Question 2 Seung Hee Lee

$$\sum_{i=1}^{n} \alpha_i \left(y_i - \sum_{j=1}^{d} x_{ij} w_j - b\right)^2$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad n \times 1$$

$$X_\ell = \underbrace{\begin{bmatrix} x_{1,1} & - & - & \cdots & 1 \\ x_{2,1} & - & - & \cdots & 1 \\ x_{3,1} & - & - & \cdots & 1 \\ \vdots & & & \ddots & \vdots \\ x_{n,1} & - & - & x_{n,d} & 1 \end{bmatrix}}_{d+1}$$

$$\vec{w}_\ell = \begin{bmatrix} w_1 \\ \vdots \\ w_d \\ 1 \end{bmatrix} \Bigg\} n+1 \times 1$$

$$\underset{w}{\text{argmin}} \sum_{i=1}^{n} a_i (y_i - x_{i\ell} \vec{w}_\ell)^2$$

$$\underset{w}{\text{argmin}} \quad A(\vec{y} - X_\ell \vec{w}_\ell)^2$$

$$\frac{d}{dw} \rightarrow \quad (-X_\ell^T)\, 2A(\vec{y} - X_\ell \vec{w}_\ell) = 0$$

$$X_\ell^T A(\vec{y} - X_\ell \vec{w}_\ell) = 0$$

$$X_\ell^T A \vec{y} - X_\ell^T X_\ell \vec{w}_\ell = 0$$

$$X_\ell^T A \vec{y} = X_\ell^T X_\ell \vec{w}_\ell$$

$$\boxed{(X_\ell^T X_\ell)^{-1} X_\ell^T A \vec{y} = \vec{w}_\ell} \quad //.$$

HW2 Seung Hee Lee

Problem 1: Linear Algebra Review

0) B be a 4×4 matrix

$$B = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_5 & x_6 & x_7 & x_8 \\ x_9 & x_{10} & x_{11} & x_{12} \\ x_{13} & x_{14} & x_{15} & x_{16} \end{bmatrix}$$

1) double column 2

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Interchange Columns 1 and 4

$$M_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

3) Halve row 1

$$M_3 = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) add row 3 to row 1

$$M_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5) Subtract row 4 from each of other rows.

$$M_5 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6) replace Column 3 by Column 4

$$M_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

7) Delete Column 2

$$M_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ a) $B \cdot M_1$

$M_5 \cdot M_4 \cdot M_3 \cdot M_2 \cdot B \cdot M_1 \cdot M_6 \cdot M_7$ //

b) $A = M_5 \cdot M_4 \cdot M_3 \cdot M_2$.

$C = M_1 \cdot M_6 \cdot M_7$

∴ $A \cdot B \cdot C$ //