P3/ Filtering

a)

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} .* \begin{bmatrix}1&0&-1\\2&0&-2\\1&0&-1\end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix}1&0&-1\\2&0&-2\\1&0&-1\end{bmatrix}$$

b) Assume a separable filter is $M \in R^{m \times m}$.

$$M = a b^T \quad \text{where } a \in R^m,\ b \in R^m$$

Before separation, we use M to convolve, $(n-m+1)^2$ arithmetic operations.

After separation, we use a, b^T to convolve, $(n-m+1) \times n \times 2 = 2n(n-m+1)$.

For example,

$$M = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} * \frac{1}{3}\begin{bmatrix}1&1&1\end{bmatrix}$$

For an $n \times n$ image, without padding

$(n-2)^2$ operations for using M before separation

$2(n-2)n$ operations after separation