

CIS 1904: Haskell

Higher-Order Patterns

Logistics

- HW 4 will be released tonight
- Autograder should show total autograded score now

Higher-order functions

Recall: functions in Haskell are *first-class*

- They can be passed around as inputs to other functions
- They can be returned as outputs of other functions

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A function that takes in OR returns another function is called *higher-order*.

Higher-order functions

Examples:

- `map :: (a -> b) -> [a] -> [b]`
- `filter :: (a -> Bool) -> [a] -> [a]`
- `fold :: (a -> b -> b) -> b -> [a] -> b`

These all take in functions and, if partially applied, return functions.

Higher-order functions

Example:

```
compose :: (b -> c) -> (a -> b) -> a -> c  
compose f g = \x -> f (g x)
```

Recall: `\x ->` syntax is used for anonymous functions, like `fun x =>` in OCaml

Higher-order functions

Example:

```
compose :: (b -> c) -> (a -> b) -> a -> c
```

```
compose f g = f . g
```

The standard library provides function composition, written as `(.)`.

Higher-order functions

Composition helps write code that is:

- Concise
- Understandable
- In keeping with the “wholemeal” programming style

```
foo :: String -> String
```

```
foo s = length (filter (== 'C') (toUpper s))
```


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foo :: String -> String
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```
foo s = length (filter (== 'C') (toUpper s))
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```
foo = length . filter (== 'C') . toUpper
```

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foo :: String -> String
```

```
foo s = length (filter (== 'C') (toUpper s))
```

```
foo = length . filter (== 'C') . toUpper
```

Partial application

```
add :: Int -> Int -> Int
```

```
add x y = x + y
```

All functions in Haskell take one argument*!

Partial application

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add :: Int -> (Int -> Int)
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```
add 0 :: Int -> Int
```

Partial application

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add x y = x + y
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All functions in Haskell take one argument*!

```
add :: Int -> (Int -> Int)
```

```
add 0 :: Int -> Int
```

Note: `(Int -> Int) -> Int` is **not** the same as `Int -> Int -> Int`.
`->` is *right-associative*.

Partial application

\rightarrow is right-associative.

$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$ is the same as $\text{Int} \rightarrow (\text{Int} \rightarrow \text{Int})$

Function *application* is left-associative.

$\text{add } 0 \ 1$ is the same as $(\text{add } 0) \ 1$

Partial application

Note: Haskell lets us write anonymous functions like `\x y z -> ...`

This is just syntactic sugar for `\x -> (\y -> (\z -> ...))`.

Similarly, `add x y = ...` is just syntactic sugar for `add = \x -> (\y -> ...)` .

Currying

```
add :: (Int, Int) -> Int
```

```
add (x,y) = x + y
```

```
add :: Int -> Int -> Int
```

```
add x y = x + y
```

These are equivalent! We call going from the first to the second *currying*, after Haskell Curry, and the reverse *uncurrying*.

Currying makes partial application easier.

Partial application

When writing functions, consider:

Which argument are you most likely to want to partially apply with?

`filter f xs` – we often want to filter multiple lists using the same criterion

`filter xs f` – we rarely want to filter the same original list using multiple criteria

Eta reduction

Eta reduction: removing unnecessary function abstractions.

$\lambda x \rightarrow f\ x$ is equivalent to f

Eta reduction

Eta reduction: removing unnecessary function abstractions.

`\x -> f x` is equivalent to `f`

```
foo :: String -> String
```

```
foo xs = map toUpper xs
```

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foo = \xs -> map toUpper xs
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Eta reduction

Eta reduction: removing unnecessary function abstractions.

`\x -> f x` is equivalent to `f`

```
foo :: String -> String
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```
foo xs = map toUpper xs
```

```
foo = \xs -> map toUpper xs
```

```
foo = map toUpper
```

Prefix Operators

`add :: Int -> Int -> Int`

- `add 0 1` – used as a prefix
- `0 `add` 1` – used as an infix

Infix Operators

`(+) :: Int -> Int -> Int`

- `(+) 0 1` – used as a prefix
- `0 + 1` – used as an infix

Haskell lets us use *operator sections*, i.e., partially apply these on either side:

- `map (+ 1) xs`
- `filter (0 <) xs`

Operator sections

`div :: Int -> Int -> Int`

`(-) :: Int -> Int -> Int`

`(>) :: Int -> Int -> Bool`

`(0 -)` – argument is on the left, so we know it's the first argument

`(> 100)` – argument is on the right, so we know it's the second argument

`div 8` – no sides to distinguish; we have to interpret it as the first argument