4x - 5y)^{-1}$
-