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## **Homework #2**

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**Possible Protocol 1 (PP1):** roll once; if get 6 then conclude the dice is not fair; if roll any other number then conclude it is fair. **Analyze PP1:** if the dice were fair, what is the probability it would be judged to be unfair? Oppositely, if the dice were unfair, what is the probability that it would be judged to be fair?

If the dice is fair, there is a  $\frac{1}{6}$  chance of being deemed unfair.

If the dice is unfair, the probability of it being deemed fair is  $\frac{5}{6}$

Probability of rolling 2 6's is  $\frac{1}{36}$

**Possible Protocol (PP2):** roll the dice 20 times. (Each person should have done this beforehand.) Group can specify a decision rule to judge that dice is fair or unfair. Consider the stats question: if fair dice are rolled 20 times, what is likely number of 6 resulting? How unusual is it, to get 1 more or less than that? How unusual is it, to get 2 more or less? 3? **Analyze PP2** including the question: if the dice were fair, what is the chance it could be judged as unfair?

We rolled dice 20 times, and we got 3 times 6 and it is very close to the number which we were expecting, which is 3.33 times. Getting one more or less is very likely to happen.

Fair:  $\frac{3}{20} = 15\%$

If you roll x7, at least once a 6 will appear (30%)

If you roll x14, 6 will appear at least twice

If you roll x21, 6 will appear at least three times

If the dice were rolled once, there'd be a  $\frac{1}{6}$  chance of it being deemed unfair

X4 - 20% probability

X100 - 22% (theoretically should be 17)

Judged to be fair- 10 to 25 - allow for a margin of error)

**Possible Protocol (PP3):** roll 100 times and specify decision rules. Some cases are easy: if every single roll comes to 6 then might quickly conclude. But what about the edge cases? Is it fair to say that every conclusion has some level of confidence attached? Where do you set

**boundaries for decisions? Analyze PP3. What is the chance that fair dice could be judged to be unfair?**

```
how_many_rolls <- 100
> sim_rolls <- sample(1:6, how_many_rolls, replace = TRUE)
if_come_up_6 <- as.numeric(lots_of_sim_rolls == 6)
> mean(if_come_up_6)
[1] 0.15
mean(as.numeric(lots_of_sim_rolls == 1))
[1] 0.18
> mean(as.numeric(lots_of_sim_rolls == 2))
[1] 0.14
> mean(as.numeric(lots_of_sim_rolls == 3))
[1] 0.18
> mean(as.numeric(lots_of_sim_rolls == 4))
[1] 0.14
> mean(as.numeric(lots_of_sim_rolls == 5))
[1] 0.21
```

Looking at the data, there's a 15% chance of the dice being judged as unfair. Theoretically, each number should have a 16.67% chance of being ruled unfairly. However, looking at the data, it shows that each number has a varying chance of being deemed unfair (ranging from 14% - 21%).

Probability of saying that fair, untouched dice is actually bad is 16.7%. What is the probability that (I) a true fair is judged not fair; (II) true, not fair dice is judged fair.

- 1) A true F judged NF
- 2) A true NF judged F

For the first one, if we roll 6 the answer is that it depends. As for example, each of us has made something with our dice, and now the probability to roll other than 6 depends on the shape and condition of our dice.

If we roll two dice, what the probability 6 comes up twice in a row, the answer is  $1/36$ , so truly fair dice can be judged as not fair is 3%.

2) What is the probability that true, not fair dice is judged as fair? It depends as well.

For example, if we roll the dice 60 times, what is the expected number of 6s?

In theory, you should roll 10 6's

- 10 = Expected value
- Standard deviation =  $10 \pm$  (standard error) 1.96
  - 1.96 is the answer to 5%

If fair, each number will appear 16.67%; anything that comes up more frequently is unfair.

Level of confidence- none

Realistically, there's no such thing as "fair" dice. Firstly, they are never exactly the same and secondly, dice are unpredictable. Statistically, if it's within 16.67%

The chance of it being ruled unfairly is any number rolled more than once in 6 rolls