





# FluidSF: A Python package for calculating turbulent flow statistics

Cassidy M. Wagner<sup>1</sup>, Ara Lee<sup>1</sup>, and Brodie Pearson<sup>1</sup>

<sup>1</sup> College of Earth, Ocean, and Atmospheric Sciences, Oregon State University, 1500 SW Jefferson Way, Corvallis, OR 97331  Corresponding author

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

## Software

- [Review](#) 
- [Repository](#) 
- [Archive](#) 

Editor: [Open Journals](#) 

## Reviewers:

- [@openjournals](#)

Submitted: 01 January 1970

Published: unpublished

## License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).

## Summary

Fluid systems are everywhere, from small-scale engineering problems to planetary- and larger-scale systems (atmosphere, ocean, galactic gas clouds). These systems are often turbulent, where motion is chaotic, unpredictable, and can only be characterized through statistical analyses. Structure functions (SFs) are one such statistical analysis technique for turbulence that requires the calculation of spatial differences in properties as a function of their separation distance. By combining and then averaging these spatial differences, various types of SF can be constructed to measure physical properties of fluid flow, such as heat and energy transfers, energy density, intermittency, etc. However, calculating SFs is often a cumbersome and computationally-intensive task tailored to the specific format of a given fluid dataset. FluidSF is a flexible Python package that can be used to diagnose and analyze various physically-informative SFs from one-, two-, or three-dimensional fluid data sets.

## Statement of need

FluidSF can construct an array of traditional and modern structure functions, and can be easily modified to calculate user-defined SFs that utilize general fluid properties, including scalars (e.g., vorticity, density) and vectors (e.g., velocity, magnetic field). FluidSF also includes several tools to process data (e.g., array shifting, binning) and diagnose useful properties (e.g., advection) for SF analysis. The flexibility of this package enables geophysical, astrophysical, and engineering applications such as: quantifying the energy cycles within Earth's ocean ([Balwada et al., 2022](#); [J. Pearson et al., 2019](#)), Earth's atmosphere ([Lindborg, 1999](#)), and Jupiter's atmosphere ([Young & Read, 2017](#)), diagnosing the intermittency of magnetohydrodynamic plasma turbulence ([Wan et al., 2016](#)) and the scaling laws of idealized 3D turbulence ([Iyer et al., 2020](#)), or measuring the characteristics of ocean surface temperature ([Schloesser et al., 2016](#)) or the anisotropy of flow over rough beds ([Coscarella et al., 2020](#)).

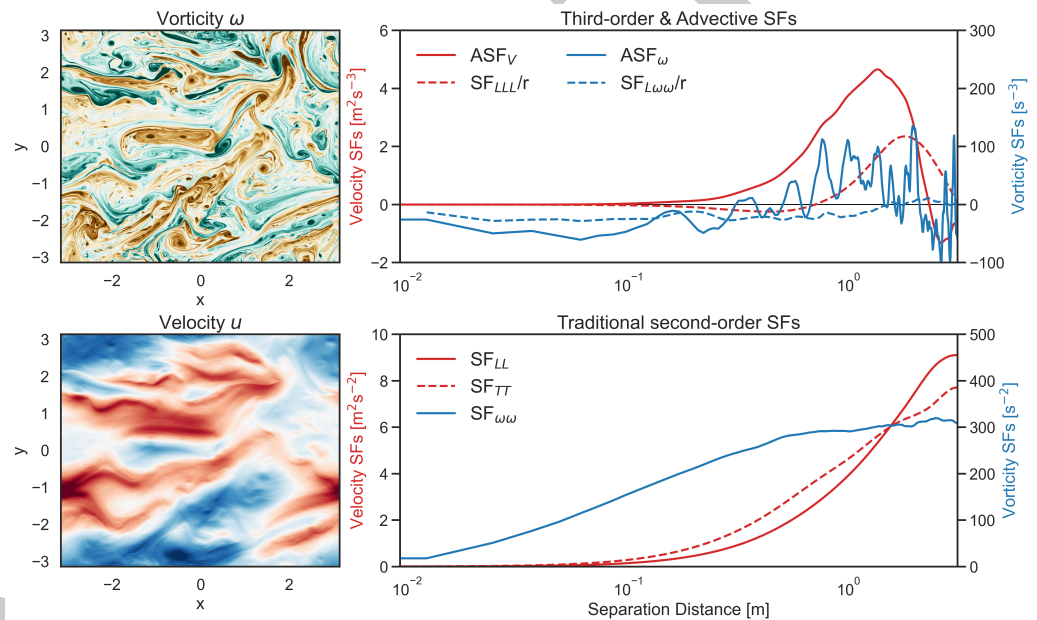
Structure functions are constructed by averaging the correlations between spatial differences of properties. For example, given an arbitrary scalar field ( $\phi$ ), we could calculate SFs such as this,

$$SF_{\phi\phi}(\mathbf{r}) = \overline{\delta\phi\delta\phi} = \overline{[\phi(\mathbf{x} + \mathbf{r}) - \phi(\mathbf{x})][\phi(\mathbf{x} + \mathbf{r}) - \phi(\mathbf{x})]} \quad (1)$$

where  $\mathbf{x}$  denotes the position of a data point,  $\delta\phi$  denotes the spatial variation of  $\phi$ , and the overline denotes an average over all positions ( $\mathbf{x}$ ). Structure functions depend on the separation vector ( $\mathbf{r}$ ), and are often analyzed with an assumption of isotropic flow statistics [ $SF(\mathbf{r}) \rightarrow SF(r = |\mathbf{r}|)$ ]. There are many types of physically useful structure functions. The example above is a second-order scalar SF (i.e., it contains two  $\delta$  terms of the scalar  $\phi$ ), but additional physical insight can be gained from third-order and higher-order scalar SFs

(three or more  $\delta$  terms), SFs that depend on vector fields such as velocity, and SFs that blend information from multiple fields.

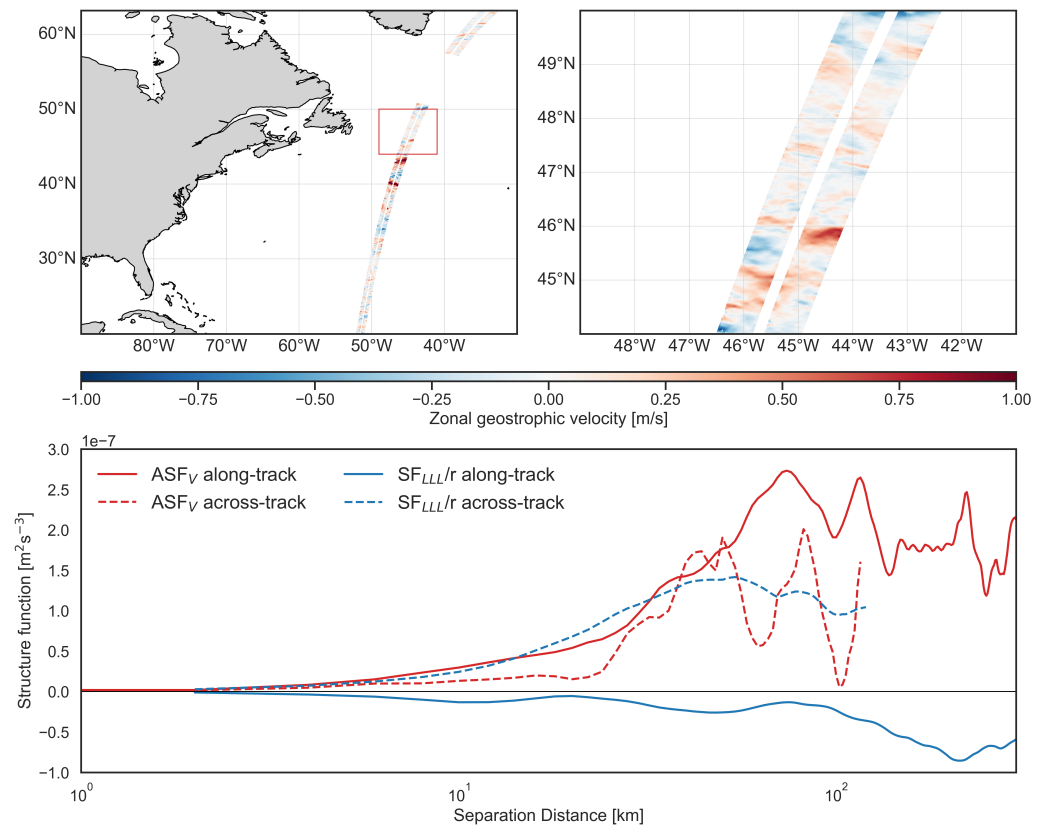
FluidSF can utilize a variety of fluid data, including datasets with one-, two-, and three-dimensional spatial data, as well as data from domains with periodic or non-periodic boundary conditions. In addition to regular Cartesian-gridded data, the software also has some support for non-uniform latitude-longitude grids (1D or 2D) but not for general curvilinear coordinates. When computing SFs that blend information from multiple fields, FluidSF assumes all variables are co-located, so care must be taken with staggered grids. Since FluidSF is written in Python, any fluid data initialized and loaded as NumPy (Harris et al., 2020) arrays can be used to calculate SFs. To demonstrate the flexibility of input data, Figure 1 shows several types of SF calculated using FluidSF for a simulation of quasi-geostrophic turbulence in a periodic domain using GeophysicalFlows.jl (Constantinou et al., 2021), while Figure 2 shows SFs calculated from satellite observations of the ocean surface made by the NASA SWOT (Surface Waves and Ocean Topography) satellite mission (Morrow et al., 2018).



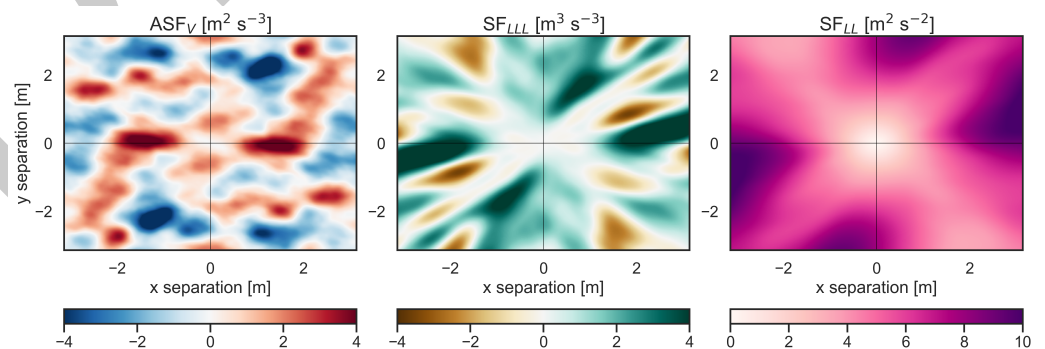
**Figure 1:** Various structure functions (SFs) calculated from a simulated 2D turbulent flow, visualized through snapshots of the vorticity field (top left) and velocity field (bottom left). The right panels show various SFs based on velocity (red lines) and vorticity (blue lines), including third-order and advective SFs (top right) and traditional second-order SFs (bottom right). The results are from the top layer snapshot of an anisotropic two-layer quasi-geostrophic simulation conducted with GeophysicalFlows.jl.

As demonstrated in Figure 1 and Figure 2, FluidSF can calculate a wide array of traditional structure functions, including  $SF_{\phi\phi}$  (Equation 1; where the scalar field in this case is vorticity  $\omega$ ), second- and third-order SFs of longitudinal velocity ( $SF_{LL} = \overline{(\delta u_L)^2}$  and  $SF_{LLL} = \overline{(\delta u_L)^3}$ ; where  $u_L = \mathbf{u} \cdot \hat{\mathbf{r}}$ ) and transverse velocity ( $SF_{TT}$  and  $SF_{TTT}$ ), and blended velocity-scalar third-order SFs ( $SF_{L\omega\omega} = \overline{\delta u_L \delta \omega \delta \omega}$ ), in addition to novel advective SFs of velocity ( $ASF_V$ ), vorticity ( $ASF_\omega$ ) and scalars (B. Pearson et al., 2021, 2025). Advective SFs require fields of the local advection, and FluidSF has a built-in function to compute these advection terms. FluidSF can calculate SFs in specific separation directions (i.e., aligned with the Cartesian co-ordinates, shown in Figure 2), and for 2D data it can diagnose maps showing how SFs vary with the magnitude and orientation of the separation vector  $\mathbf{r}$  (Figure 3). FluidSF also includes tools to make the calculation and processing of SFs easier, such as array shifting, diagnosis of the advection terms for novel SFs, decomposition of velocity into longitudinal

64 (along- $r$ ;  $u_L$ ) and transverse (across- $r$ ;  $u_T$ ) components, and data binning based on separation  
65 distance.



**Figure 2:** Velocity-based SFs calculated from satellite observations of the ocean surface in the North Atlantic. Maps of the inferred surface velocity from a satellite swath are shown in the top left. The region of data used for SF calculations is indicated by the red box and magnified on the top right. The bottom panel shows the advective (red) and third-order (blue) velocity structure functions calculated with separation vectors across the satellite swath (dashed) and along the swath (solid). Note that the velocity fields are estimated from satellite sea surface height measurements assuming geostrophic balance.



**Figure 3:** Maps showing the 2D spatial variation of various velocity structure functions. The left panel shows the advective velocity SF, the middle panel shows the third-order velocity SF, and the right panel shows the second-order velocity SF. These SFs were calculated from the same data as Figure 1.

## Related Work

FluidSF uniquely contributes to the field through a combination of expanded data support, the ability to diagnose a wide array of structure functions (including advective and blended SFs), and tools for analyzing spatial variations in SFs. FluidSF was used to develop new methods for estimating inter-scale geophysical energy fluxes (B. Pearson et al., 2025). There are several open source software packages available that calculate aspects of spatial SFs. fastSF is a parallelized C++ code designed to compute arbitrary-order SFs of velocity or scalars (but not blended) from Cartesian grids of data (Sadhukhan et al., 2021). Fuchs et al. (2022) created an open-source MATLAB toolkit that performs a variety of turbulence analyses, including arbitrary-order longitudinal-velocity SFs. A complementary and alternative method to structure functions for analyzing turbulence data is coarse-graining. FlowSieve is a primarily C++ package that uses coarse-graining to estimate ocean and atmospheric turbulence properties from Global Climate Model data (Storer & Aluie, 2023).

## Acknowledgements

The development of FluidSF was primarily supported by the National Science Foundation (NSF) under Grant No. 2023721 (Wagner & Pearson) with additional support from the NSF under Grant No. 2146910 (Lee).

## References

- Balwada, D., Xie, J.-H., Marino, R., & Feraco, F. (2022). Direct observational evidence of an oceanic dual kinetic energy cascade and its seasonality. *Science Advances*, 8(41), eabq2566. <https://doi.org/10.1126/sciadv.abq2566>
- Constantinou, N., Wagner, G., Siegelman, L., Pearson, B., & Palóczy, A. (2021). GeophysicalFlows. JI: Solvers for geophysical fluid dynamics problems in periodic domains on CPUs GPUs. *Journal of Open Source Software*, 6(60). <https://doi.org/10.21105/joss.03053>
- Coscarella, F., Penna, N., Servidio, S., & Gaudio, R. (2020). Turbulence anisotropy and intermittency in open-channel flows on rough beds. *Physics of Fluids*, 32(11). <https://doi.org/10.1063/5.0028119>
- Fuchs, A., Kharche, S., Patil, A., Friedrich, J., Wächter, M., & Peinke, J. (2022). An open source package to perform basic and advanced statistical analysis of turbulence data and other complex systems. *Physics of Fluids*, 34(10), 101801. <https://doi.org/10.1063/5.0107974>
- Harris, C. R., Millman, K. J., Walt, S. J. van der, Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., Kerkwijk, M. H. van, Brett, M., Haldane, A., Río, J. F. del, Wiebe, M., Peterson, P., ... Oliphant, T. E. (2020). Array programming with NumPy. *Nature*, 585(7825), 357–362. <https://doi.org/10.1038/s41586-020-2649-2>
- Iyer, K. P., Sreenivasan, K. R., & Yeung, P. (2020). Scaling exponents saturate in three-dimensional isotropic turbulence. *Physical Review Fluids*, 5(5), 054605. <https://doi.org/10.1103/physrevfluids.5.054605>
- Lindborg, E. (1999). Can the atmospheric kinetic energy spectrum be explained by two-dimensional turbulence? *Journal of Fluid Mechanics*, 388, 259–288. <https://doi.org/10.1017/s0022112099004851>
- Morrow, R., Blurmstein, D., & Dibarboure, G. (2018). Fine-scale altimetry and the future SWOT mission. *New Frontiers in Operational Oceanography*, 191–226. <https://doi.org/10.17125/gov2018.ch08>

- 111 Pearson, B., Pearson, J., & Fox-Kemper, B. (2021). Advective structure functions in anisotropic  
112 two-dimensional turbulence. *Journal of Fluid Mechanics*, 916, 49. <https://doi.org/10.1017/jfm.2021.247>  
113
- 114 Pearson, B., Wagner, C., Fox-Kemper, B., & Samelson, R. (2025). Estimating spectral fluxes in  
115 quasi-two-dimensional flows with advective structure functions and bessel functions. *Journal*  
116 *of Physical Oceanography*, 55(9), 1335–1352. <https://doi.org/10.1175/JPO-D-24-0211.1>
- 117 Pearson, J., Fox-Kemper, B., Barkan, R., Choi, J., Bracco, A., & McWilliams, J. C. (2019). Im-  
118 pacts of convergence on structure functions from surface drifters in the gulf of mexico. *Jour-*  
119 *nal of Physical Oceanography*, 49(3), 675–690. <https://doi.org/10.1175/jpo-d-18-0029.1>
- 120 Sadhukhan, S., Bhattacharya, S., & Verma, M. K. (2021). fastSF: A parallel code for  
121 computing the structure functions of turbulence. *Journal of Open Source Software*, 6(57),  
122 2185. <https://doi.org/10.21105/joss.02185>
- 123 Schloesser, F., Cornillon, P., Donohue, K., Boussidi, B., & Iskin, E. (2016). Evaluation of  
124 thermosalinograph and VIIRS data for the characterization of near-surface temperature  
125 fields. *Journal of Atmospheric and Oceanic Technology*, 33(9), 1843–1858. <https://doi.org/10.1175/jtech-d-15-0180.1>  
126
- 127 Storer, B. A., & Aluie, H. (2023). FlowSieve: A Coarse-Graining Utility for Geophysical Flows  
128 on the Sphere. *Journal of Open Source Software*, 8(84), 4277. <https://doi.org/10.21105/joss.04277>  
129
- 130 Wan, M., Matthaeus, W., Roytershteyn, V., Parashar, T., Wu, P., & Karimabadi, H. (2016).  
131 Intermittency, coherent structures and dissipation in plasma turbulence. *Physics of Plasmas*,  
132 23(4). <https://doi.org/10.1063/1.4945631>
- 133 Young, R. M., & Read, P. L. (2017). Forward and inverse kinetic energy cascades in jupiter's  
134 turbulent weather layer. *Nature Physics*, 13(11), 1135–1140. <https://doi.org/10.1038/nphys4227>  
135