HW4 problem 1. Seung Hee Lee

1) $h_{MPE}(x) = \text{argmax}_y\, P(y|x,\theta)$

$$P(x|y,\theta) = N(\mu_y, \Sigma_y)(x) = \frac{1}{((2\pi)^d \det(\Sigma_y))^{1/2}} e^{-\frac{1}{2}(x-\mu_y)^T \Sigma_y^{-1}(x-\mu_y)}$$

① using Baye's Rule for QDA.

$$\underset{y}{\text{argmax}}\, P(y|x,\theta) = \underset{y}{\text{argmax}} \frac{P(x|y,\theta)P(y)}{P(x)}$$

$$= \underset{y}{\text{argmax}} \frac{((2\pi)^d \det|\Sigma_y|)^{-\frac{1}{2}} e^{-\frac{1}{2}(x-\mu_y)^T \Sigma_y^{-1}(x-\mu_y)} \cdot P(y)}{P(x)}$$

② Take log

$$\underset{y}{\text{argmax}} -\frac{1}{2}\log(2\pi^d \cdot \det|\Sigma_y|) - \frac{1}{2}(x-\mu_y)^T \cdot \Sigma_y^{-1}(x-\mu_y) + \log P(y) - \log P(x)$$

② Remove coefficients

$$\underset{y}{\text{argmax}} \quad -\frac{1}{2}(x-\mu_y)^T \cdot \Sigma_y^{-1}(x-\mu_y) + \log p(y) - \frac{1}{2}\log\det|\Sigma_y|$$

③ apply $-$ to be argmin

$$\therefore\ h_{QDA} = \underset{y}{\text{argmin}}\left[\frac{1}{2}(x-\mu_y)^T \Sigma_y (x-\mu_y) + \frac{1}{2}\log\det|\Sigma_y| - \log p(y)\right]$$ //

*For LDA

Start from step② in QDA

*After taking log!

$$\underset{y}{\text{argmax}} \quad -\frac{1}{2}(x-\mu_y)^T \cdot \Sigma_y^{-1}(x-\mu_y) + \log p(y) - \frac{1}{2}\log\det|\Sigma_y|$$

$$\underset{y}{\text{argmax}} -\frac{1}{2}\left[x^T\Sigma_y^{-1}x - 2\mu_y^T\Sigma_y^{-1}x + \mu_y^T\Sigma_y^{-1}\mu_y\right] + \log p(y) - \frac{1}{2}\log\det|\Sigma_y|$$

* considering the fact that covariance in LDA are same. we can remove some values.

$$\underset{y}{\text{arg max}} -\frac{1}{2}\left[2\mu_y^T\Sigma_y^{-1}x + \mu_y^T\Sigma_y^{-1}\mu_y\right] + \log p(y)$$

*Clean up!

$$h_{LDA} = \underset{y}{\text{arg max}}\left[\left[\mu_y^T\Sigma_y^{-1}x - \frac{1}{2}\mu_y^T\Sigma_y^{-1}\mu_y\right] + \log p(y)\right]$$