problem 3

We suppose L_1 & L_2 are parallel lines in a 3-D Space, which are denoted as:

$$L_1: \vec{p} + t\vec{u}, \quad L_2: \vec{q} + t\vec{v}, \quad t \in \mathbb{R}$$

where $\vec{v} = s\vec{u}$, for $s \in \mathbb{R}$

$$\vec{p}, \vec{q}, \vec{u}, \vec{v} \in \mathbb{R}^3$$

m_1, m_2 are corresponding lines for L_1 & L_2 in an affine camera, which are represented as $\begin{bmatrix} u_1 \\ v_1 \end{bmatrix}, \begin{bmatrix} u_2 \\ v_2 \end{bmatrix}$

For $P = [x_0, y_0, z_0]^T$, there exists $A \in \mathbb{R}^{2\times 3}$.

$$A = \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 \\ 0 & f/z_0 & -fy_0/z_0^2 \end{bmatrix}$$

f is the length between plane and focal center

Such that

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = A(\vec{p} + t\vec{u}) + \vec{b}, \quad \text{where } \vec{b} \in \mathbb{R}^3$$

$$= A\vec{p} + \vec{b_1} + At\vec{u} = A\vec{p} + \vec{b_1} + tA\vec{u}$$

$$= \vec{p_1} + t(A\vec{u}) = \vec{p_1} + t\vec{u_1}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = A(\vec{q} + t\vec{v}) + \vec{b}$$

$$= A\vec{q} + \vec{b} + tA\vec{v} = \vec{q_1} + tA\vec{v}$$

$$= \vec{q_1} + tAs\vec{u} = \vec{q_1} + t(sA\vec{u})$$

$$= \vec{q_1} + t(s\vec{u_1})$$

Thus, m_1 & m_2 are parallel in an affine plane.