

equation for r_1 :

$$\frac{GM_1}{(R-r_1)^2} - \frac{GM_2}{r_1^2} = \left(\frac{M_1}{M_1+M_2} R - r_1 \right) \frac{G(M_1+M_2)}{R^3}$$

Replace $r_1 = x_1 R$

$$\frac{GM_1}{(R-x_1 R)^2} - \frac{GM_2}{x_1^2 R^2} = \left(\frac{M_1}{M_1+M_2} R - x_1 R \right) \frac{G(M_1+M_2)}{R^3}$$

$$\frac{M_1}{(R^2(1-x_1)^2)} - \frac{M_2}{R^2 x_1^2} = \left(\frac{M_1}{M_1+M_2} \cancel{R} \cdot \frac{\cancel{R} \cdot (M_1+M_2)}{R^{2+2}} - \frac{x_1 \cancel{R} (M_1+M_2)}{R^{3+1}} \right)$$

$$\frac{1}{R^2} \left[\frac{M_1}{(1-x_1)^2} - \frac{M_2}{x_1^2} \right] = \frac{1}{R^2} \left[M_1 - x_1 (M_1+M_2) \right]$$

Replace M_1 and M_2 : $\mu = \frac{M_2}{(M_1+M_2)}$ $\mu(M_1+M_2) = M_2$
 $(1-\mu)(M_1+M_2) = M_1$

$$\frac{(1-\mu)(\cancel{M_1+M_2})}{(1-x_1)^2} - \frac{\mu(\cancel{M_1+M_2})}{x_1^2} = (1-\mu)(\cancel{M_1+M_2}) - x_1(\cancel{M_1+M_2})$$

$$\frac{(1-\mu)}{(1-x_1)^2} - \frac{\mu}{x_1^2} = (1-\mu) - x_1$$

move to one side for computational purposes

$$\frac{(1-\mu)}{(1-x_1)^2} - \frac{\mu}{x_1^2} - 1 + \mu + x_1 = 0$$

equation for r_2 :

$$\frac{GM_1}{(R+r_2)^2} + \frac{GM_2}{r_2^2} = \left(\frac{M_1}{M_1+M_2} R + r_2 \right) \frac{G(M_1+M_2)}{R^3}$$

Replace $r_2 = x_2 R$

$$\frac{M_1}{(R+x_2 R)^2} + \frac{M_2}{x_2^2 R^2} = \left(\frac{M_1}{M_1+M_2} R + x_2 R \right) \cdot \frac{M_1+M_2}{R^3}$$

$$\frac{M_1}{R^2(1+x_2)^2} + \frac{M_2}{x_2^2 R^2} = \left(\frac{M_1(\cancel{M_1+M_2})}{\cancel{M_1+M_2}} \cdot \frac{\cancel{R}}{R^{2+2}} + \frac{x_2 \cancel{R} (M_1+M_2)}{R^{3+1}} \right)$$

Replace M_1 and M_2 : $\mu = \frac{M_2}{(M_1+M_2)}$ $\mu(M_1+M_2) = M_2$
 $(1-\mu)(M_1+M_2) = M_1$

$$\frac{(1-\mu)(\cancel{M_1+M_2})}{(1+x_2)^2} + \frac{\mu(\cancel{M_1+M_2})}{x_2^2} = (1-\mu)(\cancel{M_1+M_2}) + x_2(\cancel{M_1+M_2})$$

$$\frac{(1-\mu)}{(1+x_2)^2} + \frac{\mu}{x_2^2} = (1-\mu) + x_2 \longrightarrow \frac{(1-\mu)}{(1+x_2)^2} + \frac{\mu}{x_2^2} - x_2 - 1 + \mu = 0$$

equation for r_3 :

$$\frac{6M_1}{(R-r_3)^2} + \frac{6M_2}{(2R-r_3)^2} = \left(\frac{M_2}{M_1+M_2} R + R - r_3 \right) \frac{6(M_1+M_2)}{R^3}$$

Replace $r_3 = x_3 R$

$$\frac{M_1}{(R-x_3R)^2} + \frac{M_2}{(2R-x_3R)^2} = \left(\frac{M_2}{M_1+M_2} R + R - x_3R \right) \left(\frac{M_1+M_2}{R^2} \right)$$

$$\frac{M_1}{(R^2(1-x_3)^2)} + \frac{M_2}{R^2(2-x_3)^2} = \cancel{R} \left(\frac{M_2}{M_1+M_2} + 1 - x_3 \right) \left(\frac{M_1+M_2}{\cancel{R^2}} \right)$$

$$\frac{M_1}{(1-x_3)^2} + \frac{M_2}{(2-x_3)^2} = \left(\frac{M_2}{\cancel{M_1+M_2}} (\cancel{M_1+M_2}) + (M_1+M_2) - x_3(M_1+M_2) \right)$$

$$\text{Replace } M_1, M_2 : \mu = \frac{M_2}{(M_1+M_2)} \quad \mu(M_1+M_2) = M_2$$

$$(1-\mu)(M_1+M_2) = M_1$$

$$\frac{(1-\mu)(\cancel{M_1+M_2})}{(1-x_3)^2} + \frac{\mu(\cancel{M_1+M_2})}{(2-x_3)^2} = \mu(\cancel{M_1+M_2}) + (\cancel{M_1+M_2}) - x_3(\cancel{M_1+M_2})$$

$$\frac{1-\mu}{(1-x_3)^2} + \frac{\mu}{(2-x_3)^2} = \mu + 1 - x_3$$

$$\frac{1-\mu}{1-x_3^2} + \frac{\mu}{(2-x_3)^2} + x_3 - \mu - 1 = 0$$