FluidSF: A Python package for calculating turbulent flow statistics

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Summary

Fluid systems are everywhere, from small-scale engineering problems to planetary-and-larger-scale systems (atmosphere, ocean, galactic gas clouds). These systems are often turbulent, where motion is chaotic, unpredictable, and can only be characterized through statistical analyses. Spatial structure functions (SFs) are one such statistical analysis technique for turbulence, that require calculation of spatial differences in properties as a function of their separation distance. By combining and then averaging these spatial differences, various types of SFs can be constructed to measure physical properties of fluid flow, such as heat and energy transfers, energy density, intermittency etc. However, calculating SFs is often a cumbersome and computationally-intensive task tailored to the specific format of a given fluid dataset. `FluidSF` is a flexible `Python` package that can be used to diagnose and analyze various physically-informative SFs from 1-, 2-, or 3-dimensional fluid data sets.

Statement of need

`FluidSF` can construct an array of traditional and modern structure functions (SFs), and can be easily modified to calculate user-defined SFs that utilize general fluid properties, including scalars (e.g., vorticity, density) and/or vectors (e.g., velocity, magnetic field). `FluidSF` also includes several tools to process data (e.g., array shifting, binning) and diagnose useful properties (e.g., advection). The flexibility of this package enables geophysical, astrophysical, and engineering applications such as: quantifying the energy cycles within Earth's ocean (Balwada et al., 2022; J. Pearson et al., 2019), Earth's atmosphere (Lindborg, 1999), and Jupiter's atmosphere (Young & Read, 2017), diagnosing the intermittency of magnetohydrodynamic plasma turbulence (Wan et al., 2016) and the scaling laws of idealized 3D turbulence (Iyer et al., 2020), or measuring the characteristics of ocean surface temperature (Schloesser et al., 2016) or the anistropy of flow over rough beds (Coscarella et al., 2020).

Spatial SFs are constructed by averaging the correlations between spatial differences of properties. For example, given an arbitrary scalar field (ϕ), we could calculate a structure function,

$$SF_{\phi\phi}(\mathbf{r}) = \overline{\delta\phi\delta\phi} = \overline{[\phi(\mathbf{x}+\mathbf{r}) - \phi(\mathbf{x})]\,[\phi(\mathbf{x}+\mathbf{r}) - \phi(\mathbf{x})]} \tag{1}$$

where $\mathbf{x}$ denotes the position of a data point, $\delta\phi$ denotes the spatial variation of ϕ, and the overline denotes an average over all positions ($\mathbf{x}$). Structure functions depend on the separation vector ($\mathbf{r}$), and are often analyzed with an assumption of isotropic flow statistics [$SF(\mathbf{r}) \rightarrow SF(r = |\mathbf{r}|)$]. There are many types of physically-useful structure functions. The example above is a second-order scalar SF (i.e., it contains 2 δ terms of the scalar ϕ), but additional physical insight can be gained from third- & higher-order SFs (3+ δ terms), SFs

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that depend on other scalar or vector fields, and SFs that blend information from various scalar/vector fields.

`FluidSF` can utilize a variety of fluid data, including data sets with 1-, 2-, and 3-dimensional spatial data, and from domains with periodic or non-periodic boundary conditions. In addition to regular Cartesian-gridded data, the software also has some support for non-uniform latitude-longitude grids (1D or 2D). Since `FluidSF` is written in `Python`, any fluid data intialized and loaded as `NumPy` (Harris et al., 2020) arrays can be used to calculate SFs. To demonstrate the flexibility of input data, Figure 1 shows SFs calculated by `FluidSF` for a simulation of quasi-geostrophic turbulence in a periodic domain using `GeophysicalFlows.jl` (Constantinou et al., 2021), while Figure 2 shows SFs calculated from satellite observations of the ocean surface made by the NASA SWOT (Surface Waves and Ocean Topography) satellite mission (Morrow et al., 2018).

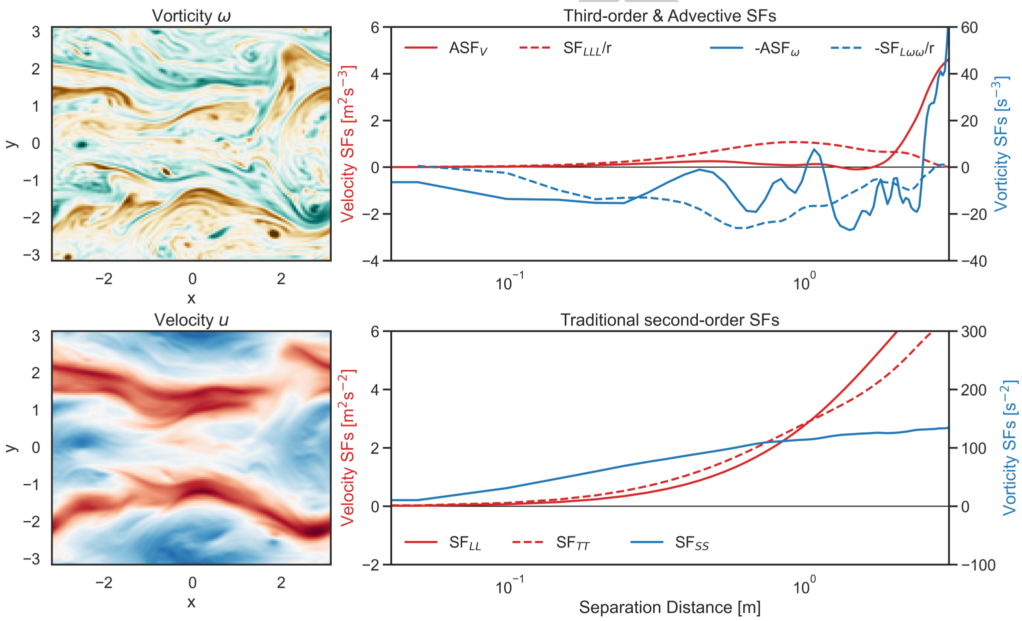

Figure 1: Structure functions from a simulation of anisotropic 2-layer quasi-geostrophic turbulence. Left two panels are snapshots of the vorticity field (top) and velocity field (bottom) used for SF calculations. Right two panels are third-order and advective SFs (top) and traditional second-order SFs (bottom). The velocity-based structure functions are indicated in red and correspond to the left y-axis whereas voriticity-based structure functions are blue and correspond to the right y-axis. Both panels have the same x-axis.

`FluidSF` can calculate a wide array of traditional and novel structure functions, including $SF_{\phi\phi}$ (Equation 1), second- and third-order SFs of longitudinal and transverse velocity, blended velocity-scalar third-order SFs, and advective SFs of velocity, vorticity and scalars (B. Pearson et al., 2021). `FluidSF` can calculate these SFs in specific directions (i.e., aligned with the Cartesian co-ordinates, shown in Figure 2), and for 2D data it can diagnose a map showing how the SFs vary with the magnitude and orientation of the separation vector $\mathbf{r}$ (Figure 3). `FluidSF` also includes tools to make the calculation and processing of SFs easier, such as array shifting, diagnosis of the advection terms for novel SFs, decomposition of longitudinal (along-$\mathbf{r}$) and transverse (across-$\mathbf{r}$) velocities, and data binning based on separation distance.

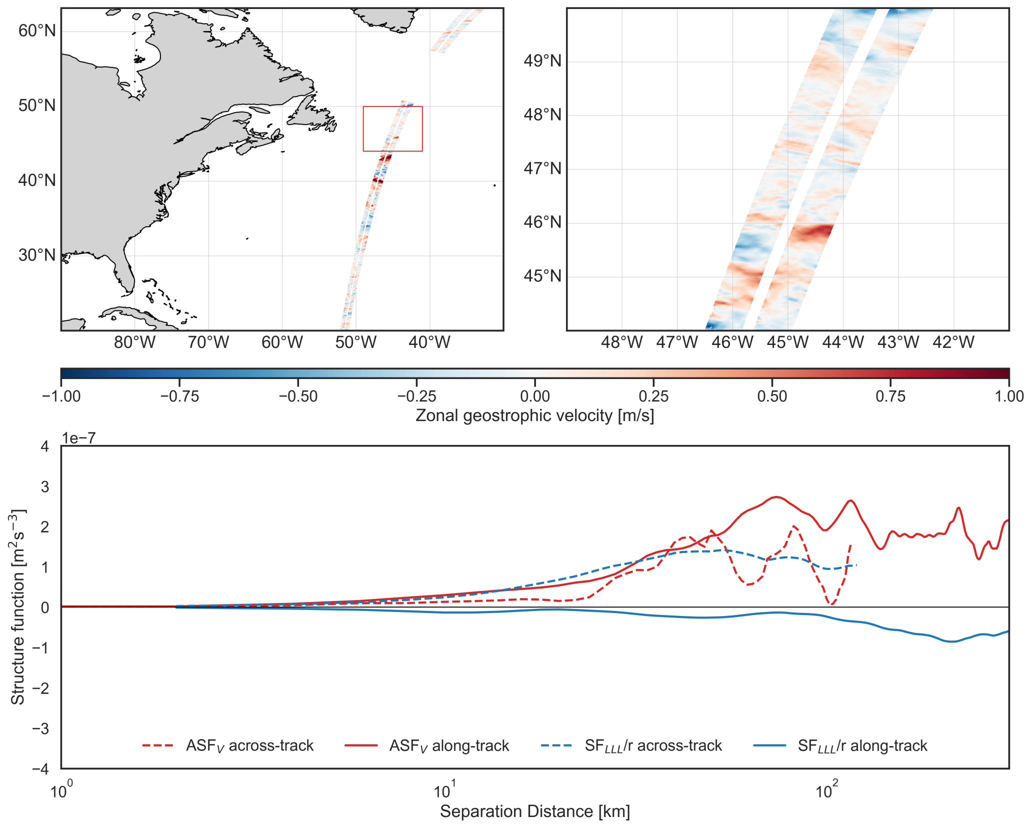

Figure 2: Structure functions calculated from satellite observations of the ocean surface. Top two panels show the zonal geostrophic velocity estimated from the satellite swath sea surface height data. In the left panel the red box indicates the region used to calculate SFs. The right panel shows the region in the red box zoomed in. The bottom panel shows the advective (red) and third-order (blue) velocity structure functions calculated across the satellite swath (dashed) and along the swath (solid).

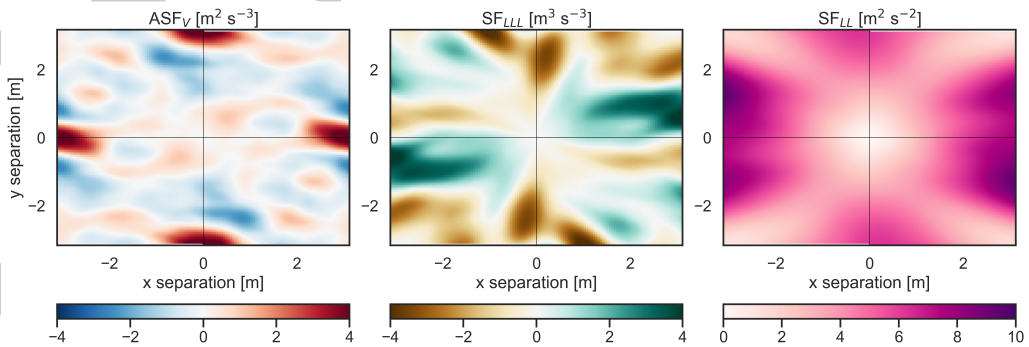

Figure 3: Maps of the 2D spatial variation of velocity structure functions. The left panel shows the advective velocity SFS, the middle panel is the third-order velocity SF, and the right panel is the second-order velocity SF. The colorbars are the value of the SF in 2D space. These SFs were calculated from the same data as Figure 1.

Related Work

`FluidSF` uniquely contributes to the field through a combination of expanded data support, the ability to diagnose a wide array of SF types (including novel and blended SFs), and tools for analyzing spatial variations in SFs. There are several open source software packages available that calculate aspects of spatial SFs. `fastSF` is a parallelized C++ code designed to compute arbitrary-order SFs of velocity or scalars (but not blended) from Cartesian grids of data (Sadhukhan et al., 2021). Fuchs et al. (2022) created an open source `MATLAB` toolkit that performs a variety of turbulence analysis, including arbitrary-order longitudinal-velocity SFs. A complimentary and alternative method to structure functions for analyzing turbulence data is coarse-graining. `FlowSieve` is a primarily C++ package that uses coarse-graining to estimate ocean and atmospheric turbulence properties from Global Climate Model data (Storer & Aluie, 2023).

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References

Balwada, D., Xie, J.-H., Marino, R., & Feraco, F. (2022). Direct observational evidence of an oceanic dual kinetic energy cascade and its seasonality. *Science Advances*, *8*(41), eabq2566.

Constantinou, N., Wagner, G., Siegelman, L., Pearson, B., & Palóczy, A. (2021). GeophysicalFlows. Jl: Solvers for geophysical fluid dynamics problems in periodic domains on CPUs GPUs. *Journal of Open Source Software*, *6*(60).

Coscarella, F., Penna, N., Servidio, S., & Gaudio, R. (2020). Turbulence anisotropy and intermittency in open-channel flows on rough beds. *Physics of Fluids*, *32*(11).

Fuchs, A., Kharche, S., Patil, A., Friedrich, J., Wächter, M., & Peinke, J. (2022). An open source package to perform basic and advanced statistical analysis of turbulence data and other complex systems. *Physics of Fluids*, *34*(10), 101801. https://doi.org/10.1063/5.0107974

Harris, C. R., Millman, K. J., Walt, S. J. van der, Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., Kerkwijk, M. H. van, Brett, M., Haldane, A., Río, J. F. del, Wiebe, M., Peterson, P., … Oliphant, T. E. (2020). Array programming with NumPy. *Nature*, *585*(7825), 357–362. https://doi.org/10.1038/s41586-020-2649-2

Iyer, K. P., Sreenivasan, K. R., & Yeung, P. (2020). Scaling exponents saturate in three-dimensional isotropic turbulence. *Physical Review Fluids*, *5*(5), 054605.

Lindborg, E. (1999). Can the atmospheric kinetic energy spectrum be explained by two-dimensional turbulence? *Journal of Fluid Mechanics*, *388*, 259–288.

Morrow, R., Blurmstein, D., & Dibarboure, G. (2018). Fine-scale altimetry and the future SWOT mission. *New Frontiers in Operational Oceanography*, 191–226.

Pearson, B., Pearson, J., & Fox-Kemper, B. (2021). Advective structure functions in anisotropic two-dimensional turbulence. *Journal of Fluid Mechanics*, *916*, 49. https://doi.org/10.1017/jfm.2021.247

Pearson, J., Fox-Kemper, B., Barkan, R., Choi, J., Bracco, A., & McWilliams, J. C. (2019). Impacts of convergence on structure functions from surface drifters in the gulf of mexico. *Journal of Physical Oceanography*, *49*(3), 675–690.

Sadhukhan, S., Bhattacharya, S., & Verma, M. K. (2021). fastSF: A parallel code for computing the structure functions of turbulence. *Journal of Open Source Software*, *6*(57),

2185. https://doi.org/10.21105/joss.02185

Schloesser, F., Cornillon, P., Donohue, K., Boussidi, B., & Iskin, E. (2016). Evaluation of thermosalinograph and VIIRS data for the characterization of near-surface temperature fields. *Journal of Atmospheric and Oceanic Technology*, *33*(9), 1843–1858.

Storer, B. A., & Aluie, H. (2023). FlowSieve: A Coarse-Graining Utility for Geophysical Flows on the Sphere. *Journal of Open Source Software*, *8*(84), 4277. https://doi.org/10.21105/joss.04277

Wan, M., Matthaeus, W., Roytershteyn, V., Parashar, T., Wu, P., & Karimabadi, H. (2016). Intermittency, coherent structures and dissipation in plasma turbulence. *Physics of Plasmas*, *23*(4).

Young, R. M., & Read, P. L. (2017). Forward and inverse kinetic energy cascades in jupiter's turbulent weather layer. *Nature Physics*, *13*(11), 1135–1140.