AMATH 569 Homework Assignment Spring 2023

Assigned: April 19, 2023
Due: April 26, 2023

1. Consider the wave equation:

$$c^2 u_{xx} - u_{tt} = 0,$$

which is a special case of the general quasi-linear equation:

$$au_{xx} + 2bu_{xy} + cu_{yy} = f, \text{ with } y = t.$$

Find the slope of each of the two characteristics:

$$\frac{dy}{dx} = -z_1 \text{ along } \alpha = \text{constant characteristic, and}$$

$$\frac{dy}{dx} = -z_2 \text{ along } \beta = \text{constant characteristic.}$$

Find the expression in terms of x and t for α and β, so that the wave equation simplifies to

$$u_{\alpha\beta} = 0$$

2. Use the Fourier transform method to solve the 2-D Laplace equation in the upper plane for the bounded solution:

$$\nabla^2 u = 0 \text{ in } y > 0, \quad -\infty < x < \infty.$$

$$u(x,0) = f(x), \quad -\infty < x < \infty.$$

Assume $f(x)$ is of compact support; $u(x,y) \to 0$ as $x \to \pm\infty$.

3. Solve the following problem in two ways:

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u, \quad 0 < t; 0 < x < \infty$$

$$u(x,0) = 0, \; u(x,t) \text{ bounded as } x \to \infty.$$

$$u(0,t) = T_0, \text{ a constant, } t > 0.$$

(a) by the method of similarity transformation. Look for the value of α such that the PDE reduces to an ode in η, $\eta = x / t^{\alpha}$;
(b) by an integral transform in t, in this case a Laplace transform (You can use a table of Laplace transform to do the inverse transform).