Option pricing with NNs and Traditional Models for European and American Options

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Abstract— **Option pricing, a critical component of financial derivatives valuation, has traditionally relied on models like Black-Scholes for European options and the Binomial tree for American options. However, the complexities of modern financial markets, especially over extended durations, reveal certain limitations in these models. This study delves into a comparative analysis of traditional models against emerging neural network models, specifically Recurrent Neural Networks (RNNs) and Long Short-Term Memory (LSTM) networks, using European (S&P 500) and American (Alphabet) options as test cases. Our results underscore the consistent superiority of RNNs in capturing intricate pricing dynamics, while LSTMs displayed promising results in short to mid-term scenarios but faced challenges in specific long-term applications. The traditional models, though foundational, displayed diminishing accuracy in certain contexts. Considering these findings, this study ardently advocates for a broader embrace of neural network models in the option pricing domain, suggesting a promising synergy between computational advancements and financial modeling. By championing the integration of these neural models, the research endeavors to pave the way for more accurate and adaptive option valuation methodologies in evolving financial landscapes.**

Keywords—Option pricing, Neural Networks, Black-Scholes Model, Binomial Tree Model

I. INTRODUCTION

A. Background and Motivation

Options, as essential financial derivatives, provide investors with the right but not the obligation to purchase or sell an underlying asset at a predetermined price [14]. This distinguishing feature endows them with strategic value in hedging, speculation, and capital allocation, rendering their accurate pricing indispensible for the stability and efficacy of the financial market. Consequently, accurate option pricing is essential for assuring effective risk management and making informed investment decisions in the financial markets. An American option is one that can be exercised at any time prior to or on the expiration date. A European option is one that can be exercised only on the expiration date. Options can also be categorised as calls or puts. The bearer of a call option has the right to purchase the underlying asset, while the holder of a put option has the right to sell the underlying asset. The Black-Scholes model, a cornerstone in the field, has long been employed for European options. Due to the numerous limitations of the BS model, individuals have been optimising it or searching for optimal methods over the past few decades.

B. The Black-Scholes Model and Its Limitations

The historic Black-Scholes (BS) model [4] stands as a testament to the early efforts to demystify option pricing, providing a groundbreaking method to value European options in 1973. It is a widely used options pricing model [6]. However, as financial markets evolve in complexity and diversity, the assumptions underlying traditional models like the BS often fall short of capturing the intricate dynamics of real-world trading scenarios. It assumes certain conditions, such as constant volatility, efficient markets, and continuous trading, which may not hold true in real-world scenarios [10]. This has spurred researchers, financiers, and economists to embark on a relentless quest for refined and more adaptable models, leading to the incorporation of stochastic volatility, jumps, and other sophisticated features. Emergence of Neural Networks in Financial Modeling. In the decades that followed, numerous extensions to the classic model were proposed. Roul [17] derived a condensed finite difference algorithm for the fractional Black-Scholes option pricing model. Analytical solutions of a time-space-fractional Black-Scholes model (TSFBSM) are presented by combining the Fractional Complex Transform (FCT) technique with a modified differential transform method [8].

C. The Binomial Tree Model and Its Limitations

The paper published by Cox et al [7] is one of the origins of the binomial approach to option pricing. The binomial tree model is a versatile instrument that facilitates the pricing of American options that permit early exercise. Based on a discrete-time framework, this model builds a binomial lattice to track potential asset price changes. Its adaptability to early exercise rights makes it especially suitable for American options. Nevertheless, it has its own disadvantages. In order to calculate the selection cost, for instance, these methods predict only a spot price or an interval price with low precision and limited adaptability.

D. Emergence of Neural Networks in Financial Modeling

Parallel to advancements in conventional option pricing models, the ascendance of computational prowess and data-centric approaches has inaugurated a new era in the field. Neural networks, as a component of this surge, are quickly becoming invaluable assets. Their ability to recognise intricate patterns and nuances provides a promising alternative to conventional models, potentially filling in the spaces left by deterministic approaches. The application of neural networks and deep learning technology in the financial field is more and more extensive. For example, Chen et al. [5] proposed to use a convolutional neural network for financial quantitative investment. Additionally, deep learning and evolutionary computing be utilised to forecast foreign exchange rates and optimise their portfolios by Pimentel [16]. Deep learning has also been applied to predict stock data and has a certain application value in generating stock income [2]. These models are based on artificial intelligence techniques and are capable of learning from historical data to make predictions or estimate values [19]. Neural networks have demonstrated success in various financial applications, including stock price prediction and risk assessment [3]. All indications, The emergence of computational capabilities and data-centric methodologies heralds a transformative era in the field of finance in the face of the continuous evolution of traditional option pricing models.

E. Neural Networks in Option Pricing

Some research has focused on improving the accuracy of European option pricing by incorporating neural networks into the modeling process [1] [11]. Initial explorations suggest that neural networks may significantly refine European option pricing accuracy. An example is the pricing of S&P-500 European call options using a Modular Neural Network (MNN) model [12]. Yang et al. [20] proposed a Gated Neural Networks for Option Pricing in 2017, and proved that this method is superior to the pricing model at that time. Numerous additional studies demonstrate the potential of neural networks for option pricing. The vast majority of research, however, focuses on the pricing of European options. Consequently, research into the potential of American alternatives is still in its infancy. More evidence is required to demonstrate that, in some instances, they may outperform conventional pricing strategies. In the ever-changing landscape of financial markets, optimising option pricing methodologies remains of paramount importance. As markets evolve and new obstacles emerge, the pursuit of a comprehensive, accurate, and dynamic framework for pricing options continues unabated.

F. Research Objectives and Questions

Given this backdrop, this dissertation aims to undertake a comparative analysis of European and American options pricing using the traditional models (Black-Scholes and binomial tree models) and neural networks. The key questions driving this research are:

- Can neural networks enhance the pricing accuracy for European options compared to the BS model?
- How do neural networks impact the pricing of American options?
- What variations exist in the pricing of call and put options between the BS model and neural network models?

G. Scope, Limitations, and Significance

While the study encompasses a broad spectrum of market conditions and options specifications, it's pivotal to recognize that neural network efficacy might be swayed by factors like model complexity and data quality. Beyond the academic sphere, this research holds profound implications for industry practitioners. It offers a fresh lens to view option pricing, potentially leading to more informed strategies in financial derivatives pricing. By understanding the strengths and weaknesses of each model, financial professionals can better navigate risk and hone their decision-making processes.

II. METHODOLOGY

The structure of this study's methodology adheres to a rigorous and methodical sequence of steps, transitioning from initial data procurement to the conclusive assessment of the neural network paradigms. This systematic architecture guarantees the rigor and resilience of the derived conclusions.

A. Data Collection and Preparation

1) Source of Datasets

The datasets were provided by the supervisor, encompassing cleared options for S&P 500 and Alphabet. The rationale behind this selection is rooted in the differential nature of options available in these assets: European options primarily dominate the S&P 500 landscape, while American options are prevalent for Alphabet. These datasets span from 2017 to 2021 which provides a rich landscape for the study.

2) Specifications & Pre-processing Steps

- Exclusion of Imminent Expiry Data: Options nearing expiration within a week were systematically excluded to maintain data relevancy.
- Train-Test Division: The conventional method for analysing the dynamics of stock prices is founded on the well-known efficient market hypothesis. This hypothesis has the consequence that it is impossible to predict future price fluctuations based on past price fluctuations. This is the rationale for dividing the combined dataset, allocating 80% for training and the remaining 20% for validation and testing. This guarantees a balanced and representative dataset for model training and evaluation.
- Duplicate Elimination: To uphold data integrity and avoid redundancy, duplicates were meticulously identified and removed.
- Call-Put Separation: The dataset was bifurcated to cater to the research objective, separating call and put options.

Term-Based Classification: Options were systematically classified based on their expiration durations: short-term (less than a month), medium-term (1-12 months), and long-term (over a year). This segmentation aids in a detailed and nuanced analysis. The distribution of options in each category is as shown in Table I.

TABLE I. SIZE OF DATASETS

Option Class / Call-Put		Term-Based Classification		
		Short Term	***Mid Term***	***Long Term***
S&P 500 Call	Train	1784772	4103542	329443
	Test	501990	1292338	93634
S&P 500 Put	Train	1909886	4179512	333879
	Test	543519	1325885	95293
Alphabet Call	Train	482591	851280	218570
	Test	120197	211807	54621
Alphabet Put	Train	498996	842153	212683
	Test	125200	211552	53193

For every term (Short, Mid, and Long), the number of call options and put options are fairly close in both the training and test datasets. This suggests a balanced representation of both call and put options in each term category, so they will disproportionately dominate the other.

3) Data Distribution Analysis

To emphasize the consistency in data distribution between our training and test sets, we utilize histogram representations. While our datasets encompass multiple variables, due to space constraints, we specifically illustrate the distribution of the "forward price" variable as it is a crucial feature in our analysis and provides a representative snapshot of our dataset.

In Figure 2, the histograms depict the frequency distributions of the "forward price" for both the training and test sets. A visual comparison accentuates the analogous distribution patterns, corroborating the appropriateness of our data split methodology. Such consistency is pivotal to ensure that the test set genuinely mirrors the characteristics of the training data, thereby leading to more accurate and reliable model validation.

4) Feature Refinement and Normalization

After the extraction of pertinent features and targets, normalization was applied using the Min-Max scaling technique. By adjusting their ranges, we ensure that the neural network models can be trained more effectively, avoiding any undue influence from variables with larger scales.

5) Input Sequences Preparation

For the construction of neural network models, specifically defined input sequences are of utmost importance. In this research, the input sequences for both the training and testing phases were formulated using identical variables, ensuring consistency in data structure and integrity across the entire study. The key variables incorporated encompass:

- Forward Price: The predetermined price at which the underlying asset can be bought or sold.
- Strike Price: The set price at which an option can be exercised.
- Time to Maturity: The time period left until the option expires.
- Risk-Free Rate: The rate of return of a hypothetical investment with no risk of financial loss.
- Option Type: This categorizes the data into call or put options, representing the right to buy or sell, respectively.

Ensuring uniformity in the input sequence variables between the training and testing datasets guarantees that the model is evaluated under the same conditions it was trained on. This uniformity is vital for the accurate assessment of model performance and robustness.

6) Output Sequences Preparation

Concomitant with each input sequence, output sequences represent the corresponding option prices. By juxtaposing the input variables with the prevailing market conditions, these option prices are deduced.

B. Traditional Models

1) Black-Scholes Model (BS)

The Black-Scholes formula determines the value of a European option based on the price of the underlying asset, its volatility, the risk-free rate, the strike price, and the time remaining until the option expires [18]. Operating on the underlying assumptions of static volatility and a log-normal distribution of asset returns, this model serves as the starting point for a plethora of advanced derivative pricing techniques. Recognizing the diverse nature of option contracts, the model was adapted to cater to different term-based subsets, each catering to its unique characteristics.

The subsequent enhancement of the BS model led to the emergence of the Black-Scholes-Merton (BSM) model, which factors in dividends. Accounting for dividends becomes essential when pricing options on dividend-paying assets, as they can significantly influence the option's value. The mathematical representation of the BSM model for European call and put options that factor in dividends is:

For a European Call Option:

$$C = SN(d_1) - Ke^{-rt}N(d_2) \tag{1.1}$$

For a European Put Option:

$$P = Ke^{-rt}N(-d_2) - SN(-d_1) \tag{1.2}$$

Where:

$$d1 = \frac{ln_K^S + \left(r + \frac{\sigma_v^2}{2}\right)t}{\sigma s\sqrt{t}} \tag{1.3}$$

$$d_2 = d_1 - \sigma_s\sqrt{t} \tag{1.4}$$

2) Binomial Tree Model

The Binomial Tree Model constitutes another classical technique extensively employed for option pricing. Operating by partitioning the time until expiration into discrete intervals, this model orchestrates the potential price trajectories of the underlying asset, culminating in a lattice or tree that portrays plausible outcomes. Its inherent flexibility equips it to adapt adeptly to the distinctive attributes of American options—particularly the provision for early exercise. This pivotal characteristic renders the Binomial Tree Model especially pertinent for the accurate valuation of such options.

A substantial enhancement to the previous binomial model lies in the meticulous parameterization of the binomial tree, with specific emphasis on the Cox-Ross-Rubinstein (CRR) model parameters. The principal modifications encompass:

- Precomputed Discount Factor: Departing from the practice of recalculating np.exp(-r * dt) within the iteration loop, this factor is precomputed as 'discount_factor'. This optimization minimizes redundant computations.
- CRR Model Parameters: The parameters u, d, and p are meticulously computed in alignment with the CRR model. This meticulous calculation approach contributes to a more precise and stable tree structure, particularly when dealing with larger numbers of time steps.
- Function Modularity: Introducing a new level of modularity, the number of time steps 'N' is passed as a default parameter within the 'generate_binomial_prices' function. This parameterization facilitates seamless adjustments and experimentation with varying tree depths, enhancing the model's adaptability to different contexts.

By incorporating these refinements, the Binomial Tree Model emerges as a more accurate and robust approach, well-equipped to address the intricate dynamics inherent in option pricing, especially within the domain of American options with their unique characteristics such as early exercise provisions.

C. Neural Network Implementations

The application of neural networks in predicting option prices entails using interconnected neurons structured into layers. This study predominantly focuses on RNN and LSTM networks, both specialized neural network architectures apt for sequential data processing.

1) Recurrent Neural Network (RNN)

RNNs are inherently designed to process sequential data, with their memory-rich structure retaining past information through interconnected cells. Each cell, upon receiving input, produces an output, creating a memory chain that captures temporal data relationships.

Mathematically, RNN operations are described as [9]:

$$h_t = \sigma_h(W_h x_t + U_h y_{t-1} + b_h) \tag{2.1}$$

$$y_t = \sigma_y(W_y h_t + b_y) \tag{2.2}$$

Variables and functions

x_t: input vector

h_t: hidden layer vector

y_t: output vector

W, U and b: parameter matrices and vector

σ_h and σ_y: Activation functions

The foundational RNN model in this study consisted of a SimpleRNN layer of 50 units, followed by a Dense layer. To accommodate more intricate temporal relationships, subsequent layers were added, enabling the model to capture deeper data nuances.

2) Long Short-Term Memory (LSTM)

LSTMs, an evolution of the RNN, are equipped to manage long-term data dependencies [13]. Their architecture, enriched with memory cells and specific gating mechanisms, can selectively process or ignore data based on its relevance.

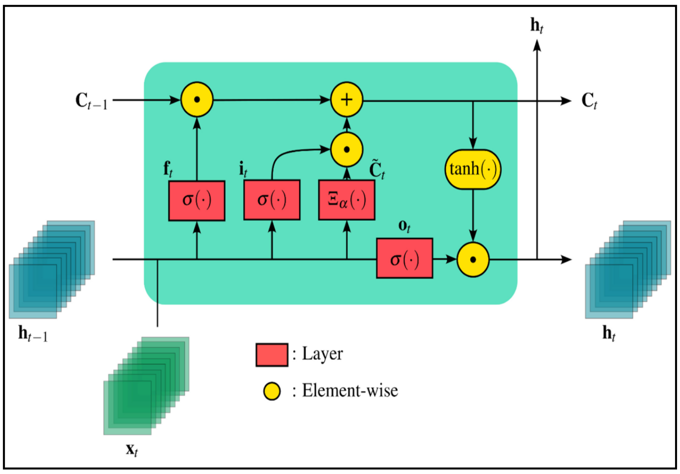

Fig. 1. Structure of the ConvLSTM cell. Adapted from Mavi et al. [15]

The underlying equations governing LSTM operations are:

$$f_t = \sigma_g(W_f x_t + U_f h_{t-1} + b_f) \tag{3.1}$$

$$i_t = \sigma_g(W_i x_t + U_i h_{t-1} + b_i) \tag{3.2}$$

$$o_t = \sigma_g(W_o x_t + U_o h_{t-1} + b_o) \tag{3.3}$$

$$\tilde{c}_t = \sigma_c(W_c x_t + U_c h_{t-1} + b_c) \tag{3.4}$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \tag{3.5}$$

where the initial values are $c_0 = 0$ and $h_0 = 0$ and the operator $\odot$ denotes the Hadamard product (element-wise product). The subscript t indexes the time step. These techniques were specifically tailored to address overfitting, enhance model complexity, and optimize convergence for improved accuracy:

- Enhanced Model Complexity: The intricacy of the LSTM units was heightened, resulting in a more intricate model architecture.
- Dropout Layers: Dropout layers were introduced to counteract the risk of overfitting by randomly deactivating neurons during training, promoting better generalization.
- Activation Function Adjustment: The "relu" activation function was replaced with kernel_regularizer=l2(0.01), aiding in regularization and addressing overfitting tendencies.
- Optimizer with Calibrated Learning Rate: A specific optimizer equipped with a carefully calibrated learning rate was defined to optimize network weights iteratively based on training data.
- Layer Augmentation: The model's complexity was augmented by increasing the initial layer size and introducing an additional LSTM layer, allowing for better capture of intricate patterns.
- Learning Rate Scheduler: A learning rate scheduler was employed, halving the learning rate every five epochs. This iterative reduction in learning rate refines training and convergence.
- Training and Validation Data Partitioning: To ensure robust model evaluation, the dataset was systematically partitioned into training and validation segments. Validation data, comprising 20% of the dataset, was held out for assessing model performance on unseen data during training.

D. Model Evaluation Metrics

To evaluate the capability of the traditional models, namely the BS and Binomial Tree models, against the deep learning architectures, RNN and LSTM in option pricing, a comprehensive array of evaluation metrics has been adopted. These metrics have been chosen to elucidate the accuracy, consistency, and predictive strengths of each model. This study employs the following evaluation benchmarks:

Mean Squared Error (MSE): The MSE measures the average squared differences between forecasted and actual option prices. This metric provides a comprehensive understanding of how closely the model's forecasts align with actual prices, demonstrating the model's pricing accuracy.

Root Mean Squared Error (RMSE): The square root of MSE, RMSE, provides information about the standard deviation of pricing errors. It allows for an in-depth look into the magnitude of the pricing deviations, indicating the typical extent by which predicted prices veer off from actual values.

R-squared (R2) Score: The R2 score encapsulates the fraction of variance in the observed option prices that is predictable from the model's forecasts. It oscillates between 0 and 1, with values tending towards 1 signifying a superior alignment between forecasted and observed prices.

An exhaustive examination of these metrics, applied to the traditional BS, Binomial Tree, RNN, and LSTM models, offers a clear perspective on their performance relative to one another. This multifaceted evaluation provides insights into the models' accuracy and predictive capacity, allowing for an informed determination of their effectiveness in capturing the intricate subtleties of the options market.

III. Comparative Analysis and Findings

In this section, I compare the pricing performance of the traditional model with the Neural Network models for both European and American options.

A. European Option Pricing Effects (SP500)

1) Call Option

a) BS model

Notably, the Mean Squared Error (MSE) for long-term options is considerably higher compared to short and mid-term options in both call and put options. This accentuates the idea that the accuracy of the Black-Scholes model diminishes as the option's time to expiration increases. The model's limitations stem from its reliance on simplifying assumptions, which become less valid over extended timeframes due to the myriad of market dynamics. Deviations from actual prices are anticipated in any model, but the pronounced MSE for long-term options underscores the need for more sophisticated approaches that consider real-world complexities.

b) LSTM model

For the call options, the loss of short and mid-term options decreases steadily across epochs which shows consistent improvement. But long-term call options, show a rapid reduction in loss during the early epochs, which could be due to the high initial errors. After increasing the complexity of the LSTM model, the performance of the short-term improved over the epochs as evident from the declining loss values. Without the learning rate scheduler, the model seems to achieve a slightly better validation loss in 10 epochs compared to the previous training (96.67 without the scheduler vs. 112.72 with the scheduler in the equivalent time frame). For mid-term, the loss values, both training and validation, are very high, in the range of 300,000+. This could indicate that the model is not fitting the data well. The learning rate does reduce after a certain point, but this reduction doesn't lead to any noticeable improvement. After removing the LearningRateScheduler and setting a batch size of 64 shows a similar pattern as before. For the long term, both training and validation loss consistently decrease across epochs, indicating that the model is learning and improving its predictions.

For the put options, all models show a rapid decline in loss in the initial epochs. This indicates that the models quickly adapt to the underlying patterns in the data. As observed previously, the absolute values of the loss are large, particularly for the long-term model. However, as mentioned before, the decreasing trend is more significant than the absolute value. The models' training time per epoch varies, with the mid-term model taking the longest time due to more data points. After complexity enhancement, LSTM validation loss improves, indicating generalized learning.

c) RNN model

In the short-term model, the two-layered RNN architecture demonstrated superior performance over the single-layered RNN, as evidenced by the lower loss, reflecting improved learning capabilities. Similarly, in the mid-term model, the two-layered RNN exhibited a more significant reduction in loss compared to its single-layered counterpart. For the long-term model, the two-layered RNN showcased slightly better performance despite an initially higher loss. Notably, by the 10th epoch, it achieved a lower loss value compared to the single-layered model, indicating its potential for enhanced learning over time.

B. American Option Pricing Effects (Alphabet)

1) Binomial Tree model

In the short term, notable improvements in R-squared values, approaching the value of 1, strongly suggest an exceptional fit of the model to the data. The corresponding MSE and RMSE values (13-14) further underscore the model's competence in accurately capturing underlying dynamics. Comparatively higher MSE and RMSE values in the mid-term point towards escalated pricing deviations. Nonetheless, the R-squared values surpassing 0.90 affirm the model's adeptness in explaining a substantial portion of the variance, signifying its robust capability to comprehend significant data trends and relationships.

In the long term, a distinct pattern emerges. The identification of an optimal N value of 300, resulting in minimized MSE, becomes evident. While the results indicate a trend of improved accuracy with larger N values, the observed differences in the outcomes are relatively marginal. It's important to acknowledge that this improvement comes at the expense of significantly higher computational costs, owing to the increased granularity of intervals. While slight variations exist among different N values, their negligible influence is evidenced by the R-squared values converging near 0.9961. This convergence highlights the model's remarkable alignment with the dataset, further reinforcing its reliability in capturing the complex nature of option pricing in extended timeframes.

Within the context of these observations, it remains imperative to deeply consider the inherent characteristics of the data. The Binomial Tree Model's unique ability to accommodate discrete intervals and simulate potential asset price trajectories renders it especially suited for scenarios where continuous-time dynamics may not holistically capture market behavior. By segmenting time into discrete steps, the model successfully mirrors the practical concept of how option prices evolve and how traders make informed decisions. Consequently, the Binomial Tree Model delivers a harmonious and intuitive representation of the intricacies inherent in real-world financial complexities.

2) LSTM model

a) Short Term

The consistent decline in both training and validation losses signifies successful learning without overfitting or underfitting concerns. The narrow gap between these losses indicates the effectiveness of the learning rate scheduler, which gradually reduces the learning rate for more refined convergence. While the enhanced LSTM model demonstrates considerable reduction in losses from the initial to final epoch, it's imperative to assess its generalization capability on a separate test dataset.

b) Mid Term

The LearningRateScheduler aids in stable convergence by modulating the learning rate during training. However, in this context, a model trained without this scheduler yields lower validation loss. This suggests that, for this specific task, a constant learning rate might be more suitable. While the absence of the LearningRateScheduler leads to faster epoch completion times, the model maintains parallel decreases in training and validation losses, indicating robust training. While the current run shows improved validation loss without the LearningRateScheduler, it's prudent to conduct multiple runs and average results to determine the optimal approach conclusively.

c) Long Term

Training time per epoch remains consistent at approximately 11ms/step, matching the earlier mid-term training pattern. Omitting the LearningRateScheduler reduces time to 7ms/step due to callback elimination. Although the LearningRateScheduler's impact is overshadowed, its role in aiding convergence remains evident. Despite the reduced learning rate, validation loss decreases consistently by Epoch 6, demonstrating the beneficial effect of learning rate adjustment. A minimal gap between training and validation losses indicates the absence of overfitting. For long-term put option data, the LSTM model performs better than the learning rate scheduler. Optimal hyperparameters like batch size (increased to 64) facilitate quicker convergence without sacrificing loss reduction. Vigilance against overfitting remains vital with continued training.

d) Compare Three Terms

LSTMs excel in sequence prediction, fitting time series in financial markets. Data characteristics vary across short, mid, and long terms. Short-term volatility makes prediction challenging due to transient market influences. Mid-term and long-term data showcase clearer trends with some mid-term volatility. Long-term LSTM outperforms mid-term, but thorough testing on unseen data is crucial for real-world prediction assessment. Learning rate tuning reduces overfitting risk by refining weight updates progressively, contingent on run results.

3) RNN model

By enhancing the RNN model with additional layers, including an extra SimpleRNN layer, its complexity increases, enabling the capture of more intricate data patterns. However, it's important to note that heightened complexity may lead to overfitting, especially without proper regularization or with limited data.

a) Call Option

The consistent training time per step (~2ms) across durations underscores the RNN model's computational efficiency. Initial epoch loss values follow a descending pattern: highest for long-term, followed by mid-term, and then short-term. This trend aligns with inherent dataset scale differences. Despite the long-term model's initial higher loss, significant convergence and reduction are noteworthy. The mid-term model converges faster, while all three models exhibit declining loss across epochs, indicating effective learning. Although long-term dataset loss values are the highest, rapid reduction signifies effective long-term pattern capture. While RNNs are faster and effective for shorter-term dependencies, issues like vanishing gradient are possible for longer sequences.

b) Put Option

The descending loss trend across all three models suggests successful convergence and learning from training data. Varying rates of loss decline among models highlight the most substantial reduction in the long-term model. This variance could stem from data nature, option price distribution differences, or distinct term length predictability. Complex models (2 RNN layers) perform better, notably in training loss reduction. Their deeper architecture captures data patterns more effectively. The RNN model demonstrates consistent accuracy, followed closely by LSTM (except for long-term predictions). Simpler yet intuitive, the Binomial model trails in prediction accuracy. The challenges in LSTM's long-term prediction effectiveness might result from overfitting, insufficient long-term data, or inherent long-term price prediction complexities. The RNN emerges as the most consistent option price predictor across terms and types, adeptly capturing dataset patterns. Contrarily, the LSTM model, while excelling in short and mid-term pricing, falters in long-term option pricing due to a negative R2 value for long-term call options.

C. Compare European and American Option Pricing

TABLE II. MSE COMPARISON OF OPTION PRICING MODELS

Option Class / Call-Put		**Term-Based Classification**		
		Short Term	***Mid Term***	***Long Term***
S&P 500 Call	BS	161.21	2349.85	80501.91
	LSTM	115.32	135.43	32539.36
	RNN	60.30	96.30	1783.39
S&P 500 Put	BS	165.82	3333.23	101716.19
	LSTM	67.16	106.38	411.22
	RNN	47.79	64.03	272.80
Alphabet Call	Binomail	185.70	1841.01	8672.05
	LSTM	43.21	1091.34	185178.45
	RNN	30.62	41.61	89.88
Alphabet Put	Binomail	173.11	1801.04	8431.24
	LSTM	36.61	54.14	17893.37
	RNN	27.31	37.69	105.40

TABLE III. RMSE COMPARISON OF OPTION PRICING MODELS

Option Class / Call-Put		**Term-Based Classification**		
		Short Term	***Mid Term***	***Long Term***
S&P 500 Call	BS	12.70	48.48	283.73
	LSTM	10.74	11.64	180.39
	RNN	7.77	9.81	42.23
S&P 500 Put	BS	12.88	57.73	318.93
	LSTM	8.20	10.31	20.28
	RNN	6.91	8.00	16.52
Alphabet Call	Binomail	13.63	42.91	93.12
	LSTM	6.57	33.04	430.32
	RNN	5.53	6.45	9.48
Alphabet Put	Binomail	13.16	42.44	91.82
	LSTM	6.05	7.36	133.77
	RNN	4.86	5.87	10.55

TABLE IV. R2 COMPARISON OF OPTION PRICING MODELS

Option Class / Call-Put		Term-Based Classification		
		Short Term	*Mid Term*	*Long Term*
S&P 500 Call	BS	0.9988	0.9923	0.8487
	LSTM	0.9991	0.9996	0.9388
	RNN	0.9996	0.9997	0.9966
S&P 500 Put	BS	0.9944	0.8956	-0.3468
	LSTM	0.9977	0.9967	0.9946
	RNN	0.9984	0.9980	0.9964
Alphabet Call	Binomail	0.9961	0.9853	0.9506
	LSTM	0.9989	0.9994	0.9948
	RNN	0.9993	0.9997	0.9995
Alphabet Put	Binomail	0.9900	0.9334	0.8459
	LSTM	0.9976	0.9974	0.9988
	RNN	0.9986	0.9988	0.9983

The efficacy of pricing models when applied to European and American options requires a comprehensive evaluation using MSE, RMSE, and R2 metrics. The values are displayed in Tables II, III, and IV. Each metric provides unique insights into the predictive ability and dependability of the models.

a) For S&P 500 Call options:

Across all durations, the RNN model consistently outperforms both the Black-Scholes (BS) and LSTM models in terms of MSE. The BS model shows limited efficiency, especially for long-term options. This indicates that the RNN model is better at capturing the complexities of both European and American call options, making it a more reliable choice for this type of data. RNN's advantage is further highlighted with RMSE values (Short: 7.77, Mid: 9.81, Long: 42.23) being notably below its counterparts. RNN achieves the highest R2 scores across all durations, emphasizing its superior predictive ability.

b) For S&P 500 Put options:

For S&P 500 Put options: RNN again exhibits the lowest MSE, highlighting its superior performance. LSTM performs better than BS for short and long-term options, but BS shows a slightly better result in the mid-term. While the negative R2 value of BS in the long term raises severe concerns about its fit.

c) For Alphabet Call options:

The RNN model vastly outperforms the other models in the short and mid-term. However, the LSTM model struggles significantly in the long term.

d) For Alphabet Put options:

The trend of RNN superiority continues, with the model showing the lowest MSE across all durations. Binomial and LSTM show competitive results, with LSTM performing slightly better in the mid-term. The consistency of the RNN model is further validated by its low RMSE scores.

Overall, considering the integrated analysis using MSE, RMSE, and R2 metrics, the RNN model emerges as the standout choice for both European (S&P 500) and American (Alphabet) options. Its consistent performance across different data sets, option types, and evaluation metrics underscores its robustness and reliability in the realm of option pricing tasks.

IV. DISCUSSION AND INTERPRETATION OF RESULTS

A. Comparison of Traditional and Neural Network Models

a) Traditional Models:

The traditional models, such as the Black-Scholes for European options and the Binomial model for American options, have been widely used in the finance industry for decades. They have specific assumptions, such as constant volatility and interest rates, that might not always hold in real-world markets. The findings indicate that these models can sometimes produce larger mean squared errors (MSE) compared to certain neural network models, especially over longer durations.

b) Neural Network Models:

Neural networks, including LSTM and RNN, are data-driven models. They learn patterns from historical data and can adjust to changing market conditions more flexibly than traditional models. The results from our study show that RNNs consistently outperformed both LSTM and traditional models across multiple durations and option types. LSTMs also showed potential, especially in specific durations and option types, but struggled in others, like long-term Alphabet calls.

B. Implications for Financial Derivatives Valuation

Given the superiority of RNNs in certain scenarios, financial analysts and traders might need to rethink their reliance solely on traditional models. Incorporating neural networks can provide more accurate option pricing, especially in volatile or less predictable market conditions.

C. Limitations

This study focuses on two specific markets (S&P 500 and Alphabet) and might not be generalizable to other markets or financial instruments. Neural networks require large amounts of data for training, making them less suitable for newer markets or instruments with limited historical data. Overfitting is a potential concern with neural networks. A model that performs exceptionally well on a specific dataset might not necessarily do so on another.

D. Future Research Directions

Investigate the performance of neural networks in other financial markets and for other types of financial derivatives. Explore the potential of other types of neural networks or machine learning models, such as convolutional neural networks or transformer architectures, in option pricing. Dive deeper into the reasons behind the underperformance of traditional models and the LSTM model in specific scenarios to improve their accuracy and reliability. Evaluate the economic implications of transitioning from traditional models to neural networks in financial markets. This includes impacts on market efficiency, liquidity, and the overall stability of financial systems.

V. CONCLUSION

The intricate dynamics of financial markets necessitate the application of robust and adaptive models, especially for the complex task of option pricing. In our detailed comparative analysis of different models for European (S&P 500) and American (Alphabet) option pricing, a few pivotal observations come to the fore.

A. *Model Dependence on Time Duration*

The Black-Scholes model, although celebrated for its foundational contribution, exhibited reduced accuracy, especially for long-term European options. This limitation resonates with its inherent simplifying assumptions, which appear less reliable over extended timeframes. On the other hand, LSTM and RNN models demonstrated their potential, adjusting adeptly to market dynamics and capturing data nuances. Notably, the LSTM model encountered challenges in the long-term American call options, manifesting a high MSE and a negative R2 value.

B. *Neural Network Models' Dynamics*

In the clash of neural networks, the RNN model emerged triumphant. Across different durations and both option types, the RNN consistently outshined its LSTM counterpart. Its rapid convergence and lowered MSE, especially for the S&P 500 options, highlight its superiority in grasping intricate pricing patterns. LSTM's strength was more noticeable in short and mid-term scenarios, though its long-term limitations, especially in American call options, are an area of concern.

C. *Binomial Tree Model's Unique Strengths*

The Binomial model, designed for American options, showcased a distinctive strength in capturing the granular nature of option pricing, especially for long durations. With R-squared values nearing 1 and minimized MSE at optimal N values, it underlined its efficacy in simulating real-world financial dynamics, especially with its innate ability to accommodate discrete intervals.

D. *Contributions to the Option Pricing Field:*

The RNN model consistently outperformed other models across various metrics and datasets, highlighting its potential as a transformative tool in option pricing. The recurrent nature of RNNs, capturing sequential data dependencies, manifests as a pivotal advantage in this domain. LSTM models showcased particular strength in short and mid-term pricing, hinting at the model's adeptness at understanding more immediate financial data sequences. This study underscores a potential limitation of the Black-Scholes model, especially with long-term options, urging the industry to reconsider its overarching reliance on this traditional method.

E. *Critical Discussion Summary*

The data's temporal nature played a pivotal role in model performance, with certain models excelling in short-term predictions and faltering in the long term. The significant challenge faced by LSTM in predicting long-term prices of Alphabet Call options, indicated by a negative R2 value, raised pertinent questions about its robustness for extended datasets.

In conclusion, while traditional models like Black-Scholes have cemented their relevance through years of industry reliance, the dynamic nature of financial markets combined with advancements in computational techniques prompts a pressing need for innovation. While each model presented unique strengths and challenges, the RNN model was the most consistent performer. It displayed a formidable capability to navigate the nuances of both European and American options across various durations. Its reliability and robustness mark it as a particularly promising tool for option pricing tasks in modern financial markets. However, it's essential to remain vigilant against potential pitfalls, such as overfitting, especially in models with heightened complexity. As financial markets evolve, continuous model evaluation and optimization will remain paramount to ensure the accuracy and relevance of pricing predictions. My findings advocate for an augmented approach, integrating the strengths of Neural Networks into the landscape of option pricing. As with any empirical study, future research should aim to validate these findings across broader datasets and continually evolving market dynamics.

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APPENDIX

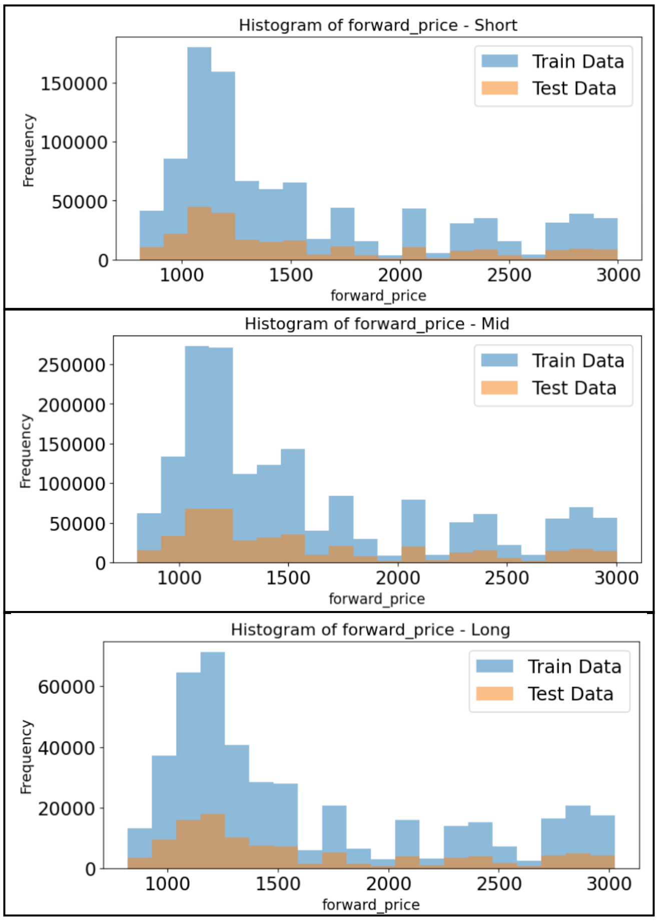

Fig. 2. Distribution of "Forward Price" Across Terms of Alphebat

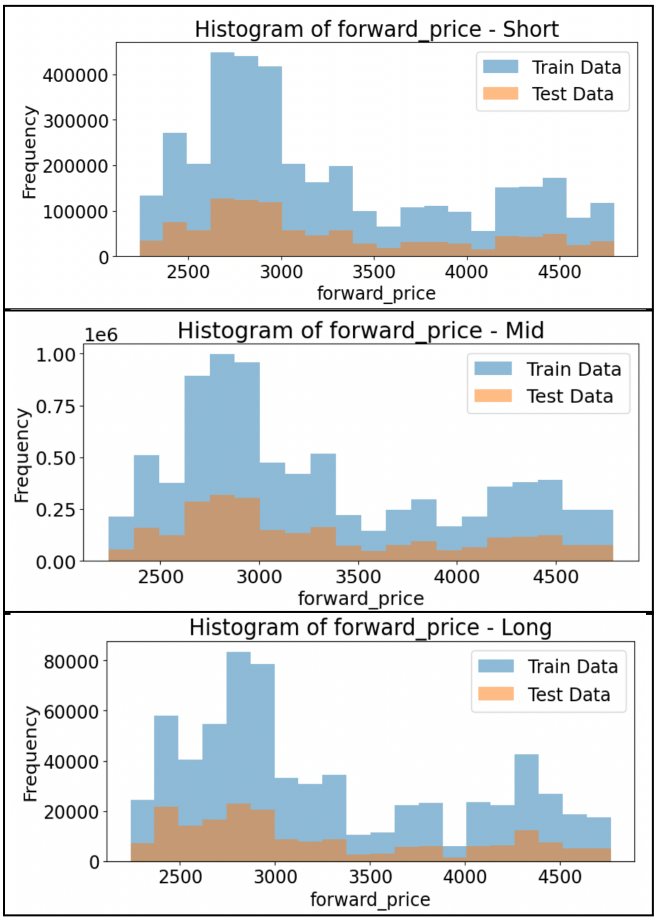

Fig. 3. Distribution of "Forward Price" Across Terms of Alphebat