Time Series Cheat Sheet

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As part of the Econometrics Cheat Sheet Project

THIS IS A WORK IN PROGRESS

NOT INTENDEND FOR GENEAL PURPOSE ATM

It is still a mess as of January 24, 2022

Basic concepts

Definitions

Time series - is a succession of quantitative observations of a phenomena ordered in time.

Stochastic process - sequence of random variables that are indexed in time.

There are types of data that makes uses of time series

Panel data - consist of a temporal series for each observation of a cross section.

Pooled cross sections - combines cross sections from different temporal periods.

Components of a time series

- Trend
- Seasonal variations (sv)
- Cyclical variations (cv)
- Residual variations (rv)

Components calculus

- Trend: linear, exponential, quadratic.

Spurious regression

Stationary processes

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Static models:
$y_t = \beta_0 + \beta_1 x_t + u_t$

Assumptions and properties

OLS model assumptions under time series

Under this assumptions, the estimators of the OLS parameters will present good properties. **Gauss-Markov assumptions extended**:

1. **Parameters linearity and weak dependence**.
   a. y_t must be a linear function of the β's.
   b. The stochastic $\{(x_t, y_t) : t = 1, 2, ...\}$ is stationary and weakly dependent.
2. **No perfect collinearity**.
   - There are no independent variables that are constant: $Var(x) \neq 0$
   - There is not an exact linear relation between independent variables.
3. **Conditional mean zero and correlation zero**.
   a. There are no systematic errors: $E(u_t|x_{1t}, ..., x_{kt}) = E(u_t) = 0, t = 1, 2, ...T \rightarrow$ **strong exogeneity** (a implies b).
   b. There are no relevant variables left out of the model: $Cov(x_{jt}, u_t) = 0$ for any $j = 1, ..., k \rightarrow$ **weak exogeneity**.
4. **Homoscedasticity**. The variability of the residuals is the same for any t: $Var(u_t|x_{1t}, ..., x_{kt}) = \sigma^2, t = 1, 2, ..., T$
5. **No auto-correlation**. The residuals do not contain information about other residuals: $Corr(u_t, u_s|x) = 0$ for any given $t \neq s$.
6. **Normality**. The residuals are independent and identically distributed: $u \sim N(0, \sigma^2)$
7. **Data size**. The number of observations available must be greater than $(k + 1)$ parameters to estimate. (IS already satisfied under asymptotic situations)

Asymptotic properties of OLS

Under the econometric model assumptions and the Central Limit Theorem:

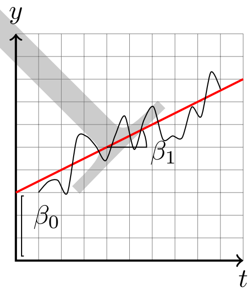
- Hold (1) to (3a): OLS is **unbiased**. $E(\hat{\beta}_j) = \beta_j$
- Hold (1) to (3): OLS is **consistent**. $plim(\hat{\beta}_j) = \beta_j$ (to (3b) left out (3a), weak exogeneity, biased but consistent)
- Hold (1) to (5): **asymptotic normality** of OLS (then, (7) is necessarily satisfied): $u \sim_a N(0, \sigma^2)$.
- Hold (1) to (5): **unbiased estimate of** σ^2. $E(\hat{\sigma}^2) = \sigma^2$
- Hold (1) to (5): OLS is BLUE (Best Linear Unbiased Estimator) or **efficient**.
- Hold (1) to (6): hypothesis testing and confidence intervals can be done reliably.

Ordinary Least Squares

Objective - minimize the Sum of Squared Residuals (SSR):

$$\text{Min} \sum_{t=1}^{T} \hat{u}_t^2, \text{ where } \hat{u}_t = y_t - \hat{y}_t$$

Regression model



Equation:
$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} u_t$

Matriciial solution: $\hat{\beta} = (X^T X)^{-1}(X^T y)$

Interpretation of coefficients

Model	Dependent	Independent	β_1 interpretation
Level-level	y	x	$\Delta y = \beta_1 \Delta x$
Level-log	y	$log(x)$	$\Delta y = (\beta_1/100)\%\Delta x$
Log-level	$log(y)$	x	$\%\Delta y = (100\beta_1)\Delta x$
Log-log	$log(y)$	$log(x)$	$\%\Delta y = \beta_1 \%\Delta x$
Quadratic	y	$x + x^2$	$\Delta y = (\beta_1 + 2\beta_2 x)\Delta x$

Error measures

Sum of Sq. Resid.: $SSR = \sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

Expl. Sum of Sq.: $SSE = \sum_{i=1}^{n}(\hat{y}_i - \overline{y})^2$

Tot. Sum of Sq.: $SST = SSE + SSR = \sum_{i=1}^{n}(y_i - \overline{y})^2$

Standard Error of the Regression: $\hat{\sigma} = \sqrt{\frac{SSR}{n-k-1}}$

Standard Error of the $\hat{\beta}$'s: $se(\hat{\beta}) = \hat{\sigma}\sqrt{(X^T X)^{-1}}$

Sqrt. of the Quadratic Mean Error: $\sqrt{\frac{\sum_{i=1}^{n}(\hat{u}_i - \overline{u})^2}{n}}$

Absolute Mean Error: $\frac{\sum_{i=1}^{n}|\hat{u}_i|}{n}$

Mean Percentage Error: $\frac{\sum_{i=1}^{n}|\hat{u}_i/y_i|}{n} \times 100$

R-squared

Is a **measure of the goodness of the fit**, Measures the **percentage of variation of y that is linearly explained by the variations of x's**:

$$R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$$

There is a version corrected by deggrees of freedom (adjusted r-squared):

$$\overline{R}^2 = 1 - \frac{n-1}{n-k-1}\frac{SSR}{SST} = 1 - \frac{n-1}{n-k-1}(1-R^2)$$

For big sample sizes: $\overline{R}^2 \approx R^2$

In time series context, r-squared tends to be very high. Although, it can be artificial (cointegration).

The F contrast

Under H_0:

$$F = \frac{SSR_r - SSR_{nr}}{SSR_{nr}}\frac{(n-k_{nr}-1)}{q} \sim F_{q,n-k_{nr}-1}$$

Where k_{nr} is the number of parameters of the non restricted model and q is the number of linear hypothesis tested.

If $F_{q,n-k_{nr}-1} < F$, there is evidence to reject the null hypothesis.

Dummy variables and structural change in time series

Dummy (or binary) variables are used for qualitative information like sex, civil state, country, etc.

- Get the **value of 1 in a given category, and 0 on the rest**.
- Are used to analyze and modeling **structural changes** in the model parameters.

If a qualitative variable have m categories, we only have to include $(m-1)$ dummy variables.

Structural change

Structural change refers to changes in the values of the parameters of the econometric model produced by the effect of different sub-populations. Structural change can be included in the model through dummy variables.

The position of the dummy variable matters:

- **On the intercept** (β_0) - represents the mean difference between the values produced by the structural change.
- **On the parameters that determines the slope of the regression line** (β_j) - represents the effect (slope) difference between the values produced by the structural change.

The Chow's structural contrast - when we want to analyze the existence of structural changes in all the model parameters, it is common to use a particular expression of the F contrast known as the Chow's contrast, where the null hypothesis is: H_0 : No structural change

Predictions

Two types of prediction:

- Of the mean value of y for a specific value of x.
- Of an individual value of y for a specific value of x.

If the values of the variables (x) approximate to the mean values ($\overline{x}$), the confidence interval amplitude of the prediction will be shorter.

Heteroscedasticity in time series

The residuals u_i of the population regression function do not have the same variance σ^2:

$$Var(u|x) = Var(y|x) \neq \sigma^2$$

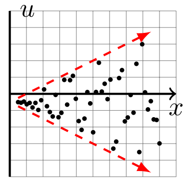
Is the **breaking of the fifth (5) econometric model assumption**.

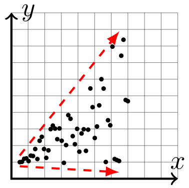
Consequences

- OLS estimators still are unbiased.
- OLS estimators still are consistent.
- OLS is **not efficient** anymore, but still a LUE (Linear Unbiased Estimator).
- **Variance estimations of the estimators are biased**: the construction of confidence intervals and the hypothesis contrast are not reliable.

Detection

- **Graphical analysis** - look for scatter patterns on x vs. u or x vs. y plots.





- **Formal tests** - White, Bartlett, Breusch-Pagan, etc. Commonly, the null hypothesis: H_0 = Homoscedasticity

DURBIN WATSON EXPLANATION AND GRAPHICAL ANALYSIS (AND MOST COMMON VALUES?)

Correction

- Use OLS with a variance-covariance matrix estimator robust to heteroscedasticity, for example, the one proposed by White.
- If the variance structure is known, make use of Weighted Least Squares (WLS) or Generalized Least Squares (GLS).
- If the variance structure is not known, make use of Feasible Weighted Least Squared (FWLS), that estimates a possible variance, divides the model variables by it and then apply OLS.
- Make assumptions about the possible variance:
  - Supposing that σ_i^2 is proportional to x_i, divide the model variables by the square root of x_i and apply OLS.
  - Supposing that σ_i^2 is proportional to x_i^2, divide the model variables by x_i and apply OLS.

- Make a new model specification, for example, logarithmic transformation.

Auto-correlation

The residual of any observation, u_t, is correlated with the residual of any other observation. The observations are not independent.

$$Corr(u_t, u_s|x) \neq 0 \text{ for any } t \neq s$$

The "natural" context of this phenomena is time series. Is the **breaking of the sixth (6) econometric model assumption**.

Consequences

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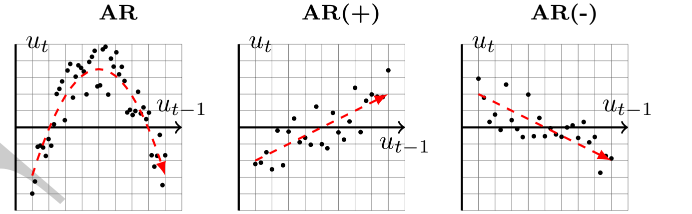
- OLS estimators still are consistent.
- OLS is **not efficient** anymore, but still a LUE (Linear Unbiased Estimator).
- **Variance estimations of the estimators are biased**: the construction of confidence intervals and the hypothesis contrast are not reliable.

Detection

- **Graphical analysis** - look for scatter patterns on u_{t-1} vs. u_t or make use of a correlogram.



- **Formal tests** - Durbin-Watson, Breusch-Godfrey, etc. Commonly, the null hypothesis: H_0 : No auto-correlation

Correction

- Use OLS with a variance-covariance matrix estimator robust to auto-correlation, for example, the one proposed by Newey-West.
- Use Generalized Least Squares. Supposing $y_t = \beta_0 + \beta_1 x_t + u_t$, with $u_t = \rho u_{t-1} + \varepsilon_t$, where $|\rho| < 1$ and ε_t is white noise.
  - If ρ is known, create a quasi-differentiated model where u_t is white noise and estimate it by OLS.
  - If ρ is not known, estimate it by -for example- the Cochrane-Orcutt method, create a quasi-differentiated model where u_t is white noise and estimate it by OLS.

Endogeneity

Instrumental Variables Methods