

Time Series Cheat Sheet

By Marcelo Moreno - King Juan Carlos University

As part of the Econometrics Cheat Sheet Project

THIS IS A WORK IN PROGRESS

NOT INTENDEND FOR GENEAL PURPOSE

Basic concepts

Definitions

Time series - is a succession of quantitative observations of a phenomena ordered in time.

There are some variations of time series:

- **Panel data** - consist of a time series for each observation of a cross section.
- **Pooled cross sections** - combines cross sections from different time periods.

Stochastic process - sequence of random variables that are indexed in time.

Components of a time series

- **Trend** - is the long-term general movement of a series.
- **Seasonal variations** - are periodic oscillations that are produced in a period equal or inferior to a year, and can be easily identified on different years (usually are the result of climatology reasons).
- **Cyclical variations** - are periodic oscillations that are produced in a period greater than a year (are the result of the economic cycle).
- **Residual variations** - are movements that do not follow a recognizable periodic oscillation (are the result of eventual non-permanent phenomena that can affect the studied variable in a given moment).

Type of time series models

- **Static models** - the relation between y and x 's is contemporary. Conceptually:

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

- **Distributed-lag models** - the relation between y and x 's is not contemporary. Conceptually:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \dots + \beta_{q+1} x_{t-q} + u_t$$

The long term cumulative effect in y when Δx is:

$$\beta_1 + \beta_2 + \dots + \beta_{q+1}$$

Assumptions and properties

OLS model assumptions under time series

Under this assumptions, the estimators of the OLS parameters will present good properties. **Gauss-Markov assumptions extended applied to time series:**

- ts1. **Parameters linearity and weak dependence.**
- a. y_t must be a linear function of the β 's.
 - b. The stochastic $\{(x_t, y_t) : t = 1, 2, \dots, T\}$ is stationary and weakly dependent.
- ts2. **No perfect collinearity.**
- There are no independent variables that are constant: $\text{Var}(x_j) \neq 0$
 - There is not an exact linear relation between independent variables.
- ts3. **Conditional mean zero and correlation zero.**
- a. There are no systematic errors: $E(u_t | x_{1t}, \dots, x_{kt}) = E(u_t) = 0 \rightarrow$ **strong exogeneity** (a implies b).
 - b. There are no relevant variables left out of the model: $\text{Cov}(x_{jt}, u_t) = 0$ for any $j = 1, \dots, k \rightarrow$ **weak exogeneity**.
- ts4. **Homoscedasticity.** The variability of the residuals is the same for any t : $\text{Var}(u_t | x_{1t}, \dots, x_{kt}) = \sigma^2$
- ts5. **No auto-correlation.** The residuals do not contain information about other residuals: $\text{Corr}(u_t, u_s | x) = 0$ for any given $t \neq s$.
- ts6. **Normality.** The residuals are independent and identically distributed: $u \sim N(0, \sigma^2)$
- ts7. **Data size.** The number of observations available must be greater than $(k+1)$ parameters to estimate. (IS already satisfied under asymptotic situations)

Asymptotic properties of OLS

Under the econometric model assumptions and the Central Limit Theorem:

- Hold (1) to (3a): OLS is **unbiased**. $E(\hat{\beta}_j) = \beta_j$
- Hold (1) to (3): OLS is **consistent**. $\text{plim}(\hat{\beta}_j) = \beta_j$ (to (3b) left out (3a), weak exogeneity, biased but consistent)
- Hold (1) to (5): **asymptotic normality** of OLS (then, (6) is necessarily satisfied): $u \sim_a N(0, \sigma^2)$.
- Hold (1) to (5): **unbiased estimate of σ^2** . $E(\hat{\sigma}^2) = \sigma^2$
- Hold (1) to (5): OLS is **BLUE** (Best Linear Unbiased Estimator) or **efficient**.
- Hold (1) to (6): hypothesis testing and confidence intervals can be done reliably.

Trends and seasonality

Trends

Two time series can have the same (or contrary) trend, that should lend to a high level of correlation. This, can provoke a false appearance of causality, the problem is in what is known as **spurious regression** (the non-fulfillment of ts3). For example, given the model:

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

where:

$$y_t = \alpha_0 + \alpha_1 \text{Trend} + e_t$$

$$x_t = \gamma_0 + \gamma_1 \text{Trend} + v_t$$

Adding a trend to the model can solve the problem:

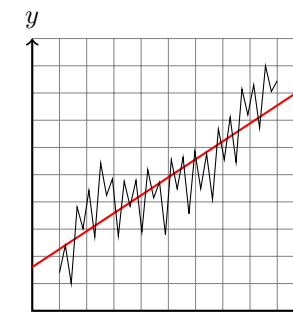
$$y_t = \beta_0 + \beta_1 x_t + \beta_2 \text{Trend} + u_t$$

The trend can be linear or non-linear (quadratic, cubic, exponential, etc.)

Seasonality

A time series with can manifest seasonality. That is, the series is subject to a seasonal variations or pattern, usually related to climatology conditions.

For example, the GDP is usually higher in summer and lower in winter.



This problem is part of what is known as **spurious regression** (the non-fulfillment of ts3). A seasonal adjustment can solve the problem.

A simple seasonal adjustment could be creating stational binary variables and adding them to the model. For example, for quarterly series:

$$y_t = \beta_0 + \beta_1 Q2_t + \beta_2 Q3_t + \beta_3 Q4_t + \beta_4 x_{1t} + \dots + \beta_k x_{kt} + u_t$$

Another way is to seasonally adjust (sa) the variables, and then, do the regression with the final regression:

$$z_t = \beta_0 + \beta_1 Q2_t + \beta_2 Q3_t + \beta_3 Q4_t + e_t \rightarrow (sa)\tilde{e}_t = (sa)\tilde{z}_t$$
$$(sa)\tilde{y}_t = \beta_0 + \beta_1 (sa)\tilde{x}_{1t} + \dots + \beta_k (sa)\tilde{x}_{kt} + u_t$$

There are much better and complex methods to seasonally adjust a time series.

Stationarity

Strong exogeneity is violated in static models and distributed-lag models.

Stationary and non-stationary process

- Stationary process - is the one in that the probability distribution are stable in time. If any collection of random variables are taken, and are shifted h periods, the joint probability distribution should be unaltered.
 - Strong stationarity - if the mean is constant in time.
 - Weak stationarity - $Cov(x_t, x_{t+h})$ only depends on h , not of t .
- Non-stationary process - for example, a series with trend, at least the mean change with the time (at least).

Stationarity is good, β_j stand unchanged in time.

Weak-dependent time series

How close the relationship between x_t and x_{t+h} is as the temporal distance between the series increases (h). (it is good, can achieve Central Limit Theorem). It is different from the concept of stationarity.

Weak dependence: when x_t and x_{t+h} are almost independent as h increases to infinity.

When $Corr(x_t, x_{t+h}) \rightarrow 0$ when $h \rightarrow infinity$

Moving average process order 1

MA(1)

iid (independent and identically distributed)

$$x_t = e_t + \alpha_1 e_{t-1}, t = 1, 2, \dots, T$$

is stationary and weak-dependent

Auto-regressive process order 1

AR(1)

iid

$$y_t = \rho_1 y_{t-1} + u_t$$

If $|\rho_1| < 1$ in AR(1)

stable process

is stationary and weak dependent

A series with a trend cannot be stationary, but can be weak dependent (and OK if the model have included a

Trend variable)

Dummy variables and structural change in time series

Dummy (or binary) variables are used for qualitative information like sex, civil state, country, etc.

- Get the **value of 1 in a given category, and 0 on the rest.**
- Are used to analyze and modeling **structural changes** in the model parameters.

If a qualitative variable have m categories, we only have to include $(m - 1)$ dummy variables.

Structural change

Structural change refers to changes in the values of the parameters of the econometric model produced by the effect of different sub-populations. Structural change can be included in the model through dummy variables.

The position of the dummy variable matters:

- **On the intercept (β_0)** - represents the mean difference between the values produced by the structural change.
- **On the parameters that determines the slope of the regression line (β_j)** - represents the effect (slope) difference between the values produced by the structural change.

The Chow's structural contrast - when we want to analyze the existence of structural changes in all the model parameters, it is common to use a particular expression of the F contrast known as the Chow's contrast, where the null hypothesis is: H_0 : No structural change (non-asymptotic correlation)

Predictions

Two types of prediction:

- Of the mean value of y for a specific value of x .
- Of an individual value of y for a specific value of x .

If the values of the variables (x) approximate to the mean values (\bar{x}), the confidence interval amplitude of the prediction will be shorter.

Heteroscedasticity in time series

The residuals u_i of the population regression function do not have the same variance σ^2 :

$$Var(u|x) = Var(y|x) \neq \sigma^2$$

Is the **breaking of the fifth (5) econometric model**

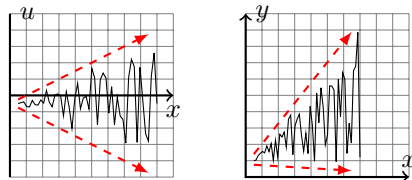
assumption.

Consequences

- OLS estimators still are unbiased.
- OLS estimators still are consistent.
- OLS is **not efficient** anymore, but still a LUE (Linear Unbiased Estimator).
- **Variance estimations of the estimators are biased:** the construction of confidence intervals and the hypothesis contrast are not reliable.

Detection

- **Graphical analysis** - look for scatter patterns on x vs. u or x vs. y plots.



- **Formal tests** - White, Bartlett, Breusch-Pagan, etc. Commonly, the null hypothesis: H_0 = Homoscedasticity

DURBIN WATSON EXPLANATION AND GRAPHICAL ANALYSIS (AND MOST COMMON VALUES?)

Correction

- Use OLS with a variance-covariance matrix estimator robust to heteroscedasticity, for example, the one proposed by White.
- If the variance structure is known, make use of Weighted Least Squares (WLS) or Generalized Least Squares (GLS).

- If the variance structure is not known, make use of Feasible Weighted Least Squared (FWLS), that estimates a possible variance, divides the model variables by it and then apply OLS.
- Make assumptions about the possible variance:
 - Supposing that σ_i^2 is proportional to x_i , divide the model variables by the square root of x_i and apply OLS.
 - Supposing that σ_i^2 is proportional to x_i^2 , divide the model variables by x_i and apply OLS.
- Make a new model specification, for example, logarithmic transformation.

Auto-correlation

The residual of any observation, u_t , is correlated with the residual of any other observation. The observations are not independent.

$$\text{Corr}(u_t, u_s | x) \neq 0 \text{ for any } t \neq s$$

The “natural” context of this phenomena is time series. Is the **breaking of the sixth (6) econometric model assumption**.

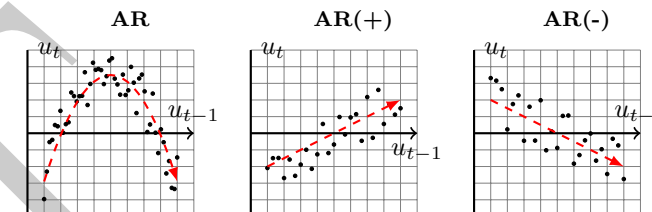
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Detection

- **Graphical analysis** - look for scatter patterns on u_{t-1} vs. u_t or make use of a correlogram.



- **Formal tests** - Durbin-Watson, Breusch-Godfrey, etc. Commonly, the null hypothesis: H_0 : No auto-correlation

Correction

- Use OLS with a variance-covariance matrix estimator robust to auto-correlation, for example, the one proposed by Newey-West.
- Use Generalized Least Squares. Supposing $y_t = \beta_0 + \beta_1 x_t + u_t$, with $u_t = \rho u_{t-1} + \varepsilon_t$, where $|\rho| < 1$ and ε_t is white noise.
 - If ρ is known, create a quasi-differentiated model where u_t is white noise and estimate it by OLS.
 - If ρ is not known, estimate it by -for example- the Cochrane-Orcutt method, create a quasi-differentiated model where u_t is white noise and estimate it by OLS.

Endogeneity