Cosmology

- Carroll & Ostlie Chapter 29
- Sparke & Gallagher Chapter 8.2
- **Ryden Chapter 2, 3, 4**

The developing theory of the origin, evolution and fate of the universe.

Cosmology describes how matter is distributed at early times, and defines properties for those various components, which allows us to develop models for how the universe evolves.

Cosmological Principle: On average, over large scales, the universe is homogeneous and isotropic.

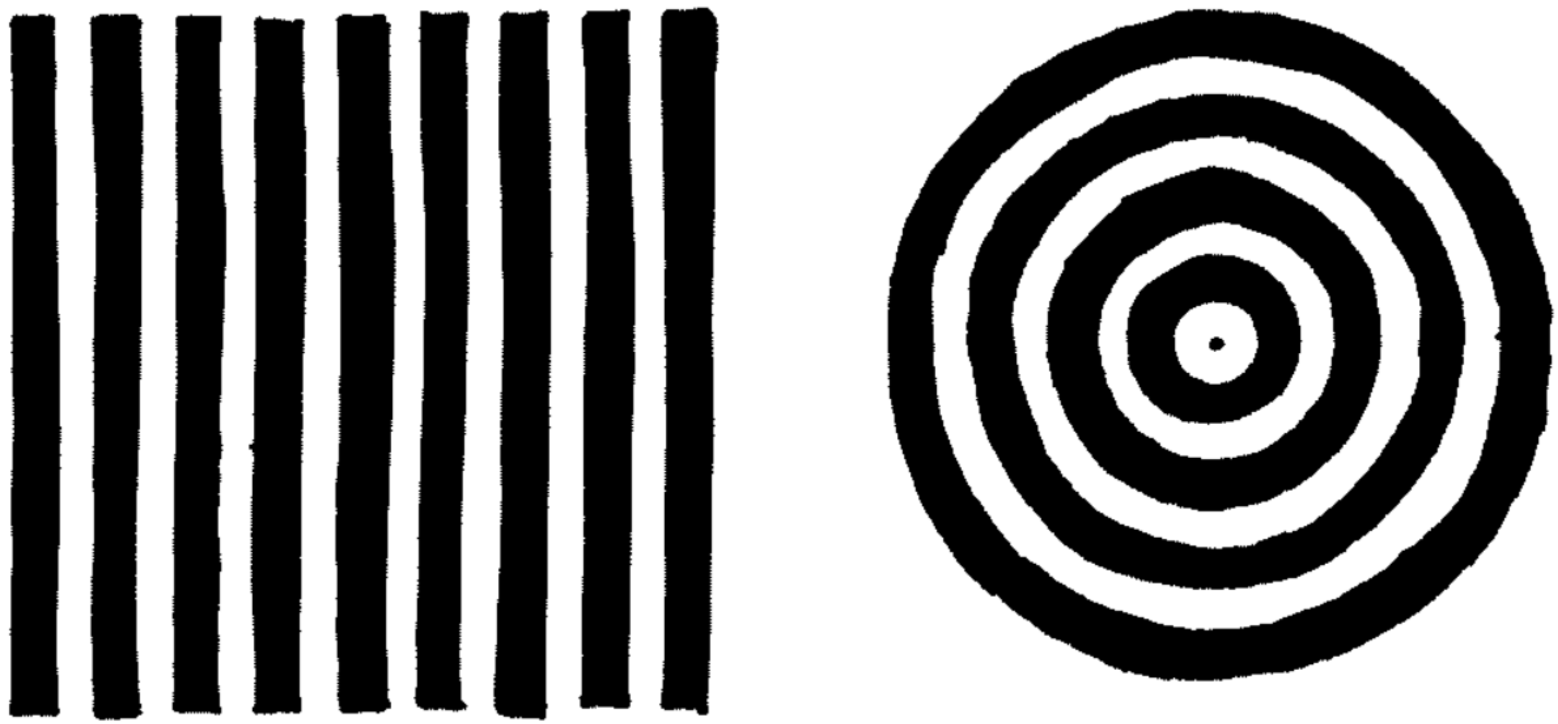
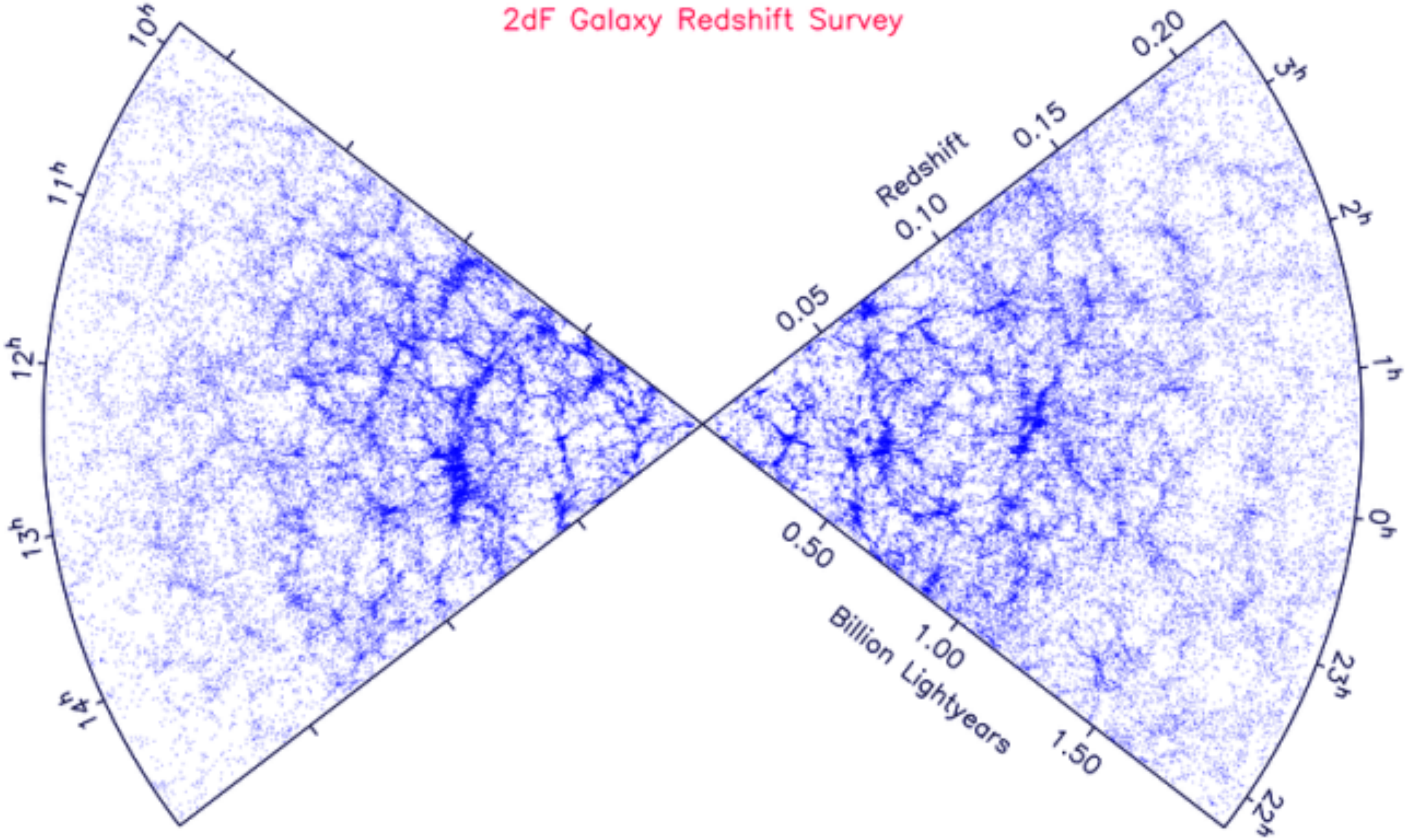


Figure 2.3: (a) A pattern which is anisotropic, but which is homogeneous on scales larger than the stripe width. (b) A pattern which is isotropic about the origin, but which is inhomogeneous.

Ryden

Cosmological Principle: We are not located in a special location in the universe. Consequence is that either the universe is static, or it has purely radial motions.



We know the universe isn't static:

Vesto Slipher first discovered the Doppler shift in the spectra of galaxies, Hubble used this to prove the universe is expanding (1929)

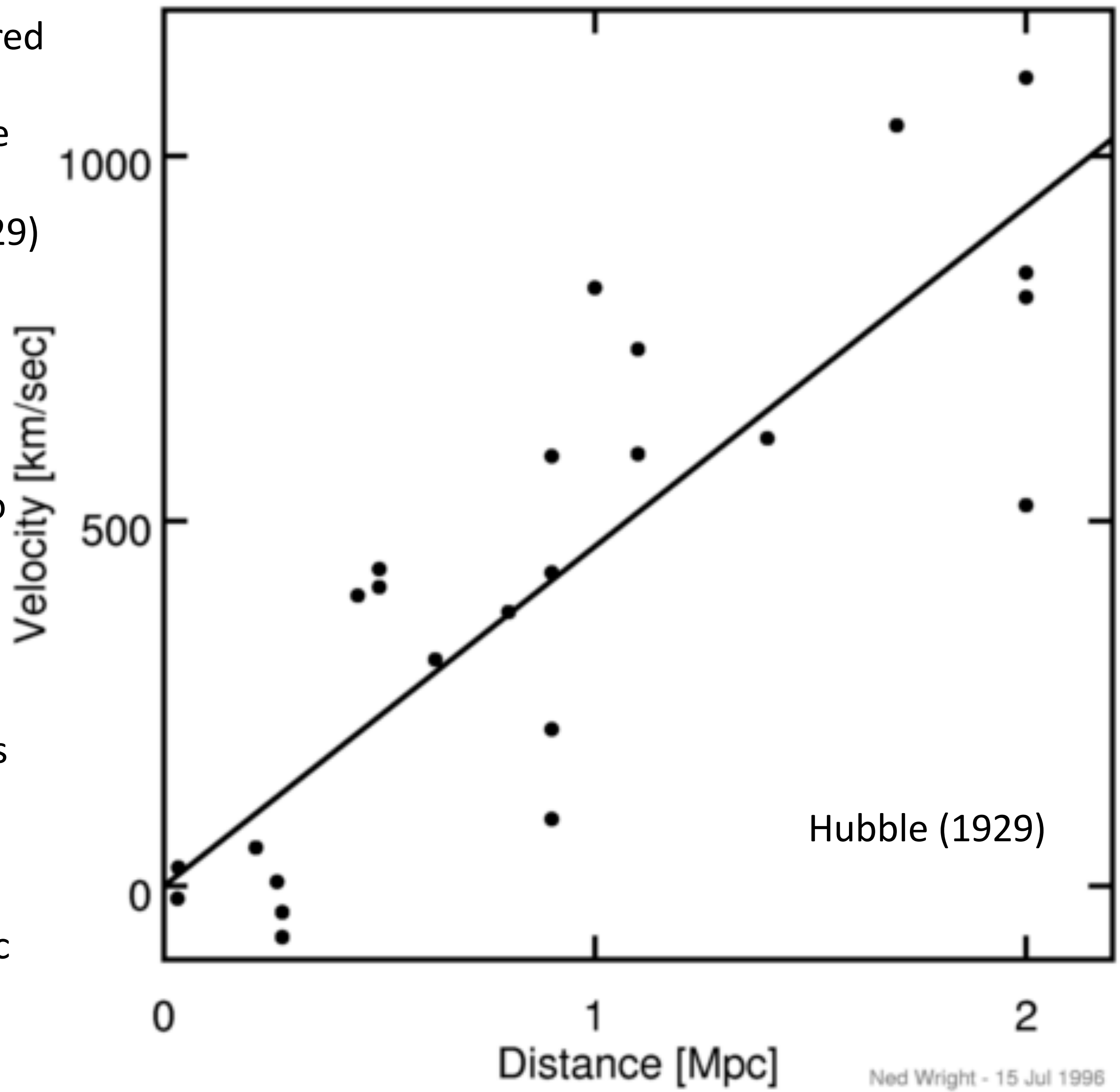
Hubble's Law:

the *radial velocities* of galaxies are proportional to their distance.

$$\mathbf{V = H_0\, r}$$

The Hubble Constant (H_0) is defined as the slope of the fitted line.

Here, H_0 = 464 km/sec/Mpc



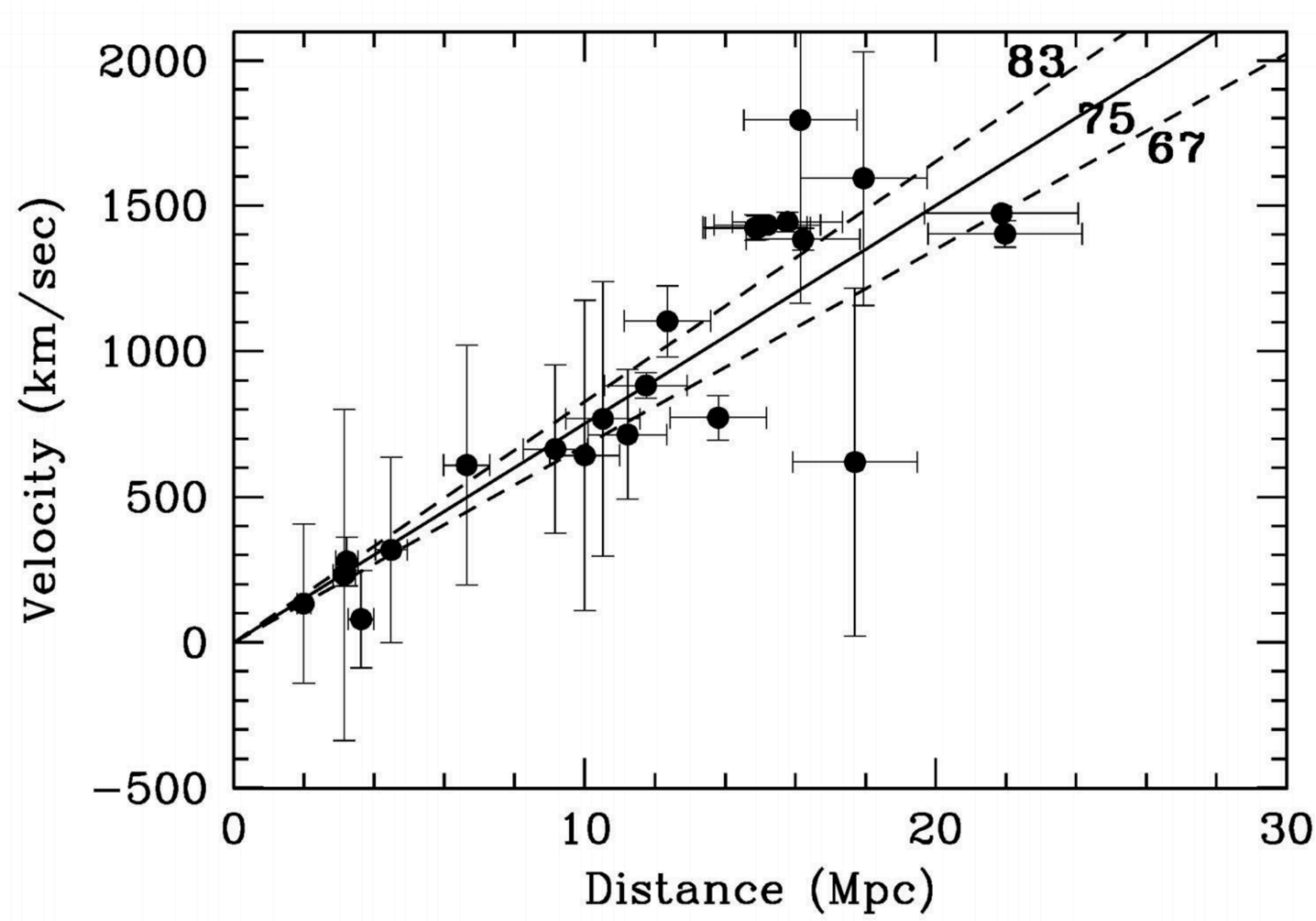


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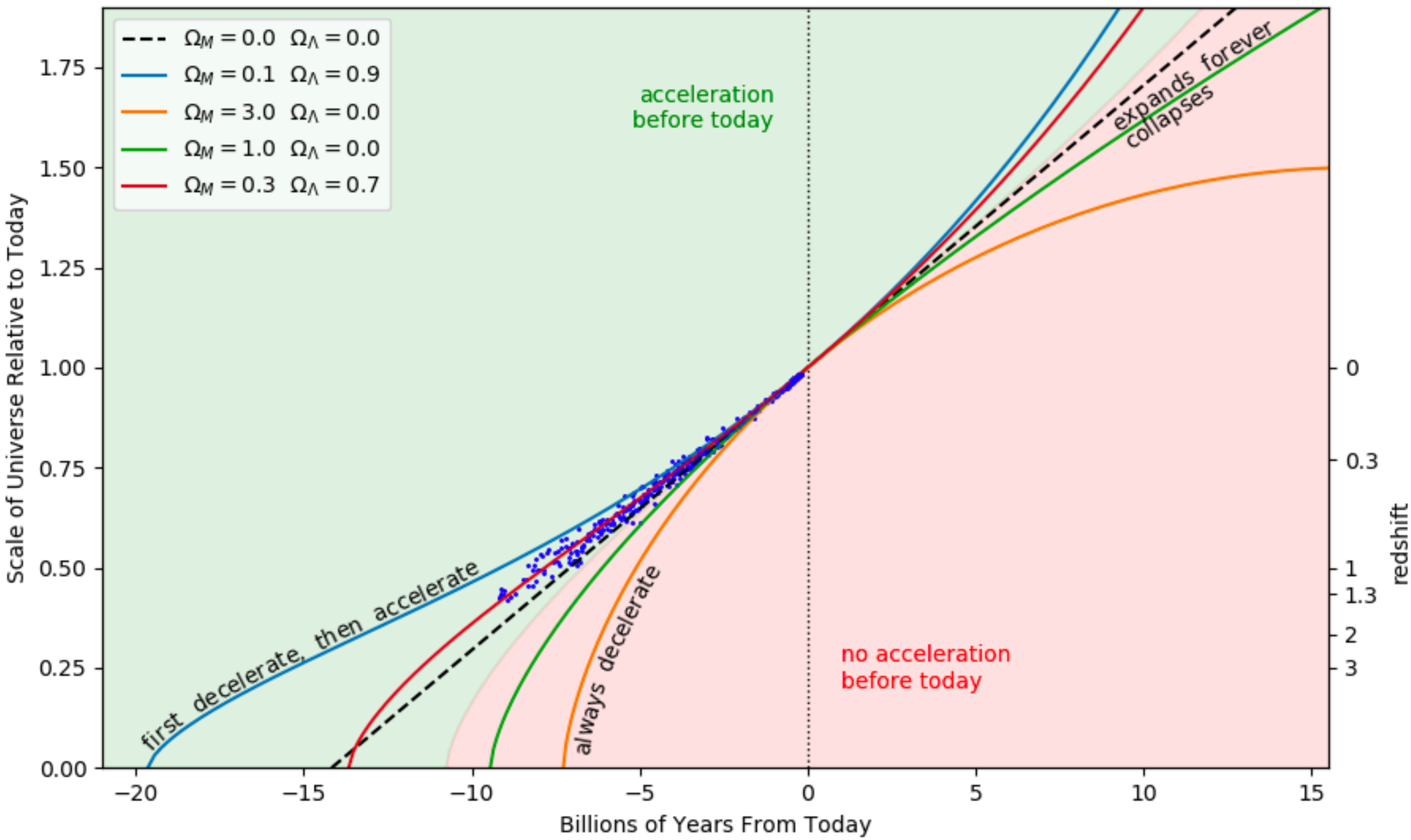
Figure 2.5: A more modern version of Hubble's plot, showing cz versus distance. In this case, the galaxy distances have been determined using Cepheid variable stars as standard candles, as described in Chapter 6. (from Freedman, et al. 2001, ApJ, 553, 47)

And the rate of this expansion is currently accelerating ….

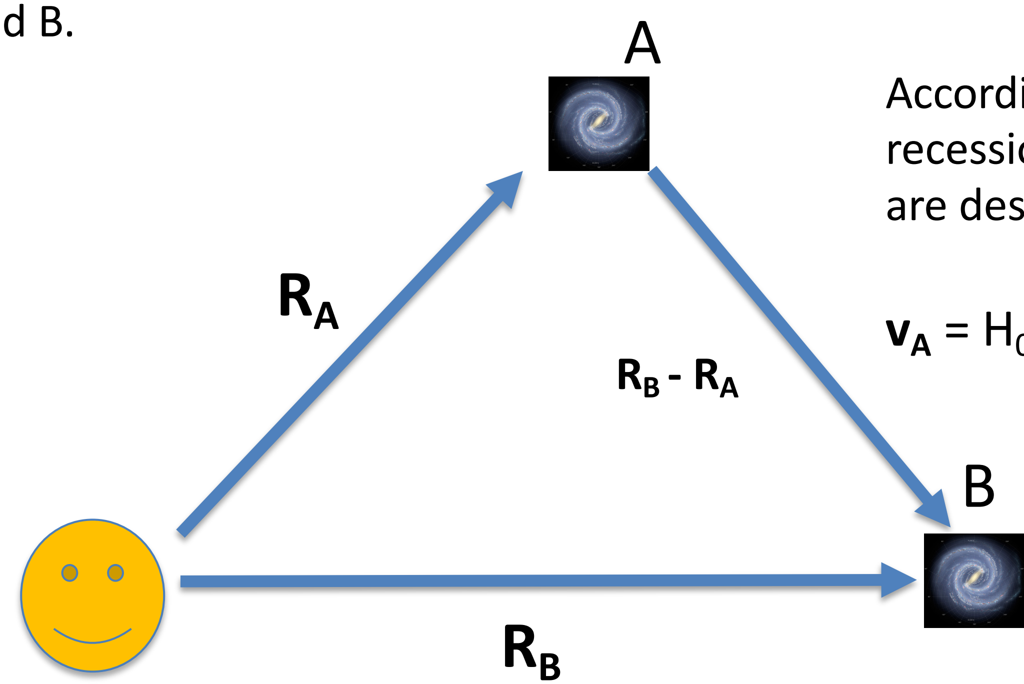
Updated: H_0 = 100 h km/s/Mpc h = 0.7 at present day (z=0)

e.g. SNe data Perlmutter + 1999 ApJ, 517



The Cosmological Principle implies that: the expansion of the universe appears the same to all observers at all locations.

Imagine an observer on Earth who is measuring the radial velocities of two other galaxies, called A and B.



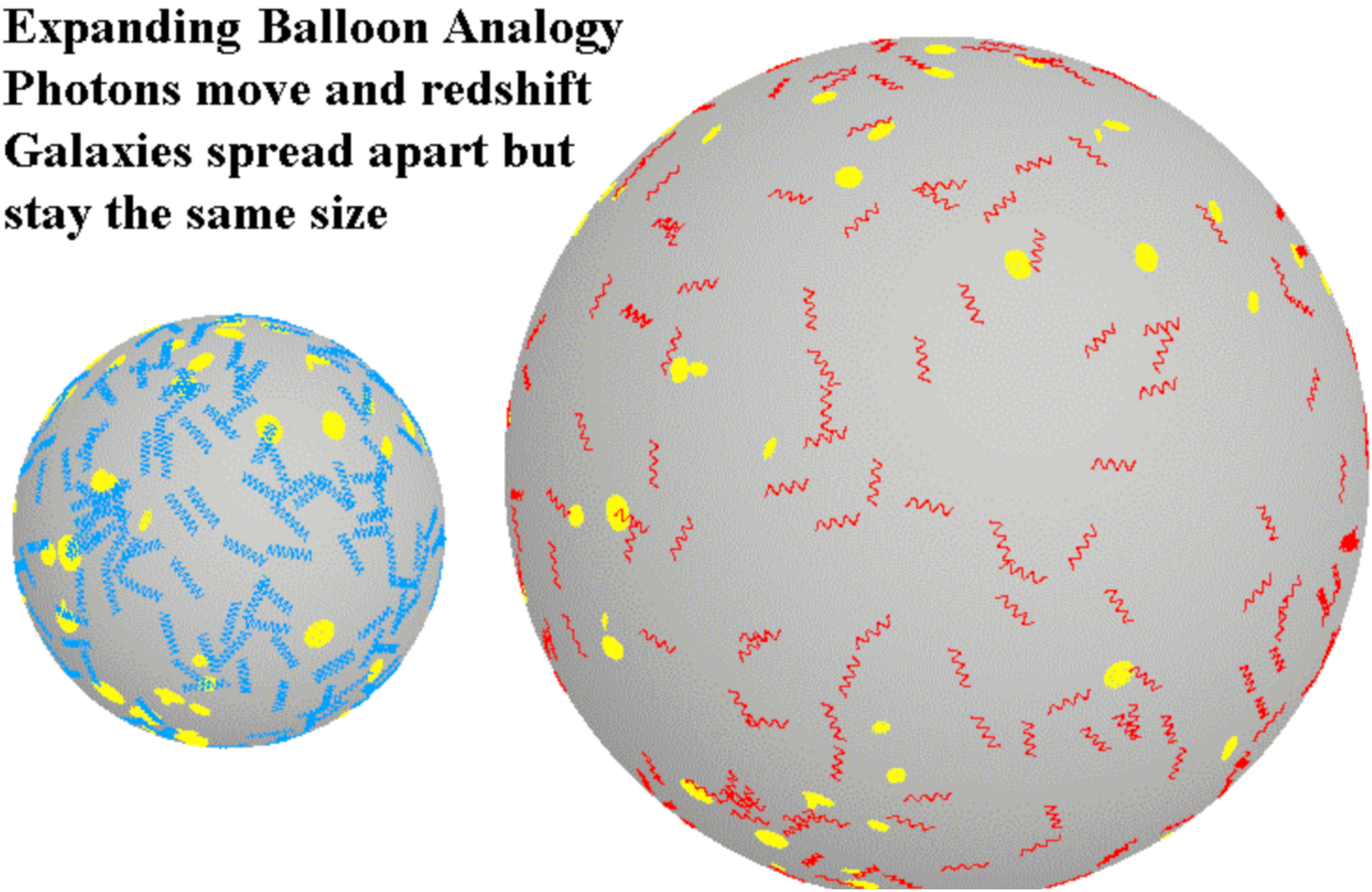
According to Hubble's Law, the recessional velocity of Galaxies are described by the two vectors

$\mathbf{v_A} = H_0\, \mathbf{R_A}$ and. $\mathbf{v_B} = H_0\, \mathbf{R_B}$

The recessional velocity of Galaxy B as seen by an observer on Galaxy A is:

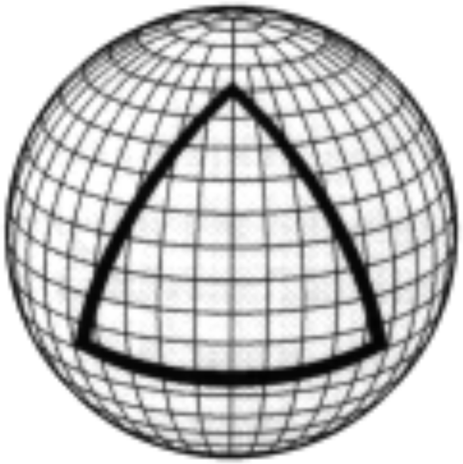
$$\mathbf{v_B} - \mathbf{v_A} = H_0\, \mathbf{R_B} - H_0\, \mathbf{R_A} = H_0\, (\mathbf{R_B} - \mathbf{R_A})$$ This is the same Hubble Law!

Cosmological Principle: the expansion of the universe appears the same to all observers at all locations.



Metric: A metric defines the distance between two points in space, accounting for topology

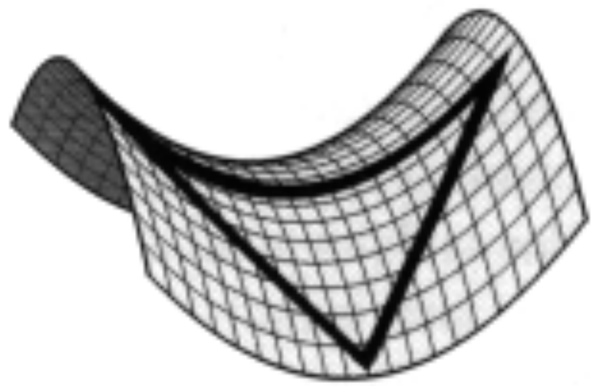
Consider three spatial geometries:



Positive Curvature

Closed Geometry

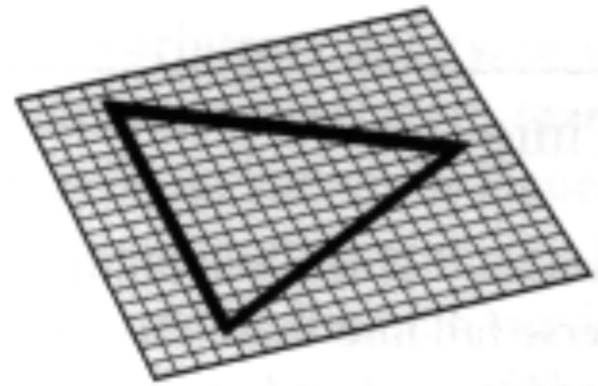
Angles > 180^{o}



Negative Curvature

Open Geometry

Angles < 180^{o}



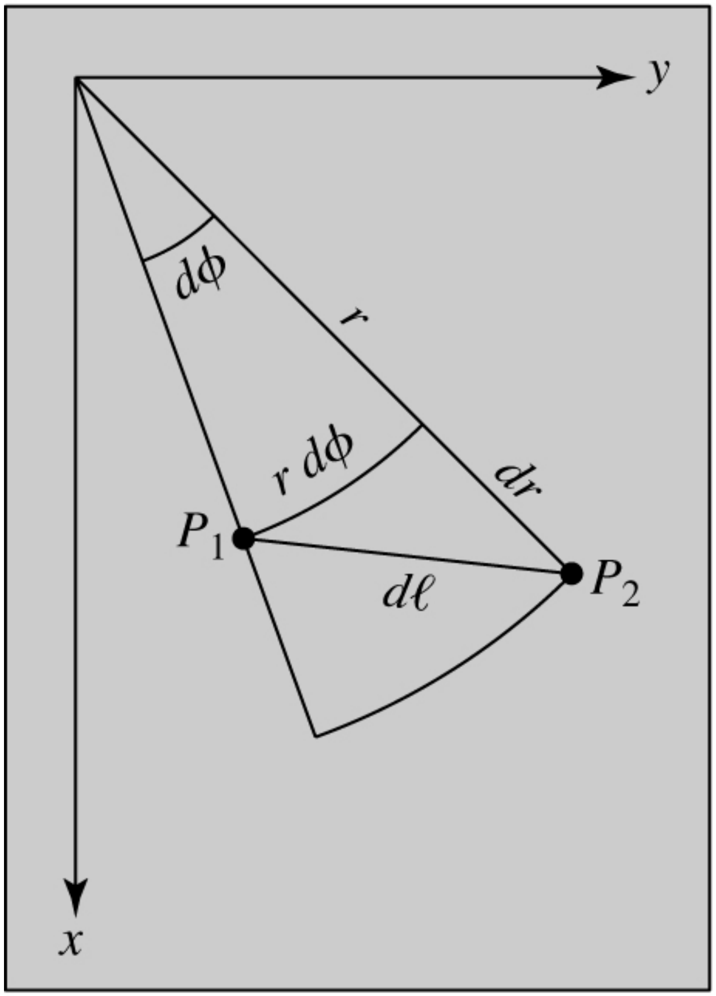
Flat Curvature

Flat Geometry

Angles = 180^{o}

Computing distances depends strongly on the assumed geometry of space.

Metric: 2D Flat Geometry



(a) Carroll & Ostlie Figure 29.18

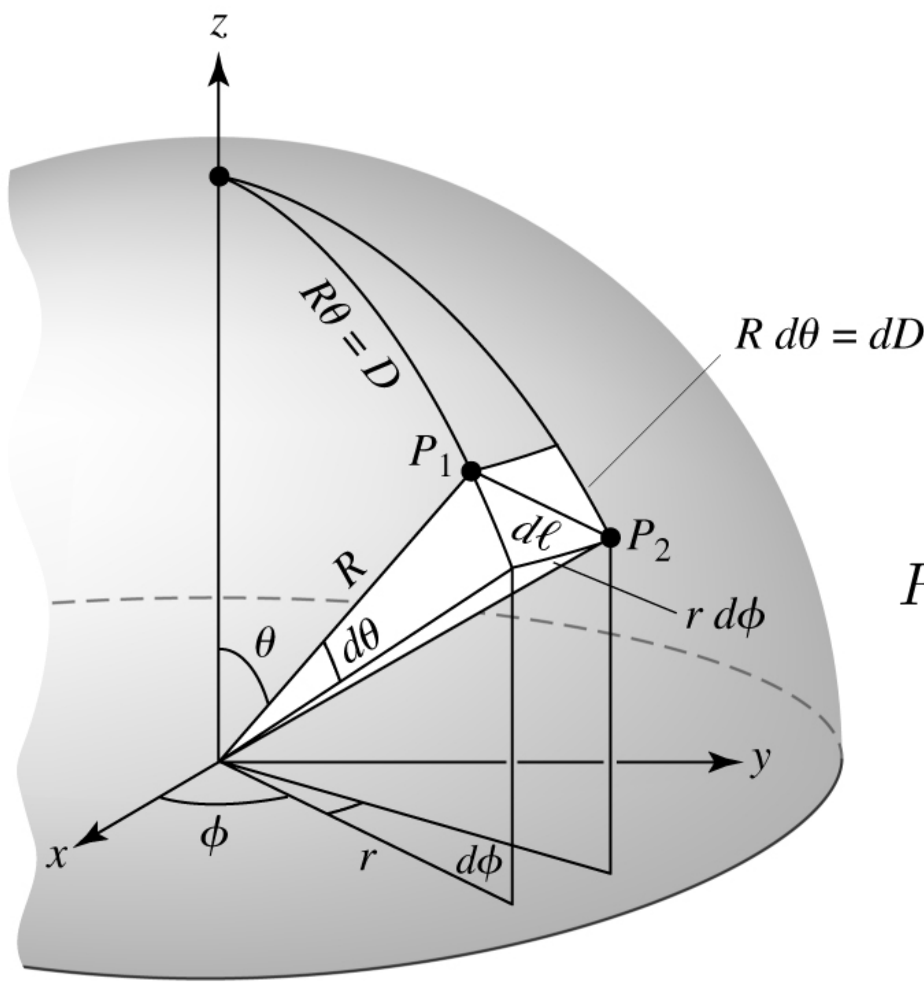
Consider a distance element *dl* connecting two points in space, P1 and P2 .

In Polar coordinates:

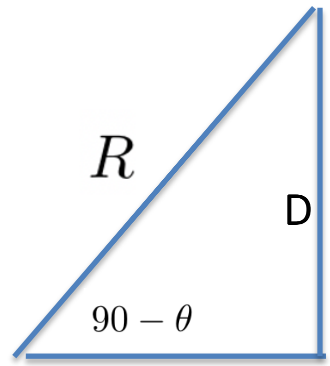
$$d\ell^2 = dr^2 + r^2 d\phi^2$$

2D Curved Geometry

$$d\ell^2 = R^2 d\theta^2 + r^2\, d\phi^2$$



(b)



$$R^2 = D^2 + r^2$$

$$\sqrt{R^2 - r^2}$$

$$r = Rsin\theta, \text{ so } dr = Rcos\theta d\theta$$

$$Rd\theta = \frac{dr}{sin(90-\theta)} = \frac{dr}{cos\theta}$$

$$= \frac{Rdr}{\sqrt{R^2 - r^2}} = \frac{dr}{\sqrt{1 - r^2/R^2}}$$

$$d\ell^2 = \left(\frac{dr}{\sqrt{1 - r^2/R^2}}\right)^2 + (rd\phi)^2$$

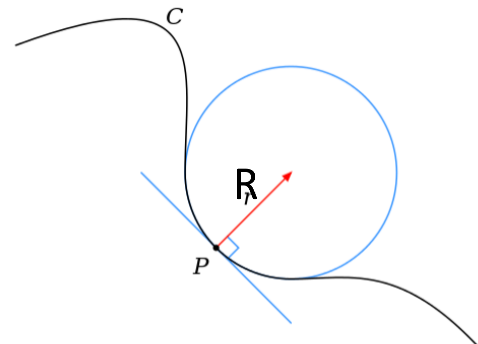
Carroll & Ostlie Figure 29.18
dl as measured for the surface of a spherical shell

(polar coordinates)

$$\text{Curvature } K = 1/R^2$$

$$d\ell^2 = \left(\frac{dr}{\sqrt{1 - r^2 K}}\right)^2 + (r d\phi)^2$$

Positive Curvature: K = 1
Negative Curvature : K = -1
Flat Curvature : K = 0



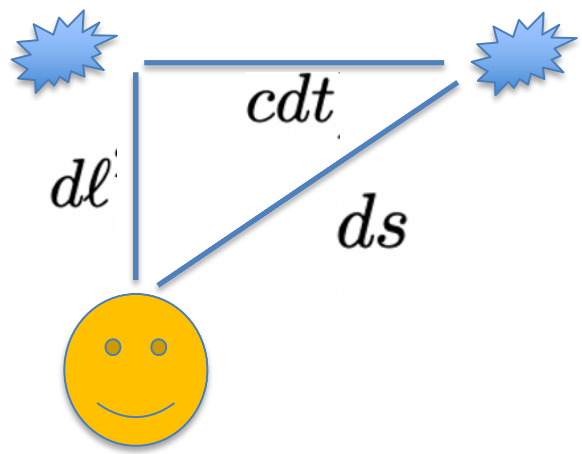
Metric: 3D Curved Geometry

Switch from Polar to Spherical Coordinates: $r = \sqrt{x^2 + y^2 + z^2}$

$$d\ell^2 = \left(\frac{dr}{\sqrt{1 - r^2 K}}\right)^2 + (r d\theta)^2 + (r sin\theta d\phi)^2$$

Space Time

Consider two events that occur at the same location, but at two different points in time.



$$ds^2 = (cdt)^2 - d\ell^2 = (cdt)^2 - \left[\left(\frac{dr}{\sqrt{1 - r^2K}} + (rd\theta)^2 + (rsin\theta d\phi)^2\right)\right]$$

If K =0 this is called the Minkowski Metric

In an expanding universe, however, *dl* will change during this period of time. This is why we need to account for time in the metric.

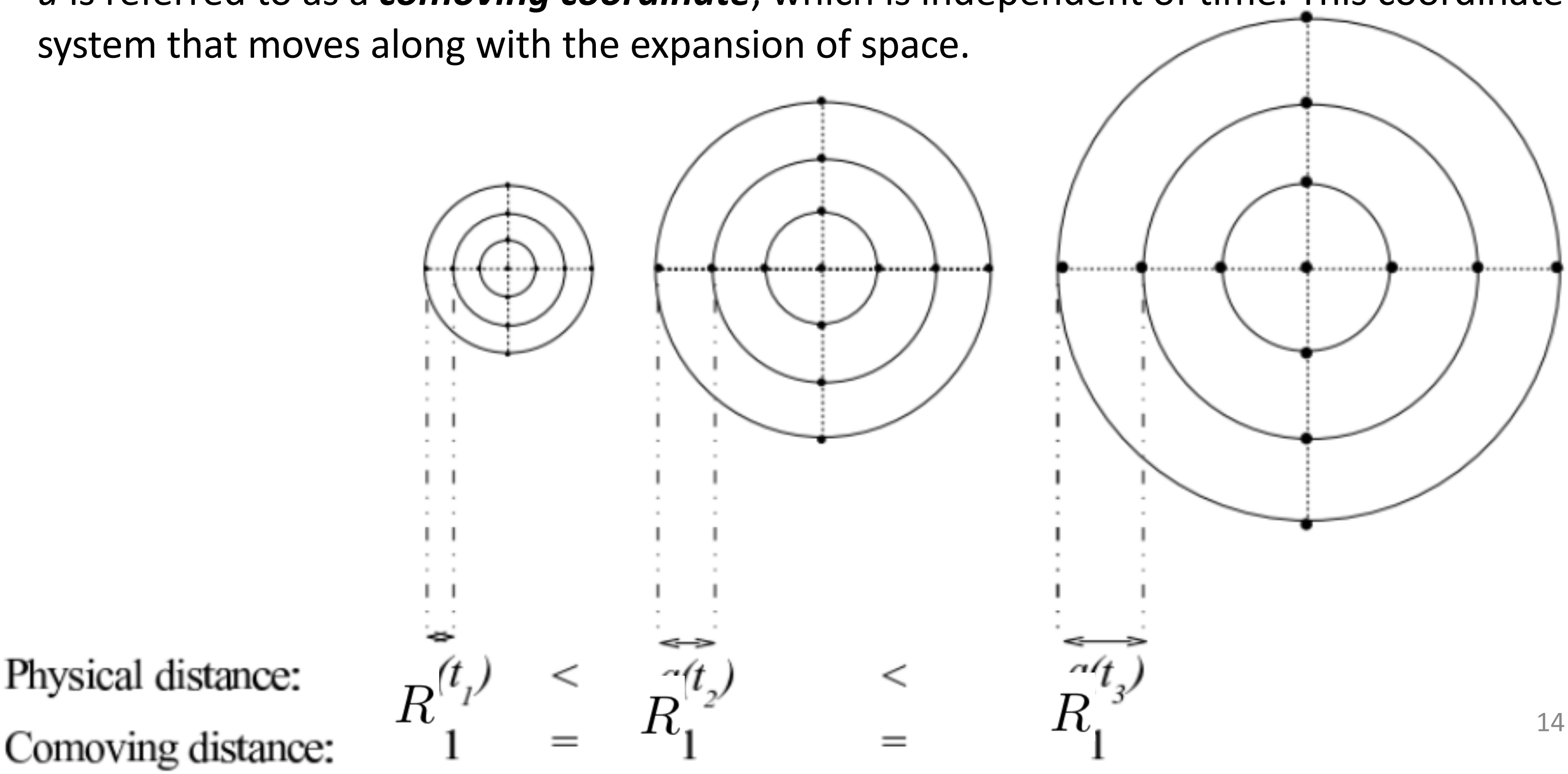
Scale Factor

Let's define our radial distance element in terms of the expansion of the universe.

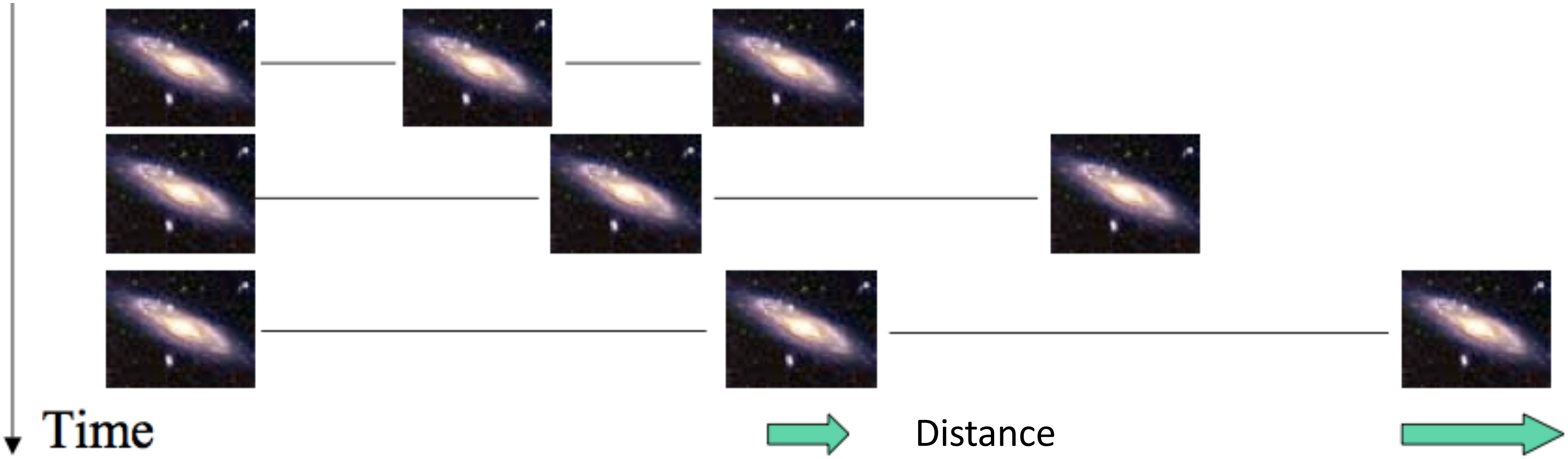
$$r = R(t)u$$

Where R(t) is the ***scale factor***, which defines the rate of expansion at any point in time. In Ryden this is written as a(t)

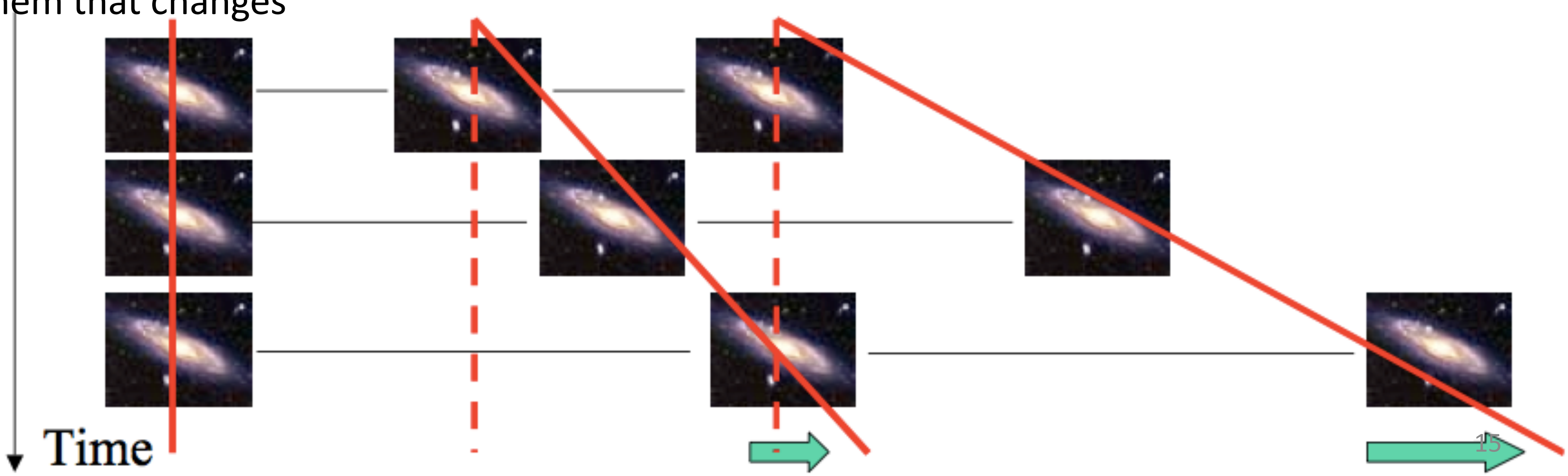
u is referred to as a ***comoving coordinate***, which is independent of time. This coordinate system that moves along with the expansion of space.



We observe galaxies receding from us



Comoving coordinates: Galaxies are not moving.
Galaxies remain at the same position in comoving coordinates – it is the space between them that changes



Robertson-Walker Metric

3D Curved metric $$ds^2 = (cdt)^2 - d\ell^2 = (cdt)^2 - \left[\left(\frac{dr}{\sqrt{1-r^2K}} + (rd\theta)^2 + (rsin\theta d\phi)^2\right)\right]$$

Replacing r with R(t)u

$$ds^2 = (cdt)^2 - R(t)^2 \underbrace{\left[\left(\frac{du}{\sqrt{1-u^2K}}\right)^2 + (ud\theta)^2 + (usin\theta d\phi)^2\right]}_{d\ell^2}$$

Proper Distance (*ds² = dl²*): distance between two events that occur at the same time (dt=0). I.e. the distance that would be measured by a ruler at the time they are observed.

This metric is used to measure the interval between two events in a curved (K), variably expanding (R(t)) space time.

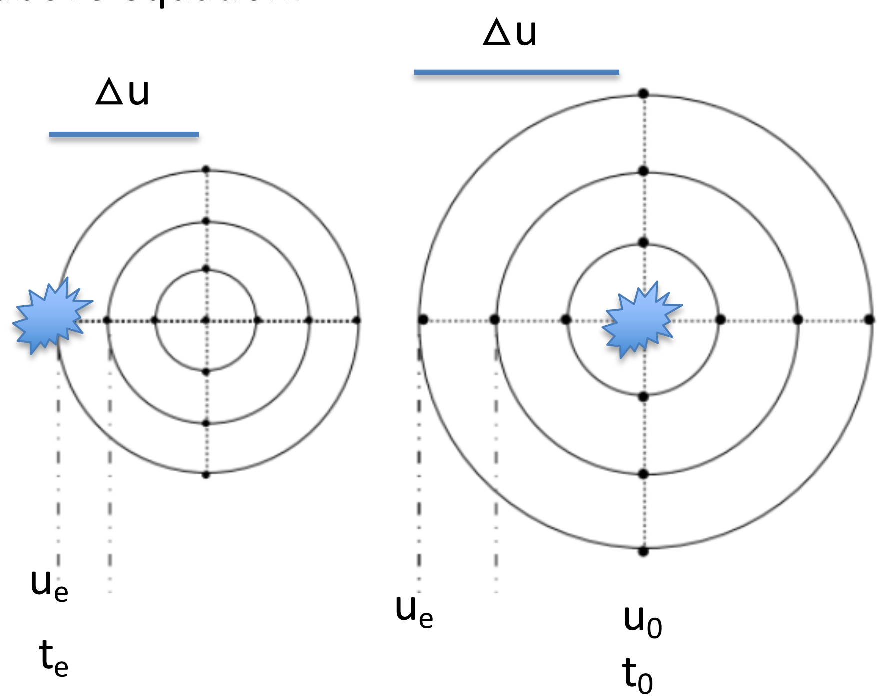
Proper Time

Light rays travel paths with $ds^2 = 0$, called null geodesics. $c^2dt^2 = d\ell^2$

Light rays travel on radial paths, where $(\theta = 0,\ \phi = 0)$ $$cdt = R(t)\frac{du}{\sqrt{1-Ku^2}}$$

Consider a photon that leaves its source at comoving coordinate u_e at time t_e and reaches us at t_0, u_0. Integrate both sides of the above equation.

$$c\int_{t_e}^{t_0}\frac{dt}{R(t)} = \int_{u_e}^{u_0}\frac{du}{\sqrt{1-ku^2}}$$



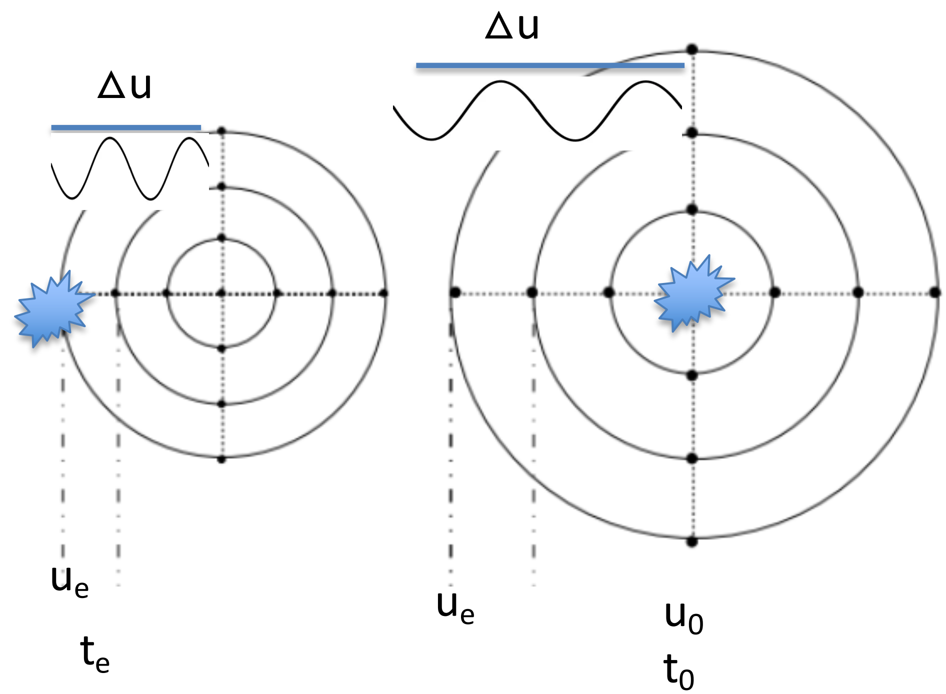
Consider another photon leaving at a later time $t_e + \Delta t_e$. It arrives at $t_0 + \Delta t_0$.
The comoving distance is the same!

$$\int_{t_e+\Delta t_e}^{t_0+\Delta t_0}\frac{dt}{R(t)} = \boxed{\int_{t_e}^{t_0}\frac{dt}{R(t)}}$$

This is called the **Proper Time,** the time delay between two events that occur over the same comoving distance

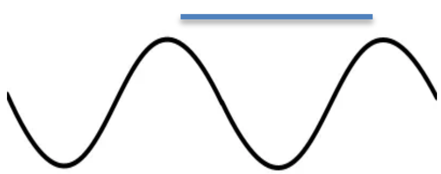
Redshift

$$\int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{dt}{R(t)} = \int_{t_e}^{t_0} \frac{dt}{R(t)}$$



This means that $$\frac{\Delta t_e}{R(t_e)} = \frac{\Delta t_0}{R(t_0)}$$

At Emission: Δt_e



$\lambda_e = c\ \Delta t_e$

This photon is received with wavelength: $\lambda_0 = c\ \Delta t_0\ = c\ \Delta t_e\ \ R(t_0)\ /\ R(t_e)$

The wavelength increases by the ratio of the scale factor.

$$\boxed{1 + z = \frac{\lambda_{obs}}{\lambda_e} = \frac{R(t_0)}{R(t_e)} = \frac{1}{R(t_e)}.}$$

$$\boxed{R(t) = \frac{1}{1+z}}$$

Dimensionless scale factor

Hubble Parameter

Hubble Law (local): $v_r = H_0\, r$

Distance to galaxy in terms of comoving coordinates: $r = R(t)u$

The velocity v_r is then $\dfrac{dr}{dt} = \dfrac{dR}{dt}u$ where u is constant in time.

$$v_r = \dot{R}u = \frac{\dot{R}}{R}r$$

Where u was replaced with r/R

$$H(t) = \frac{\dot{R}}{R}$$

Time evolving Hubble Parameter

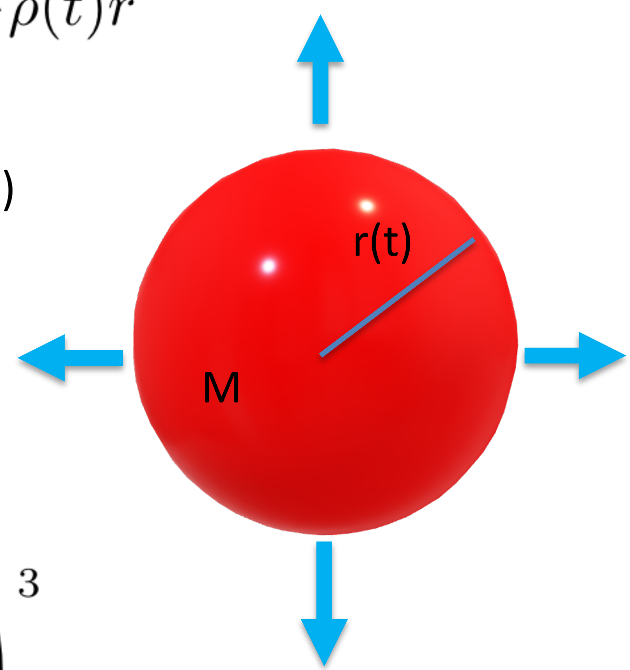
So the Hubble Parameter is the fractional rate of expansion of the universe.
It tells you about the rate at which the scale factor is increasing over time.

The time evolution of the scale factor

Consider a homogeneous spherical density distribution of radius **r(t)** that is expanding uniformly.

The rate at which the sphere can expand is limited by the mass and energy within it.
The expansion will be halted by gravity. We need to determine the expansion rate needed to counteract gravity.

$$\frac{d^2r}{dt^2} = -\frac{GM(<r)}{r^2} = -\frac{4\pi G}{3}\rho(t)r$$



Changing to comoving units, r = R(t)u

(divide out u)

$$\frac{d^2r}{dt^2} = \ddot{R}(t) = -\frac{4\pi G}{3}\rho(t)R(t)$$

The mass is constant, but the volume is growing

$$M = 4/3\pi\rho_o R_o^3 = 4/3\pi\rho R^3$$

So the density can be written as:

$$\rho(t) = \rho_o\left(\frac{R_o}{R(t)}\right)^3$$

Combine the acceleration equation with the density equation

$$\ddot{R}(t) = -\frac{4\pi G}{3}\rho(t)R(t) \;= -\frac{4\pi G}{3}\frac{\rho_o R_o^3}{R^2(t)}$$

Multiply both sides by dR/dt

$$\frac{1}{2}\frac{d}{dt}[\dot{R}^2(t)] = \dot{R}(t)\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_o R_o^3}{R^2(t)}\;\dot{R}(t)$$

Integrate both sides over t

$$\dot{R}^2(t) = \frac{8\pi G}{3}\rho_o\frac{R_o^3}{R} + C$$

where $C \;= -Kc^2$

Putting back $\rho(t) = \rho_o\left(\frac{R_o}{R(t)}\right)^3$

First Friedmann Equation

$$\boxed{\dot{R}^2(t) \;= \frac{8\pi G}{3}\rho(t)R^2(t) \;-Kc^2}$$

First Friedmann Equation & Density

First Friedmann Equation

$$\boxed{\dot{R}^2(t) = \frac{8\pi G}{3}\rho(t)R^2(t) - Kc^2}$$

Total Mass Density = Mass + Radiation + Cosmological Constant (Dark Energy)

$$(\rho_m + \rho_{rad} + \rho_\Lambda)$$

Modifying so that Density refers to *Energy Density*, ($E= mc^2$) : mass + radiation + dark energy

$$\boxed{\dot{R}^2(t) = \frac{8\pi G}{3c^2}\epsilon(t)R^2(t) - Kc^2}$$