

FluidSF: A Python package for calculating turbulent flow statistics

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Summary

Fluid systems are everywhere, from small-scale engineering problems to planetary-and-larger-scale systems (atmosphere, ocean, galactic gas clouds). These systems are often turbulent, where motion is chaotic, unpredictable, and can only be characterized through statistical analyses. Spatial structure functions (SFs) are one such statistical analysis technique for turbulence, that require calculation of spatial differences in properties as a function of their separation distance. By combining and then averaging these spatial differences, various types of SF can be constructed to measure physical properties of fluid flow, such as heat and energy transfers, energy density, intermittency etc. However, calculating SFs is often a cumbersome and computationally-intensive task tailored to the specific format of a given fluid dataset. FluidSF is a flexible Python package that can be used to diagnose and analyze various physically-informative SFs from 1-, 2-, or 3-dimensional fluid data sets.

Statement of need

FluidSF can construct an array of traditional and modern structure functions (SFs), and can be easily modified to calculate user-defined SFs that utilize general fluid properties, including scalars (e.g., vorticity, density) and/or vectors (e.g., velocity, magnetic field). FluidSF also includes several tools to process data (e.g., array shifting, binning) and diagnose useful properties (e.g., advection). The flexibility of this package enables geophysical, astrophysical, and engineering applications such as: quantifying the energy cycles within Earth's ocean (Balwada et al., 2022; J. Pearson et al., 2019), Earth's atmosphere (Lindborg, 1999), and Jupiter's atmosphere (Young & Read, 2017), diagnosing the intermittency of magnetohydrodynamic plasma turbulence (Wan et al., 2016) and the scaling laws of idealized 3D turbulence (Iyer et al., 2020), or measuring the characteristics of ocean surface temperature (Schloesser et al., 2016) or the anisotropy of flow over rough beds (Coscarella et al., 2020).

Spatial SFs are constructed by averaging the correlations between spatial differences of properties. For example, given an arbitrary scalar field (ϕ), we could calculate a structure function,

$$SF_{\phi\phi}(\mathbf{r}) = \overline{\delta\phi\delta\phi} = \overline{[\phi(\mathbf{x} + \mathbf{r}) - \phi(\mathbf{x})][\phi(\mathbf{x} + \mathbf{r}) - \phi(\mathbf{x})]} \quad (1)$$

where \mathbf{x} denotes the position of a data point, $\delta\phi$ denotes the spatial variation of ϕ , and the overline denotes an average over all positions (\mathbf{x}). Structure functions depend on the separation vector (\mathbf{r}), and are often analyzed with an assumption of isotropic flow statistics [$SF(\mathbf{r}) \rightarrow SF(r = |\mathbf{r}|)$]. There are many types of physically-useful structure functions. The example above is a second-order scalar SF (i.e., it contains 2 δ terms of the scalar ϕ), but additional physical insight can be gained from third- & higher-order SFs (3+ δ terms), SFs

that depend on other scalar or vector fields, and SFs that blend information from various scalar/vector fields.

FluidSF can utilize a variety of fluid data, including data sets with 1-, 2-, and 3-dimensional spatial data, and from domains with periodic or non-periodic boundary conditions. In addition to regular Cartesian-gridded data, the software also has some support for non-uniform latitude-longitude grids (1D or 2D). Since FluidSF is written in Python, any fluid data initialized and loaded as NumPy (Harris et al., 2020) arrays can be used to calculate SFs. To demonstrate the flexibility of input data, Figure 1 shows SFs calculated by FluidSF for a simulation of quasi-geostrophic turbulence in a periodic domain using GeophysicalFlows.jl (Constantinou et al., 2021), while Figure 2 shows SFs calculated from satellite observations of the ocean surface made by the NASA SWOT (Surface Waves and Ocean Topography) satellite mission (Morrow et al., 2018).

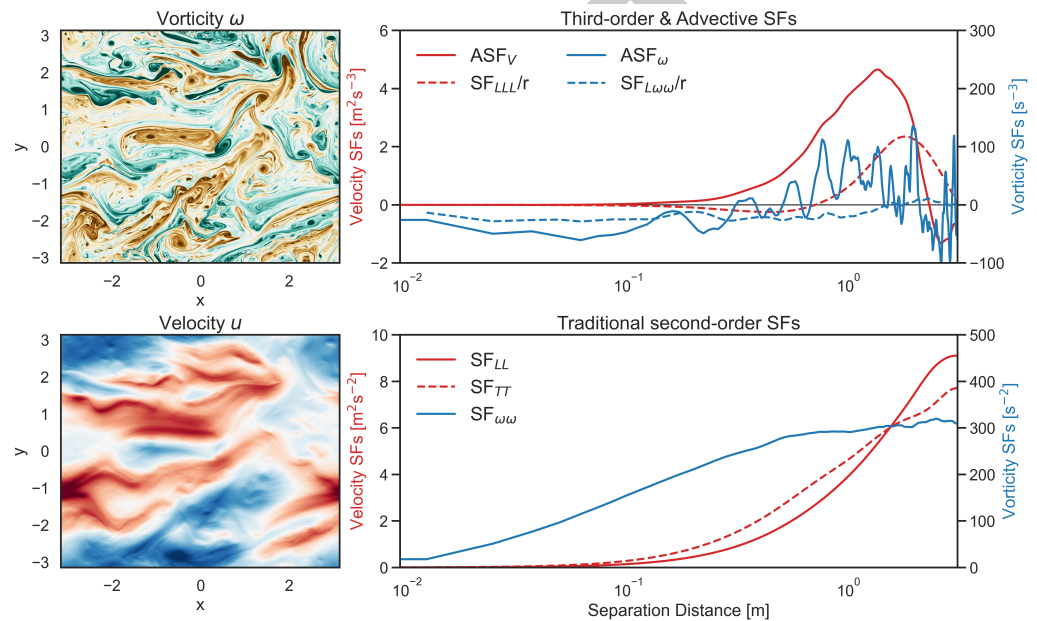


Figure 1: Various structure functions (SFs) calculated from a simulated 2D turbulent flow, visualized through snapshots of the vorticity field (top left) and velocity field (bottom left). The right panels show various SFs based on velocity (red lines) and vorticity (blue lines), including third-order and advective SFs (top right) and traditional second-order SFs (bottom right). The results are from the top layer snapshot of an anisotropic 2-layer quasi-geostrophic simulation conducted with GeophysicalFlows.jl.

FluidSF can calculate a wide array of traditional and novel structure functions, including $SF_{\phi\phi}$ (Equation 1), second- and third-order SFs of longitudinal and transverse velocity, blended velocity-scalar third-order SFs, and advective SFs of velocity, vorticity and scalars (B. Pearson et al., 2021). FluidSF can calculate these SFs in specific directions (i.e., aligned with the Cartesian co-ordinates, shown in Figure 2), and for 2D data it can diagnose maps showing how SFs vary with the magnitude and orientation of the separation vector \mathbf{r} (Figure 3). FluidSF also includes tools to make the calculation and processing of SFs easier, such as array shifting, diagnosis of the advection terms for novel SFs, decomposition of longitudinal (along- \mathbf{r}) and transverse (across- \mathbf{r}) velocities, and data binning based on separation distance.

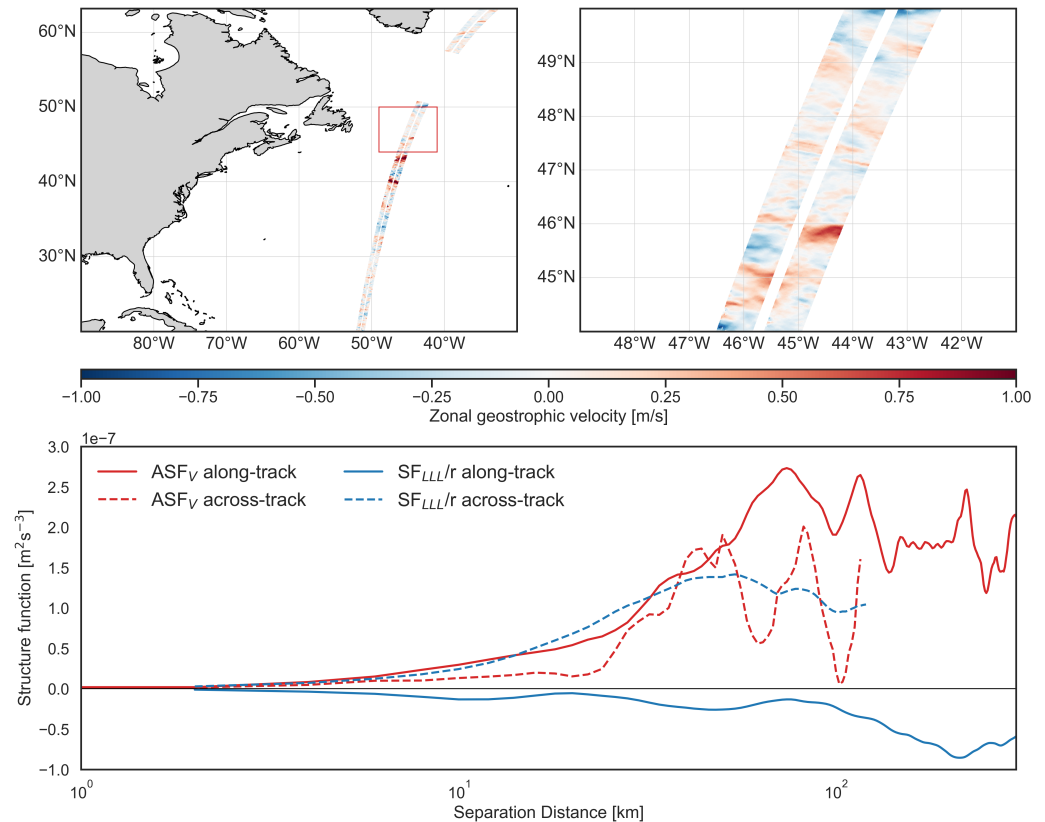


Figure 2: Velocity-based SFs calculated from satellite observations of the ocean surface in the North Atlantic. Maps of the inferred surface velocity from a satellite swath are shown in the top left. The region of data used for SF calculations is indicated by the red box and magnified on the top right. The bottom panel shows the advective (red) and third-order (blue) velocity structure functions calculated with separation vectors across the satellite swath (dashed) and along the swath (solid). Note the velocity fields are estimated from satellite sea surface height measurements assuming geostrophic balance.

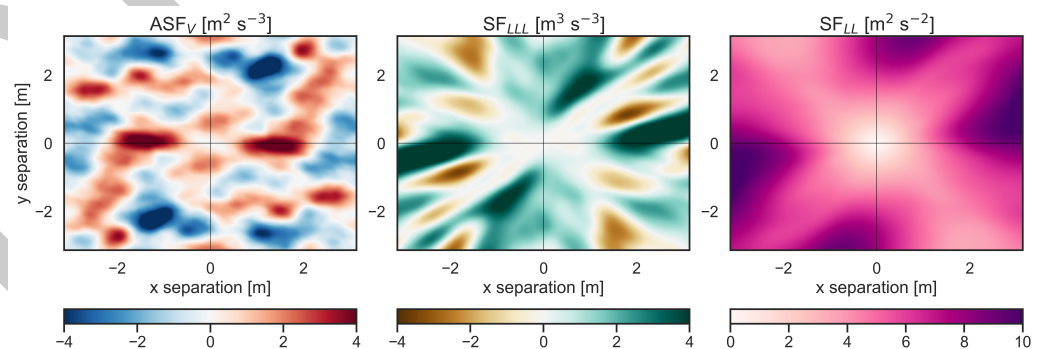


Figure 3: Maps showing the 2D spatial variation of various velocity structure functions. The left panel shows the advective velocity SF, the middle panel is the third-order velocity SF, and the right panel is the second-order velocity SF. These SFs were calculated from the same data as Figure 1.

59 Related Work

60 FluidSF uniquely contributes to the field through a combination of expanded data support, the
61 ability to diagnose a wide array of SF types (including novel and blended SFs), and tools for
62 analyzing spatial variations in SFs. FluidSF was used to develop new methods for estimating

inter-scale geophysical energy fluxes (B. Pearson et al., 2024). There are several open source software packages available that calculate aspects of spatial SFs. fastSF is a parallelized C++ code designed to compute arbitrary-order SFs of velocity or scalars (but not blended) from Cartesian grids of data (Sadhukhan et al., 2021). Fuchs et al. (2022) created an open source MATLAB toolkit that performs a variety of turbulence analysis, including arbitrary-order longitudinal-velocity SFs. A complimentary and alternative method to structure functions for analyzing turbulence data is coarse-graining. FlowSieve is a primarily C++ package that uses coarse-graining to estimate ocean and atmospheric turbulence properties from Global Climate Model data (Storer & Aluie, 2023).

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