Understanding the Limiting Factors of Topic Models via Posterior Contraction Analysis

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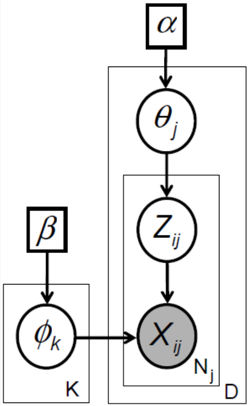
Outline

1 Limiting Factors of the LDA

2 Experiments

Latent Dirichlet allocation (LDA)

ϕ_k: word-topic distributions; θ_j: topic proportion;
Z_{ij}: topic indicators; X_{ij}: observed words



$$\phi_k|\beta \sim \text{Dirichlet}(\beta)$$
$$\theta_j|\alpha \sim \text{Dirichlet}(\alpha)$$
$$Z_{ij}|\theta_j \sim \text{Categorical}(\theta_j)$$
$$X_{ij}|\{\phi_k\}, Z_{ij} \sim \text{Categorical}(\phi_{Z_{ij}})$$

Motivations & Contributions

- Common questions from non-experts in LDA:
  - is my data topic-model friendly?
  - why did LDA fail on my data?
  - how many documents do I need to learn 100 topics?
- This paper provides theory to describe how the following limiting factors affect convergence of the LDA:
  - # documents
  - lengths of documents
  - # topics
  - Dirichlet hyper-parameters

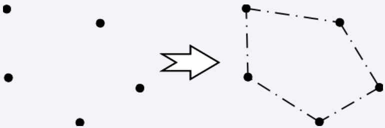
Problem setting

- In LDA, docs are generated from K topics $\phi = (\phi_1, \cdots, \phi_K)$.
- Each doc is associated with a topic proportion vector $\theta_d \in \triangle^{K-1}$
  - equivalently, each doc uniquely corresponds to a word probability vector $\eta_d = \sum_{k=1}^{K} \theta_{dk}\phi_k$
  - observed words are generated from these $\{\eta_d\}$'s, represented with a $D \times N$ matrix
- Problem:
  - how fast (rate) does the posterior distribution of $\{\phi_k\}$'s converge to the true value as D and N approach infinity?

Latent topic polytope in LDA

- Study convergence of individual topic-word distribtion?
  - identifiability problems in LDA: *e.g.*, the label-switching issue
- To avoid such problems, instead of studying individual topics, the *topic polytope* is used as a representation of topic structures in LDA:
  - given topics $\{\phi_k\}_{k=1}^K$, the topic polytope is defined as the convex hull of $\{\phi_k\}$:

$$G(\Phi) \triangleq \text{conv}(\phi_1, \cdots, \phi_K) \tag{1}$$



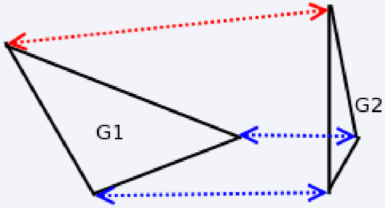
Distance between topic polytopes

- To compare 2 different models, distance between topics need to be defined.
- Define distance between two topic polytopes G_1 and G_2:

$$d_{\mathscr{M}}(G_1, G_2) \triangleq \max\{d(G_1, G_2), d(G_2, G_1)\}, \text{ ,where} \quad (2)$$

$$d(G_1, G_2) = \max_{\phi_1 \in \mathrm{extr}(G_1)} \min_{\phi_2 \in \mathrm{extr}(G_2)} \|\phi_1 - \phi_2\|_2 \quad (3)$$

where 'extr' means the *extreme points* (topics in LDA).
- Equivalent to the well-known Hausdorff metric in convex geometry under mild assumptions.



Posterior contraction analysis

- Posterior contraction analysis describes how fast the posterior of a given subset of data convergence to the true posterior distribution.
- This paper uses posterior contraction analysis to analyze the impact of limiting factors in LDA, *e.g.*, #docs, #topics, lengths of docs.
- In the following:
  - K^*: true #topics
  - K: #topics in a model
  - D: # docs
  - N: document length (assume same document length)

Contraction of the posterior of topic polytope

- Assume mild regularity conditions such that (formal descriptions omitted):
  - topic polytopes are not degenerated or collapsing
  - the prior is dense enough in the space of parameters

Theorem

Let the Dirichlet parameters for topic proportions $\alpha_k \in (0,1]$, and assume either one of the following holds:

(A1) $K = K^*$, *i.e., the true #topics is known;*

(A2) *the Euclidean distance between every pair of topics is bounded from below by a known positive constant* r_0.

then as $D \to \infty$ *and* $N \to \infty$ *such that* $N \geq \log D$, *for some* $C > 0$ *independent of* N *and* D:

$$\Pi(d_{\mathscr{M}}(G, G^*) \leq C\delta_{D,N}) \to 1 \ , \tag{4}$$

where $\delta_{D,N} = (\frac{\log D}{D} + \frac{\log N}{N} + \frac{\log N}{D})^{1/2}$, $\Pi(\cdot)$ *means under the posterior distribution.*

Some observations on the convergence rate

$$\Pi(d_{\mathscr{M}}(G,G^*) \leq C\delta_{D,N}) \to 1,\ \delta_{D,N} = (\frac{\log D}{D} + \frac{\log N}{N} + \frac{\log N}{D})^{1/2}$$

- The proof of the theorem requires $N \geq \log D$.
- Convergence rate: $\max\{(\frac{\log N}{N})^{1/2}, (\frac{\log D}{D})^{1/2}, (\frac{\log N}{D})^{1/2}\}$, $(\frac{\log N}{D})^{1/2}$ does not play a noticeable role empirically (might be an artifact due to the proof techniques).
- The actually rate might be faster since this is an upper bound (there is an lower bound $\Omega(\frac{1}{DN})$ not given here).
- The rate does not depend on #topics K, meaning if K is known or the topics are well-seperated, the inference is statistically efficient.
- In practice, the overfitted setting is preferred, *e.g.*, $K \gg K^*$, which is considered in the following.

Contraction of the posterior of topic polytope

- When neither of (A1) and (A2) hold, the rate is much worse:
  - (A1) $K = K^*$, *i.e.*, the true #topics is known;
  - (A2) the Euclidean distance between every pair of topics is bounded from below by a known positive constant r_0.

Theorem

Under the same conditions as the previous theorem, except that none of the conditions (A1) and (A2) holds, then for $K^* < K \leq |V|$*, we have*

$$\Pi(d_{\mathscr{M}}(G, G^*) \leq C\delta_{D,N}) \to 1 \ , \tag{5}$$

where $\delta_{D,N} = \left(\frac{\log D}{D} + \frac{\log N}{N} + \frac{\log N}{D}\right)^{\frac{1}{2(K-1)}}$.

- This means the convergence is very slow, depending on K.
- It is said underfitting ($K < K^*$) will result in a persistent error even with infinite data, thus not considered.

Outline

1 Limiting Factors of the LDA

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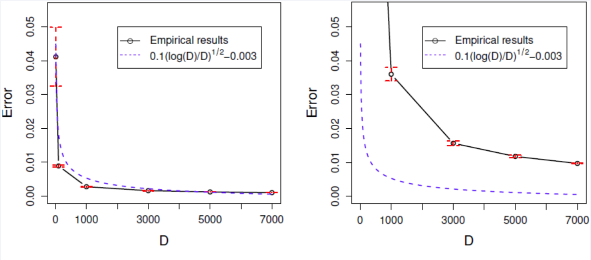
On synthetic data

- Generate data from an LDA with $K^* = 3$, $V = 5000$, symmetric Dirichlet prior for topic proportions and word-topic distributions to being 1 and 0.01, respectively.
- Variation of parameters: #docs D, length of docs N, Dirichlet hyperparameter for topic-word distributions β, #topics K.
- Use collapsed Gibbs sampler.

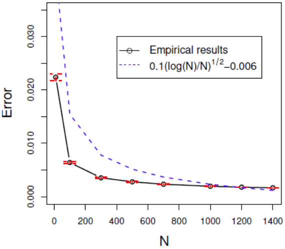
Fixing N: theoretical upper bound: $\propto \left(\frac{\log D}{D}\right)^{\frac{1}{2}}$



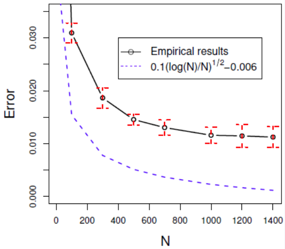
(a) $K = K^*, \beta = 0.01$ (b) $K > K^*, \beta = 0.01$

(c) $K = K^*, \beta = 1$ (d) $K > K^*, \beta = 1$

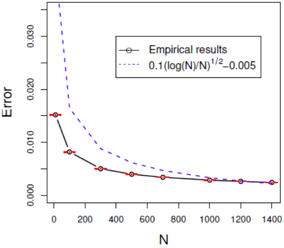
Fixing D: theoretical upper bound: $\propto \left(\frac{\log N}{N}\right)^{\frac{1}{2}}$



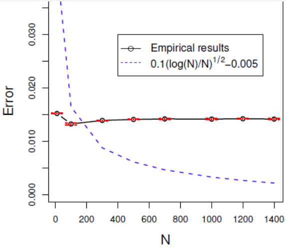
(a) $K = K^*, \beta = 0.01$



(b) $K > K^*, \beta = 0.01$

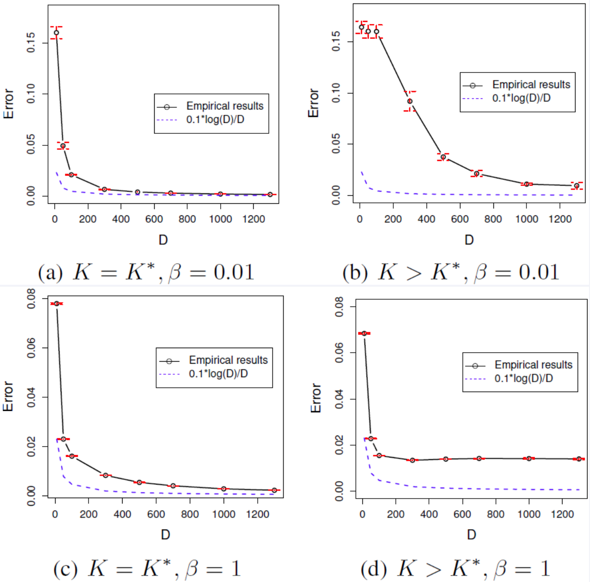


(c) $K = K^*, \beta = 1$



(d) $K > K^*, \beta = 1$

Increasing $N = D$: theoretical upper bound: $\propto \left(\frac{\log N}{N}\right)^{\frac{1}{2}}$



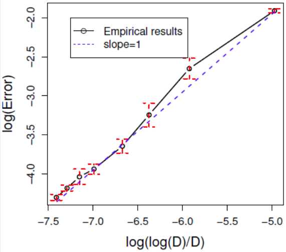
(a) $K = K^*, \beta = 0.01$

(b) $K > K^*, \beta = 0.01$

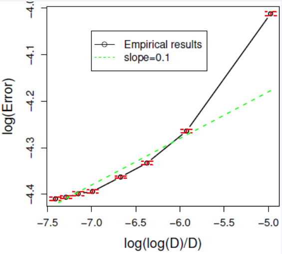
(c) $K = K^*, \beta = 1$

(d) $K > K^*, \beta = 1$

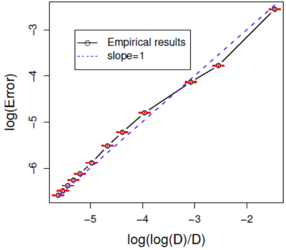
Compared with theoretically asymptotic error rates



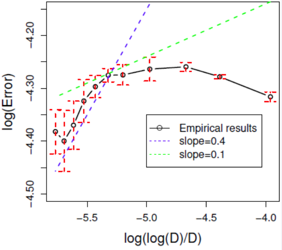
(a) Fixed N, $K = K^*$



(b) Fixed N, $K > K^*$

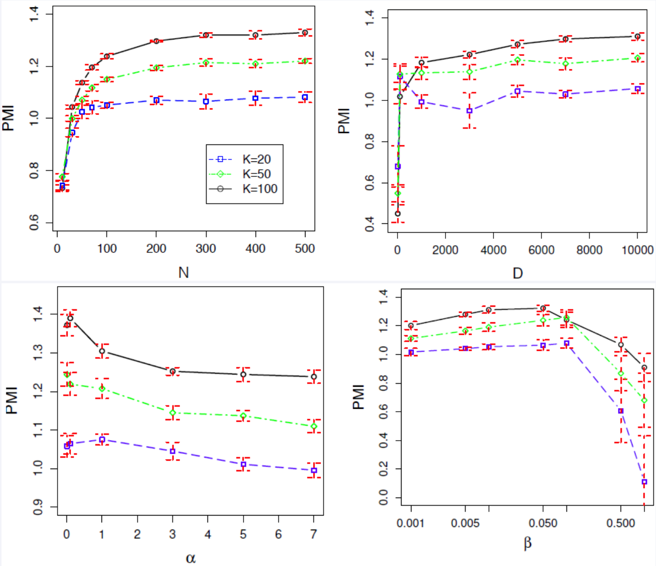


(c) $D = N$, $K = K^*$

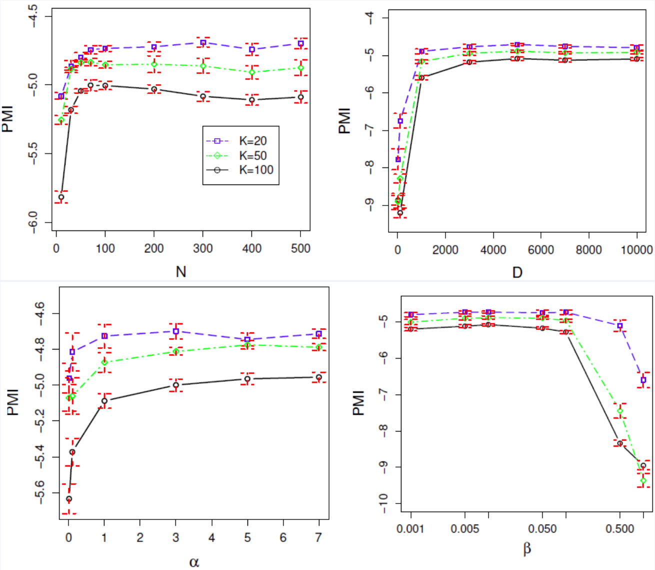


(d) $D = N$, $K > K^*$

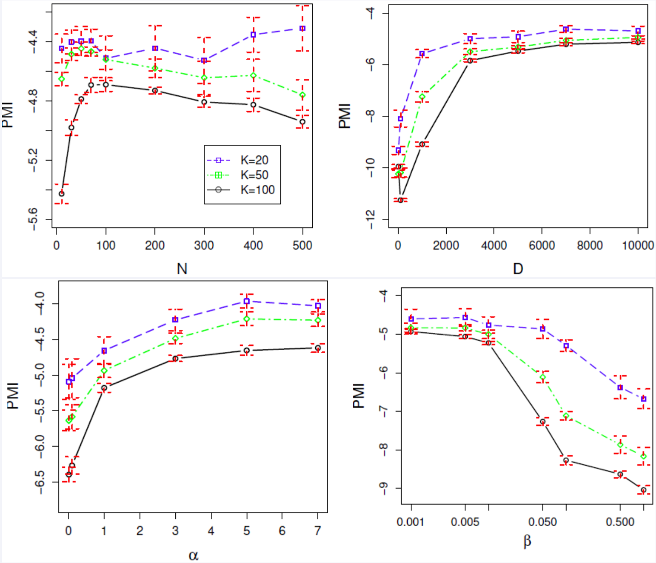
Real data: Wikipedia



Real data: New York Times



Real data: Twitter



Implications and guidelines for LDA

- #docs plays the most important role:
  - it is theoretically impossible to guarantee identification of topics from a small docs
  - once sufficient docs are provided, further increasing the number might not help significantly, unless document lengths are also increased
- poor performance when lengths of docs are too short, even if there are a lot of docs.
- when over fitting ($K \gg K^*$), convergence rates might deteriorate quickly.
- the LDA performs well when the underlying topics are well-seperated.
- if each doc is associated with few topics, the Dirichlet hyperparameter should be set to small

Thanks for your attention!!!