

AMATH 569 Homework Assignment #1 Spring 2023

Assigned: Wednesday, April 5, 2023

Due: Wednesday, April 12, 2023

1. Solve the PDE:

$$\frac{\partial}{\partial t}u + u \frac{\partial}{\partial x}u = 0, \quad -\infty < x < \infty, \quad t > 0,$$

subject to the initial condition:

$$u(x, 0) = u_0(x) = \begin{cases} -1, & -\infty < x \leq -a, \quad a > 0. \\ \frac{x}{a}, & -a < x < a \\ 1, & a \leq x < \infty \end{cases}$$

2. Consider the initial value problem in infinite domain:

$$\frac{\partial}{\partial t}u + u \frac{\partial}{\partial x}u = 0$$

$$u(x, 0) = u_0(x), \text{ where } u_0(x) = \begin{cases} 1, & x \leq 0 \\ 1-x, & 0 < x < 1 \\ 0, & x \geq 1. \end{cases}$$

- (a) Find where and when a shock first forms.
- (b) Solve the problem and sketch or plot the solution before when a shock first forms.
- (c) Find the shock speed using the Rankine-Hugoniot condition.
- (d) Solve the problem and sketch or plot the solution after the shock has formed.

3. Solve the PDE:

$$\frac{\partial}{\partial t}u + u \frac{\partial}{\partial x}u = u, \quad -\infty < x < \infty, \quad t > 0,$$

subject to the initial condition:

$$u(x, 0) = u_0(x) = 2x, \quad 0 \leq x \leq 2.$$

Where in the x-t plane is the solution valid?