4.7.3

$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma_k^2}(x-\mu_k)^2}}{\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma_l^2}(x-\mu_l)^2}}$$

to find $p_k(x)$ is largest, we can only consider numerator.

$$\log\left[\pi_k \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma_k^2}(x-\mu_k)^2}\right]$$

Also, as the log function is monotonally increasing, it's equivalent to find k for which

$$\log(\pi_k) - \cancel{\log(\sqrt{2\pi}\,\sigma)} - \left(\frac{1}{2\sigma_k^2}\right)(x-\mu_k)^2$$

equivalent $\equiv$ $\log(\pi_k) - \left(\frac{1}{2\sigma_k^2}\right)(x-\mu_k)^2$

$$= \log \pi_k - \frac{1}{2\sigma_k^2} x^2 + \frac{\mu_k}{\sigma_k^2} x - \frac{\mu_k^2}{2\sigma_k^2}$$

for this expession, we can't remove $-\frac{1}{2\sigma_k^2} x^2$ because we no longer assume $\sigma_1 = \sigma_2 = \cdots = \sigma_k$

therefore, the Bayes' classfier has quadratic term

4.7.7

$P(Yes) = 0.8 \quad P(No) = 1 - 0.8 = 0.2$

$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$

$P(Yes \mid x=4)$

$$= \frac{P(x=4 \mid Yes) \cdot P(Yes)}{P(x=4 \mid Yes) \cdot P(Yes) + P(x=4 \mid No) \cdot P(No)}$$

$$= \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(4-10)^2}{2 \cdot 36^2}} \times 0.8}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(4-10)^2}{2 \cdot 36^2}} \times 0.8 + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(4-0)^2}{2 \cdot 36^2}} \times 0.2}$$

$$= \frac{\exp\left(-\frac{1}{2 \cdot 36^2}(4-10)^2\right) \cdot 0.8}{\exp\left(-\frac{1}{2 \cdot 36^2}(4-10)^2\right) \times 0.8 + \exp\left(-\frac{1}{2 \cdot 36^2}(4-0)^2\right) \times 0.2}$$

$\approx \quad 0.752$