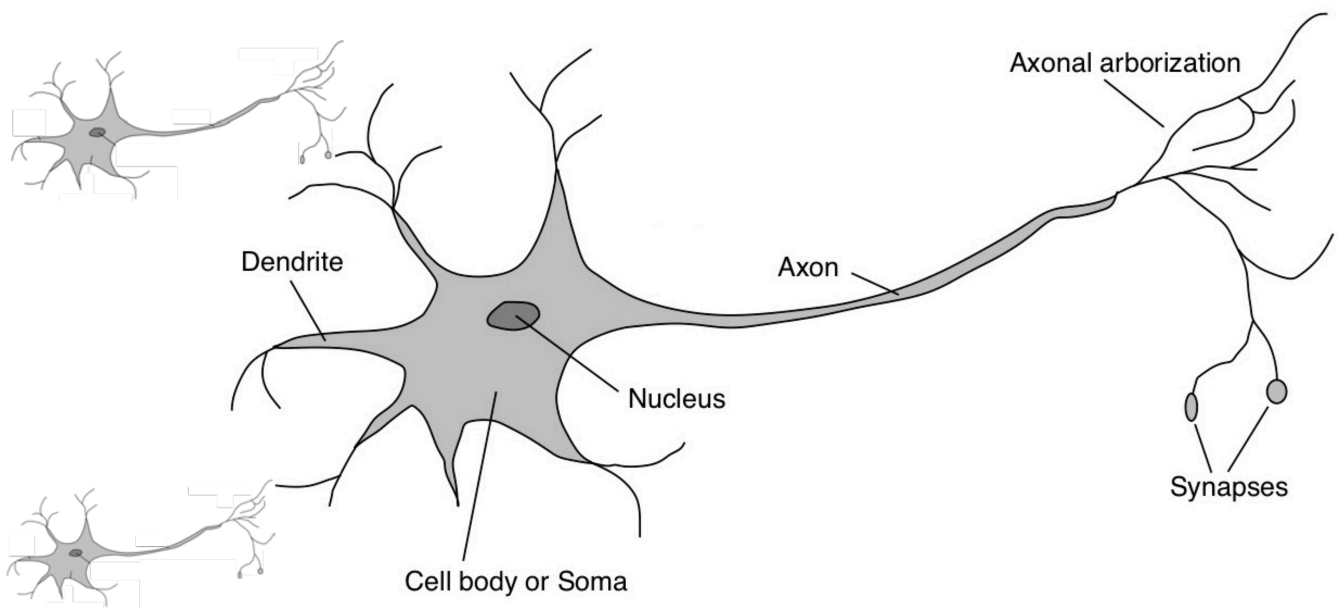
CS 380: Artificial Intelligence

Lecture 15: Neural Networks

Some materials adapted from Russell & Norvig textbook slides: http://aima.cs.berkeley.edu
and from https://en.wikipedia.org/wiki/Artificial_neuron

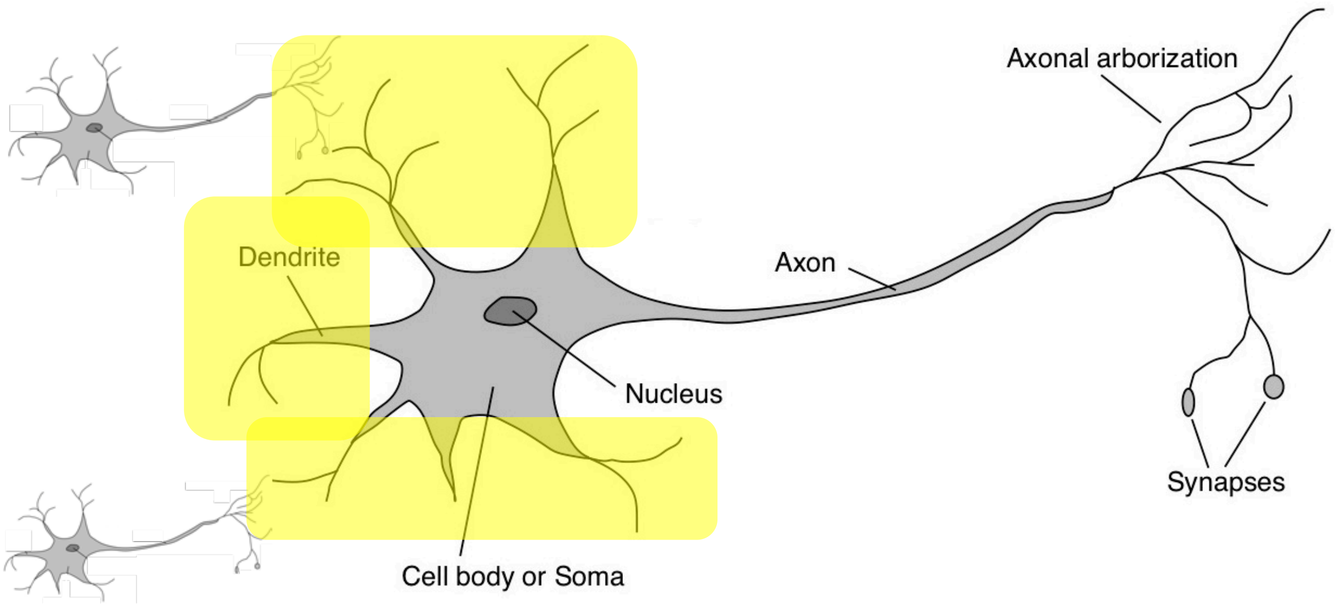
The Human Brain

- Our brains are very, very complicated
  - 10^{11} neurons (>20 types), 10^{14} synapses, 1-10 ms cycles
  - Signals are noisy "spike trains" of electrical potential



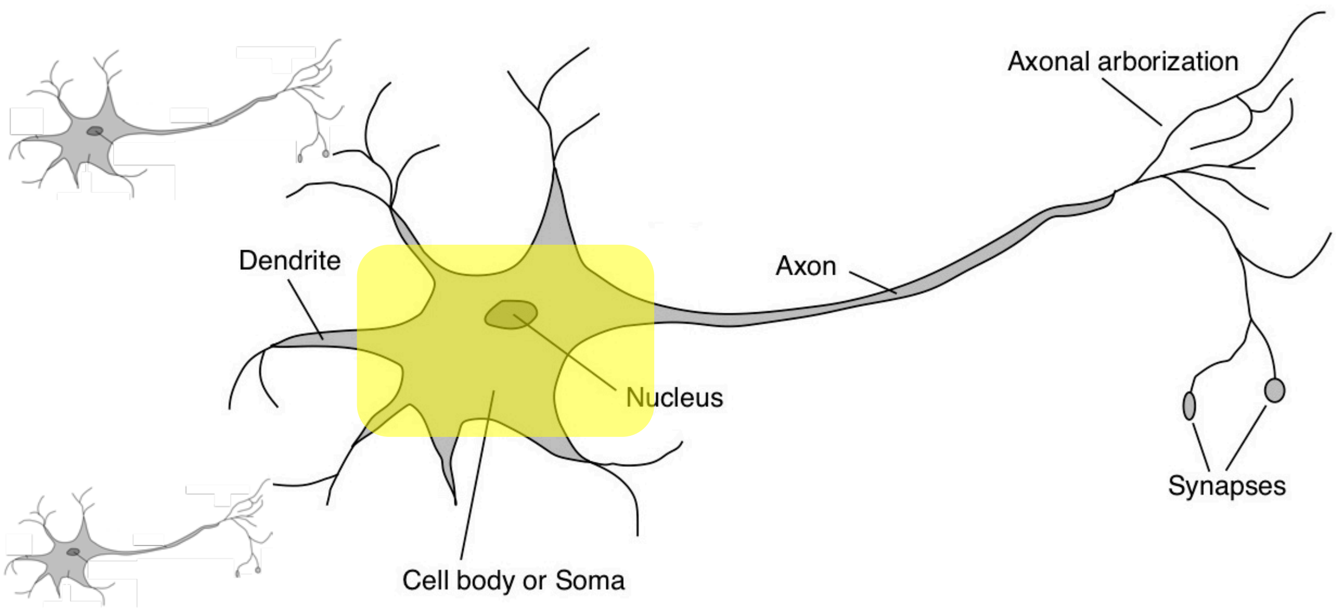
A Human Neuron

- Dendrites
  - Act as the “input vector” from >1000 neighbor neurons
  - Able to perform "multiplication" by a "weight value"



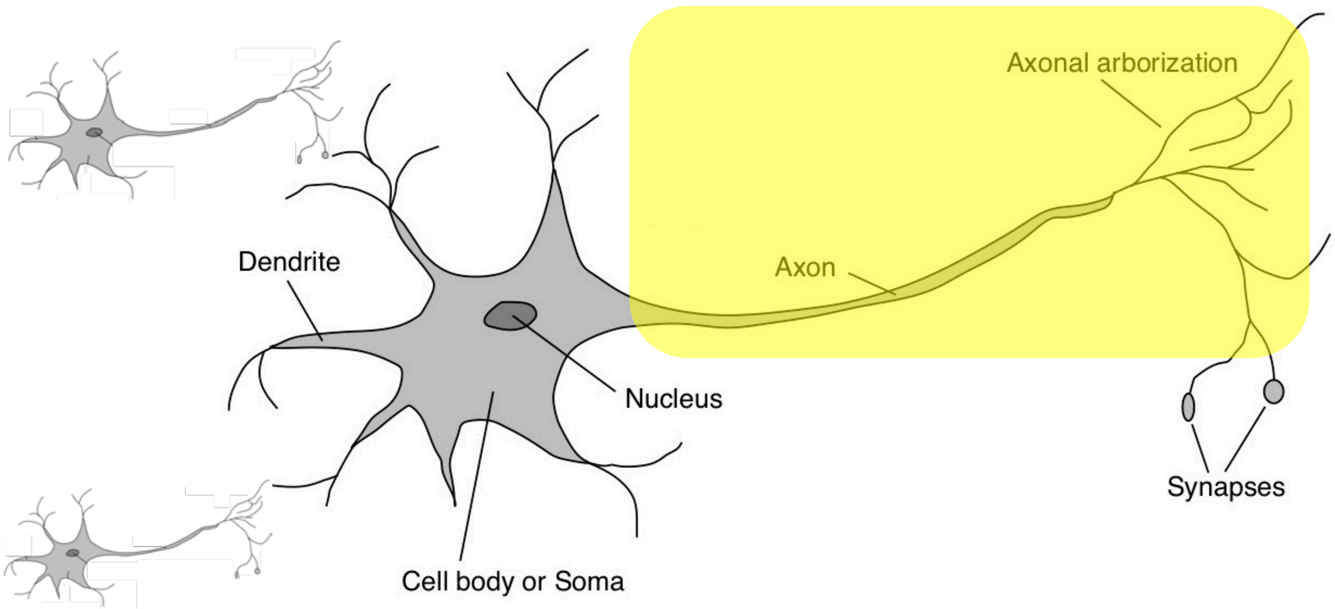
A Human Neuron

- Soma
  - Acts as a “summation function”
  - Aggregates excitatory (+) and inhibitory (-) signals



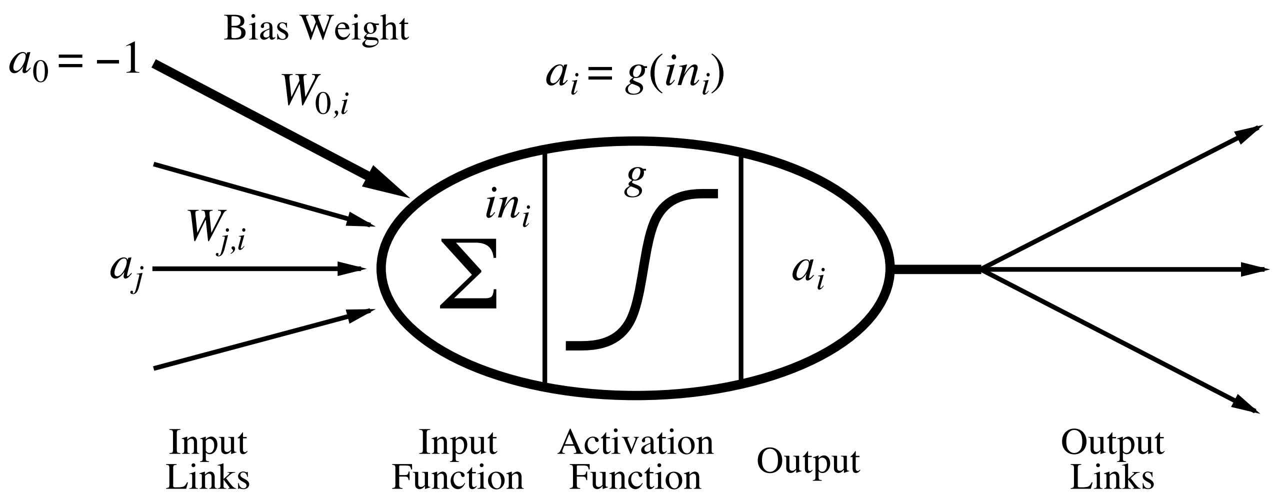
A Human Neuron

- Axon
  - Samples electric potential in the soma
  - If above "threshold", sends a pulse to neighbor neurons



An Artificial Neuron

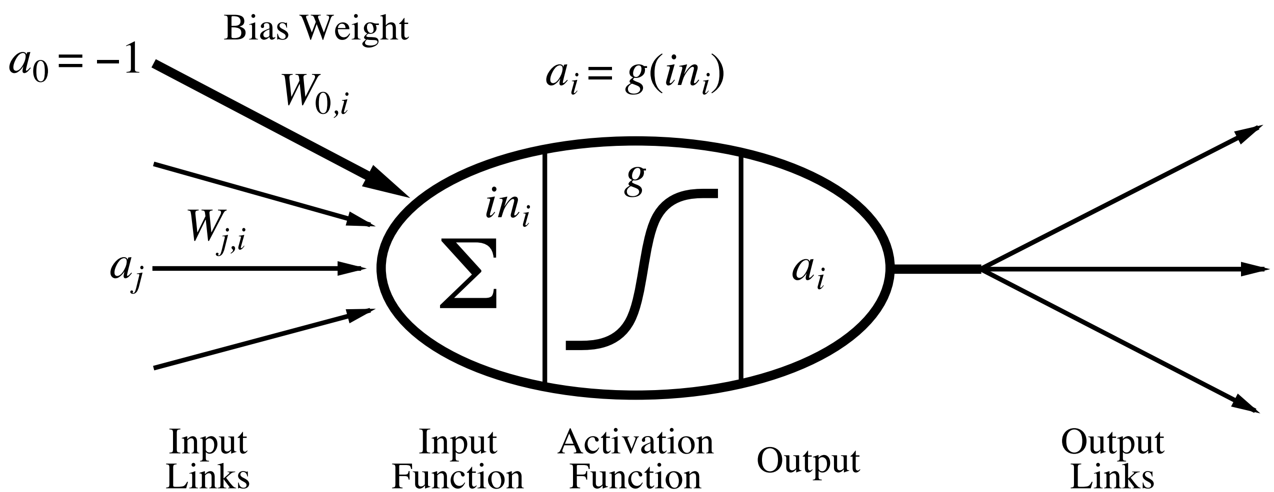
- A computational neuron (e.g., McCulloch–Pitts)



  - Inputs: $a_0, a_1, \ldots, a_j, \ldots$
  - Output: $a_i = g(\sum_j W_{j,i} a_j)$

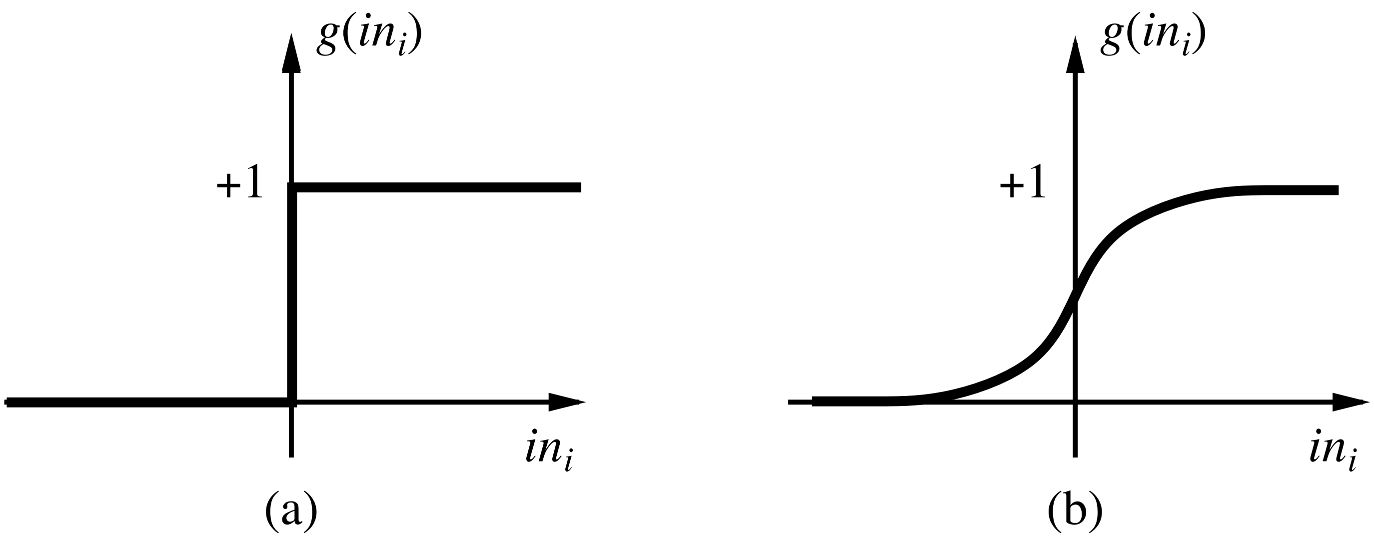
An Artificial Neuron

- A computational neuron (e.g., McCulloch–Pitts)



  - A gross oversimplification of real neurons!
  - But the goal is to develop an understanding of what networks of simple units can do together

Activation functions



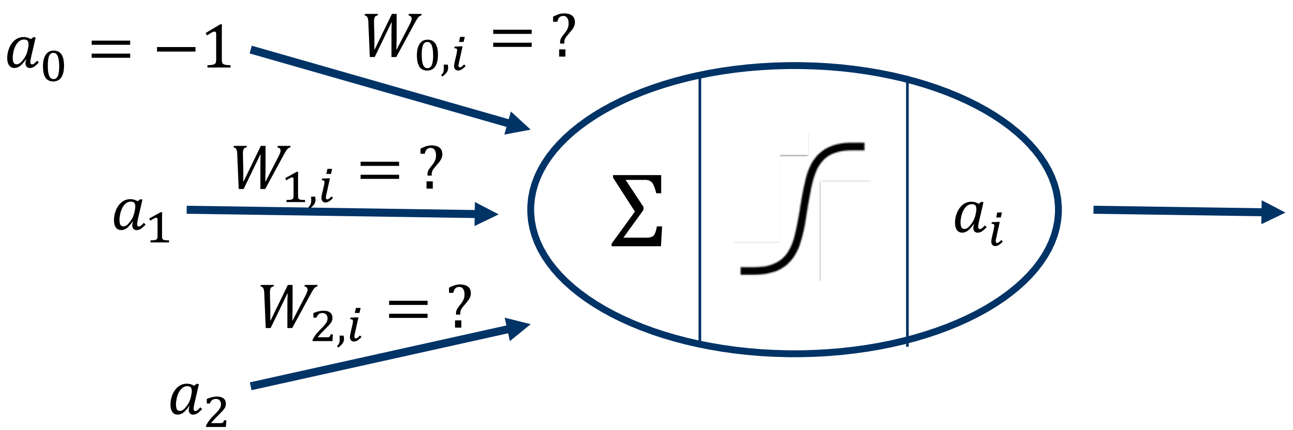
(a) is a step function or threshold function

(b) is a sigmoid function $1/(1+e^{-x})$

Changing the bias weight $W_{0,i}$ moves the threshold location

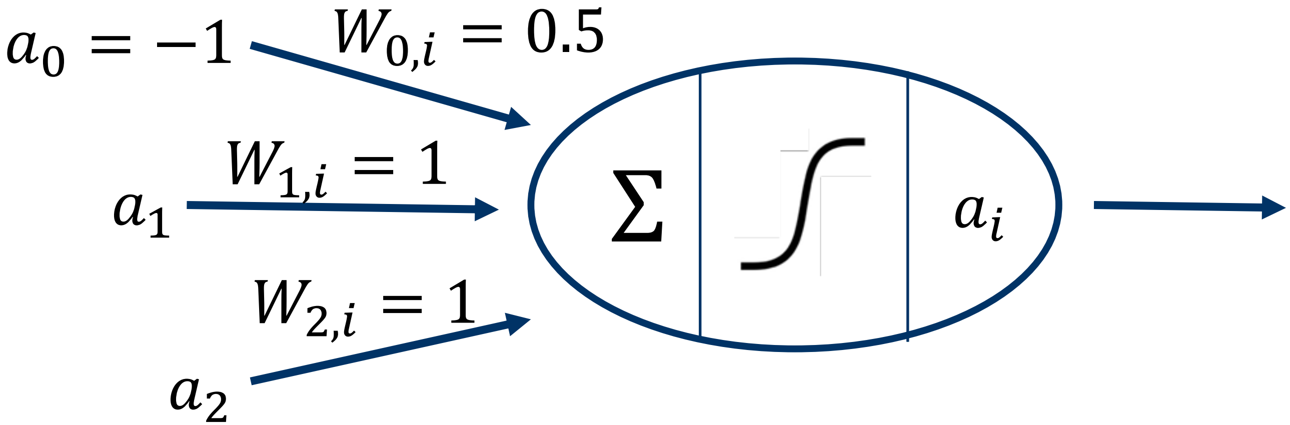
Logical Functions

- How can we implement (a_1 OR a_2)?



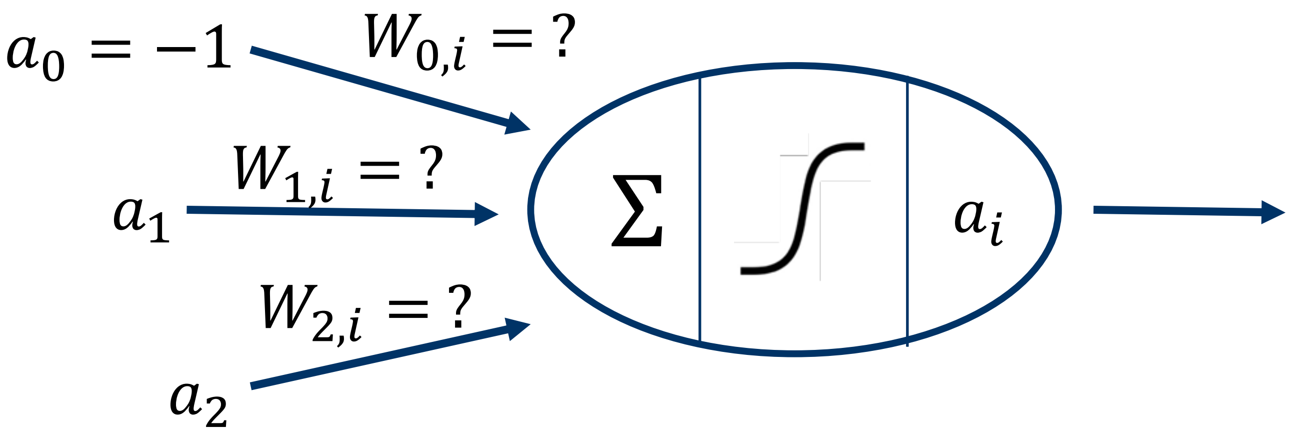
Logical Functions

- How can we implement (a_1 OR a_2)?



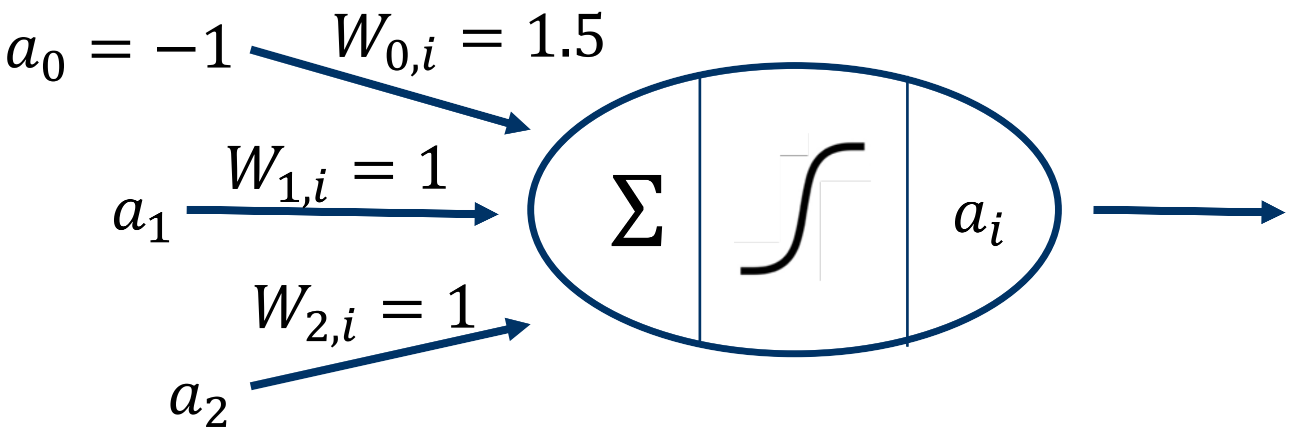
Logical Functions

- How can we implement (a_1 AND a_2)?



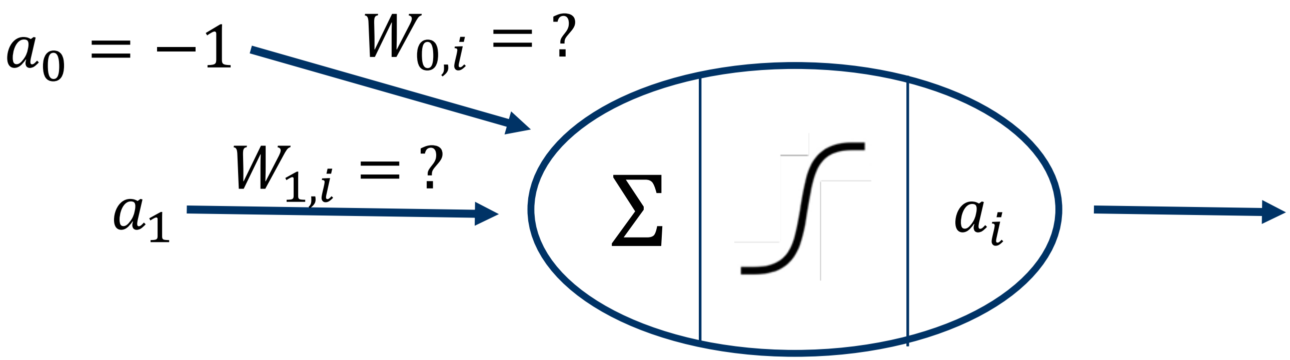
Logical Functions

- How can we implement $(a_1 \text{ AND } a_2)$?



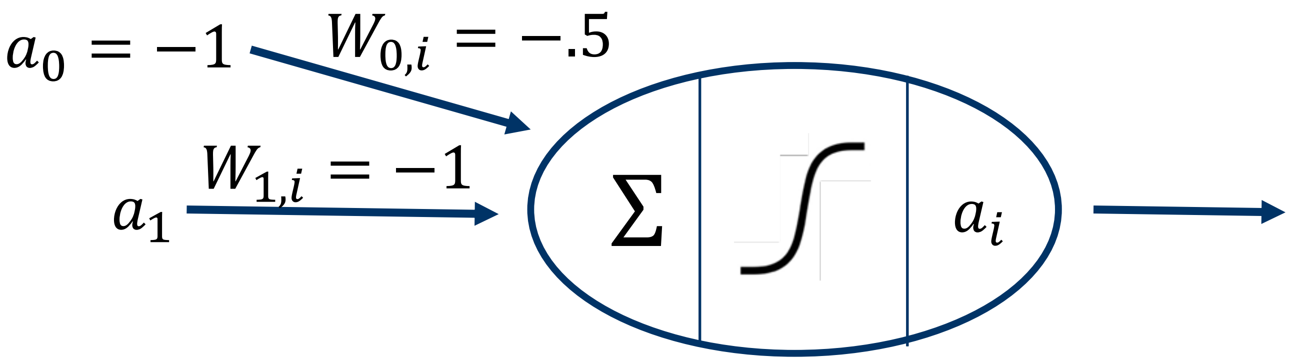
Logical Functions

- How can we implement (NOT a_1)?



Logical Functions

- How can we implement (NOT a_1)?



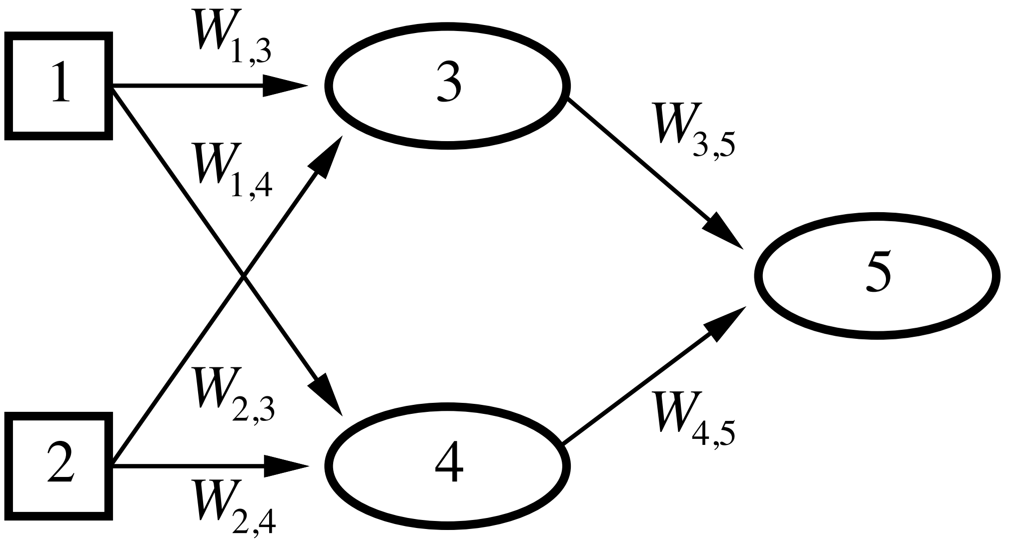
Network structures

Feed-forward networks:
- single-layer perceptrons
- multi-layer perceptrons

Feed-forward networks implement functions, have no internal state

Recurrent networks:
- Hopfield networks have symmetric weights ($W_{i,j} = W_{j,i}$)
  $g(x) = \text{sign}(x)$, $a_i = \pm 1$; **holographic associative memory**
- Boltzmann machines use stochastic activation functions,
  $\approx$ MCMC in Bayes nets (Monte Carlo Markov Chains)
- recurrent neural nets have directed cycles with delays
  $\Rightarrow$ have internal state (like flip-flops), can oscillate etc.

Feed-forward example

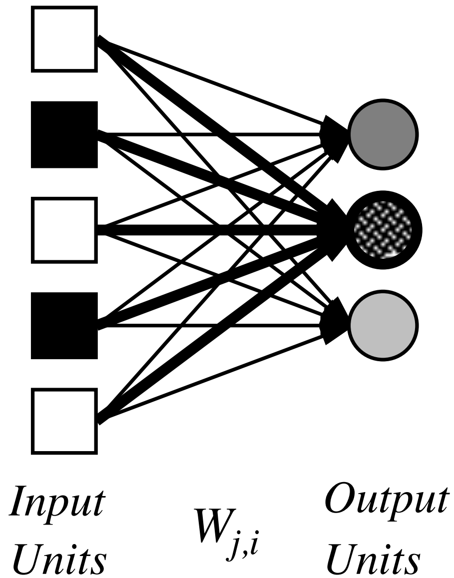


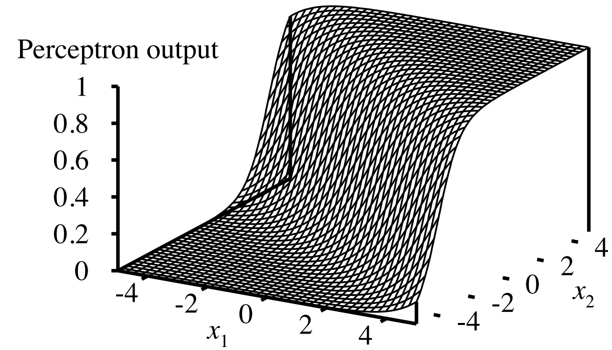
Feed-forward network = a parameterized family of nonlinear functions:

$$
\begin{aligned}
a_5 &= g(W_{3,5} \cdot a_3 + W_{4,5} \cdot a_4) \\
&= g(W_{3,5} \cdot g(W_{1,3} \cdot a_1 + W_{2,3} \cdot a_2) + W_{4,5} \cdot g(W_{1,4} \cdot a_1 + W_{2,4} \cdot a_2))
\end{aligned}
$$

Adjusting weights changes the function: do learning this way!

Single-layer perceptrons





Output units all operate separately—no shared weights

Adjusting weights moves the location, orientation, and steepness of cliff

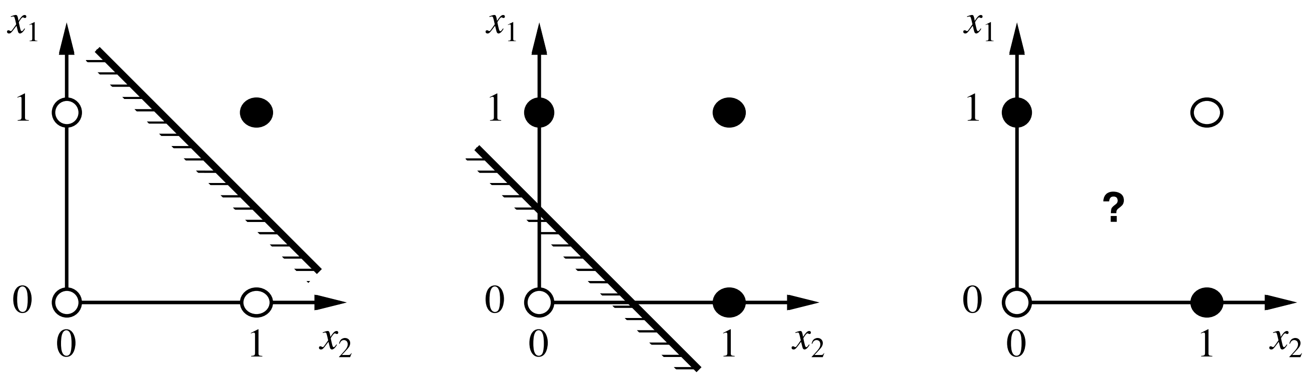
Expressiveness of perceptrons

Consider a perceptron with g = step function (Rosenblatt, 1957, 1960)

Can represent AND, OR, NOT, majority, etc., but not XOR

Represents a linear separator in input space:

$\Sigma_j W_j x_j > 0$ or $\mathbf{W} \cdot \mathbf{x} > 0$



(a) x_1 **and** x_2 (b) x_1 **or** x_2 (c) x_1 **xor** x_2

Minsky & Papert (1969) pricked the neural network balloon

Perceptron learning

Learn by adjusting weights to reduce error on training set

The squared error for an example with input $\mathbf{x}$ and true output y is

$$E = \frac{1}{2}Err^2 \equiv \frac{1}{2}(y - h_{\mathbf{W}}(\mathbf{x}))^2 \ ,$$

where h() is the "function" implemented by the network

Perform optimization search by gradient descent:

$$\begin{aligned}\frac{\partial E}{\partial W_j} &= Err \times \frac{\partial Err}{\partial W_j} = Err \times \frac{\partial}{\partial W_j}\left(y - g(\sum_{j=0}^{n} W_j x_j)\right) \\ &= -Err \times g'(in) \times x_j\end{aligned}$$

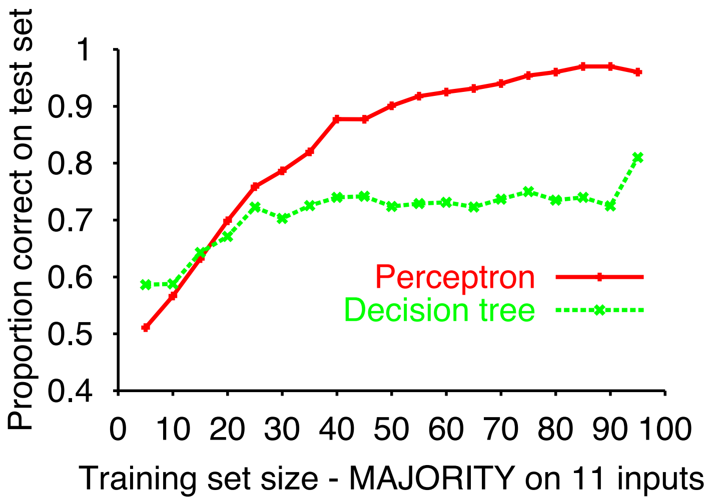
Simple weight update rule:

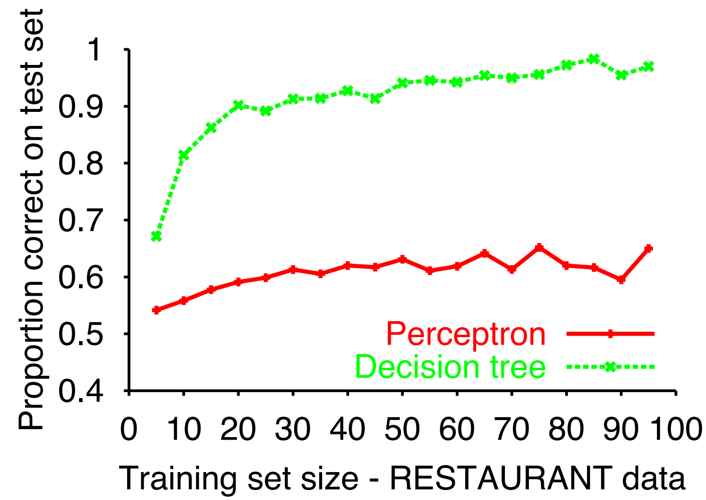
$$W_j \leftarrow W_j + \alpha \times Err \times g'(in) \times x_j$$

(positive)
E.g., $+$ve error $\Rightarrow$ increase network output
$\Rightarrow$ increase weights on $+$ve inputs, decrease on -ve inputs

Perceptron learning contd.

Perceptron learning rule converges to a consistent function
for any linearly separable data set



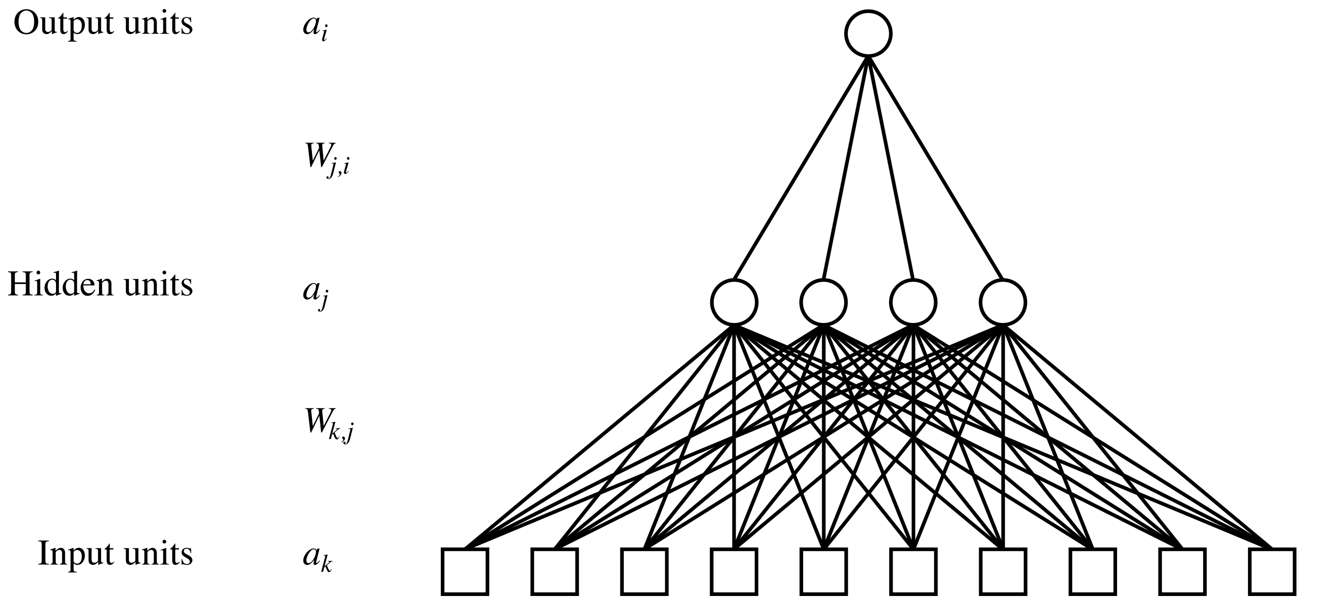


Perceptron learns majority function easily, DTL is hopeless
(Decision Tree Learning)

DTL learns restaurant function easily, perceptron cannot represent it

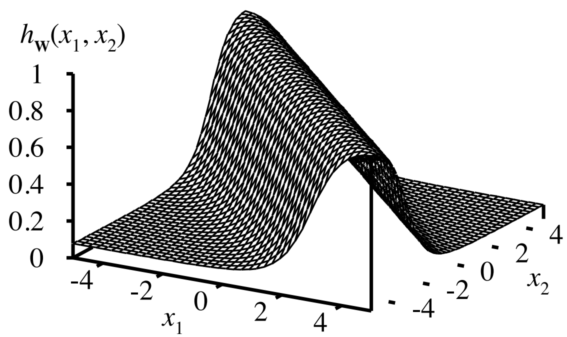
Multilayer perceptrons

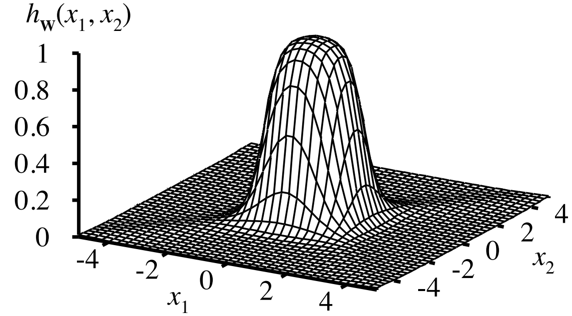
Layers are usually fully connected;
numbers of hidden units typically chosen by hand



Expressiveness of MLPs

All continuous functions w/ 2 layers, all functions w/ 3 layers





Combine two opposite-facing threshold functions to make a ridge

Combine two perpendicular ridges to make a bump

Add bumps of various sizes and locations to fit any surface

Proof requires exponentially many hidden units (cf DTL proof)

Back-propagation learning

Output layer: same as for single-layer perceptron,

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times \Delta_i$$

where $\Delta_i = Err_i \times g'(in_i)$

Hidden layer: **back-propagate** the error from the output layer:

$$\Delta_j = g'(in_j) \sum_i W_{j,i} \Delta_i \ .$$

Update rule for weights in hidden layer:

$$W_{k,j} \leftarrow W_{k,j} + \alpha \times a_k \times \Delta_j \ .$$

(Most neuroscientists deny that back-propagation occurs in the brain)

Back-propagation derivation

The squared error on a single example is defined as

$$E = \frac{1}{2}\sum_i (y_i - a_i)^2 \ ,$$

where the sum is over the nodes in the output layer.

$$\begin{aligned}
\frac{\partial E}{\partial W_{j,i}} &= -(y_i - a_i)\frac{\partial a_i}{\partial W_{j,i}} = -(y_i - a_i)\frac{\partial g(in_i)}{\partial W_{j,i}} \\
&= -(y_i - a_i)g'(in_i)\frac{\partial in_i}{\partial W_{j,i}} = -(y_i - a_i)g'(in_i)\frac{\partial}{\partial W_{j,i}}\left(\sum_j W_{j,i}a_j\right) \\
&= -(y_i - a_i)g'(in_i)a_j = -a_j\Delta_i
\end{aligned}$$

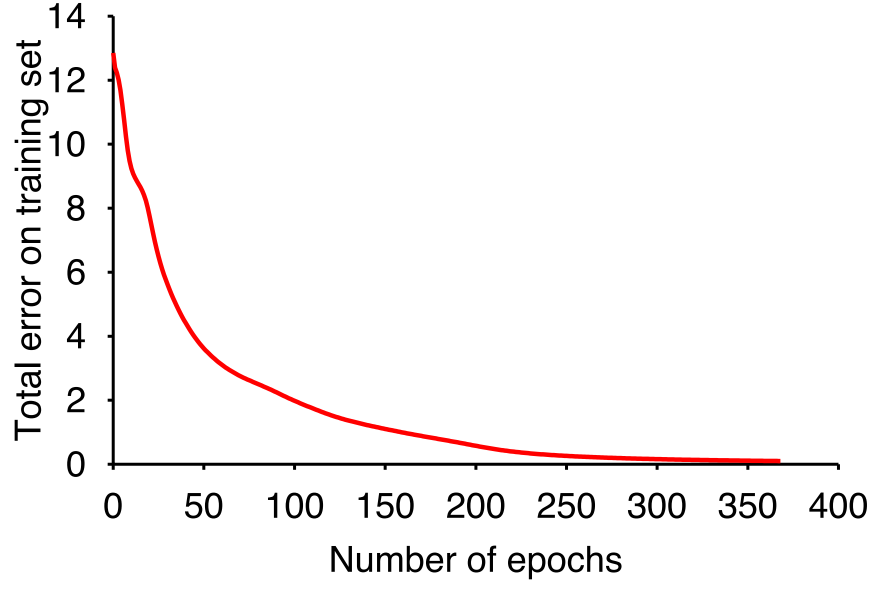
Back-propagation derivation contd.

$$
\begin{aligned}
\frac{\partial E}{\partial W_{k,j}} &= -\sum_i (y_i - a_i)\frac{\partial a_i}{\partial W_{k,j}} = -\sum_i (y_i - a_i)\frac{\partial g(in_i)}{\partial W_{k,j}} \\
&= -\sum_i (y_i - a_i) g'(in_i)\frac{\partial in_i}{\partial W_{k,j}} = -\sum_i \Delta_i \frac{\partial}{\partial W_{k,j}}\left(\sum_j W_{j,i} a_j\right) \\
&= -\sum_i \Delta_i W_{j,i}\frac{\partial a_j}{\partial W_{k,j}} = -\sum_i \Delta_i W_{j,i}\frac{\partial g(in_j)}{\partial W_{k,j}} \\
&= -\sum_i \Delta_i W_{j,i} g'(in_j)\frac{\partial in_j}{\partial W_{k,j}} \\
&= -\sum_i \Delta_i W_{j,i} g'(in_j)\frac{\partial}{\partial W_{k,j}}\left(\sum_k W_{k,j} a_k\right) \\
&= -\sum_i \Delta_i W_{j,i} g'(in_j) a_k = -a_k \Delta_j
\end{aligned}
$$

Back-propagation learning contd.

At each epoch, sum gradient updates for all examples and apply

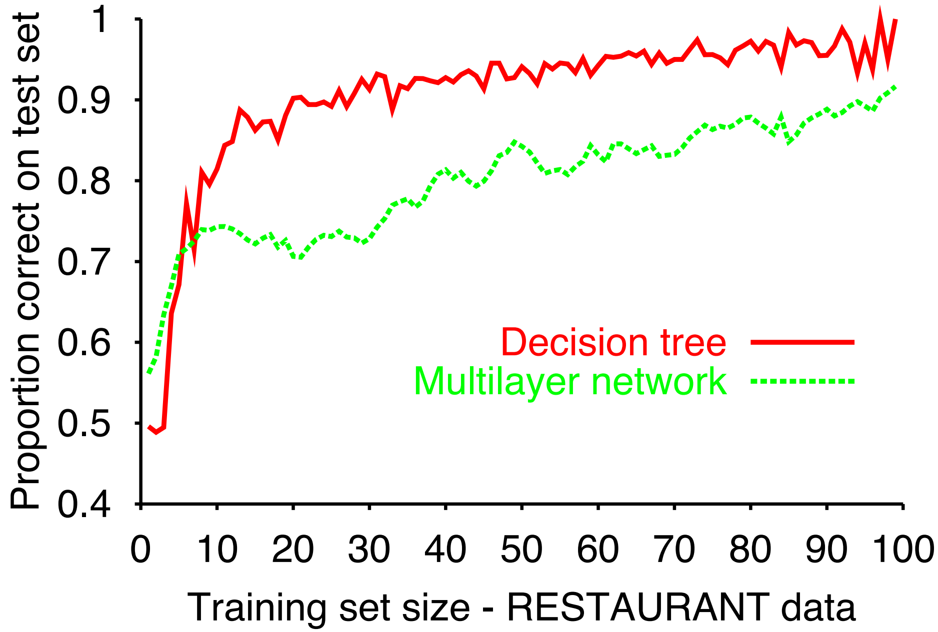
Training curve for 100 restaurant examples: finds exact fit



Typical problems: slow convergence, local minima

Back-propagation learning contd.

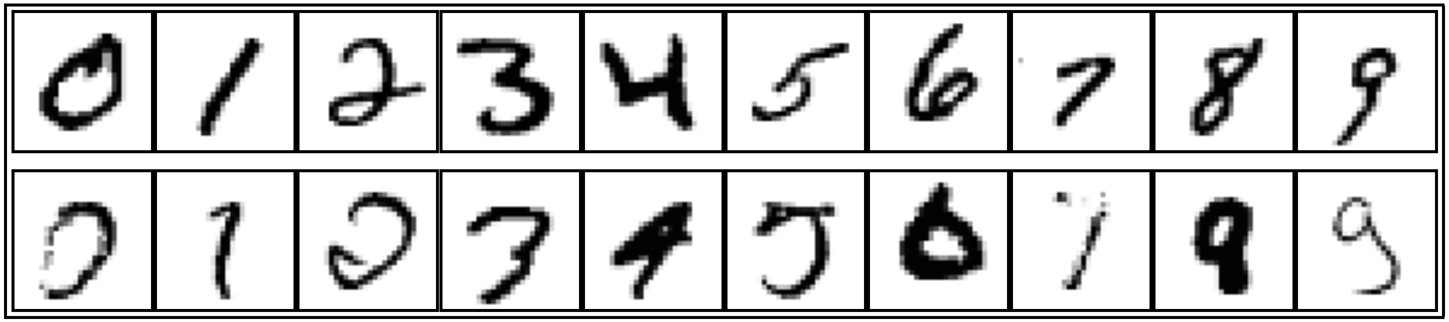
Learning curve for MLP with 4 hidden units:



MLPs are quite good for complex pattern recognition tasks,
but resulting hypotheses cannot be understood easily

Critical drawback of this approach in general!

Handwritten digit recognition



3-nearest-neighbor = 2.4% error (Store all images, find nearest image(s))

400–300–10 unit MLP = 1.6% error

LeNet: 768–192–30–10 unit MLP = 0.9% error ("Convolutional" neural net)

Current best (kernel machines, vision algorithms) $\approx$ 0.6% error

Summary

- Most brains have lots of neurons; does each neuron ≈ linear–threshold unit ??
- Perceptrons (one-layer networks) are insufficiently expressive
- Multi-layer networks are sufficiently expressive
  - can be trained by gradient descent, i.e., backpropagation
- Many potential applications
  - speech, driving, handwriting, fraud detection, etc.
- Engineering, cognitive modeling, and neural-system modelling subfields have largely diverged