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#%%
import numpy as np

def y(X):
    if X == 1:
        return -1
    elif X == 2:
        return 0
    elif X == 3:
        return 1

def mc_integral(T, num_samples):
    pi = np.array([1/3, 1/3, 1/3]) # stationary distribution
    X_t = np.random.choice(3, num_samples, p=pi)+1 # initialize X_t
    with stationary distribution
        integral_sum = 0
        for t in range(num_samples):
            X_tau = np.random.choice(3, num_samples, p=pi)+1 # sample
            X_tau from stationary distribution
            integral_sum += y(X_t[t]) * y(X_tau[t])
        return integral_sum / num_samples

T = 10000
num_samples = 10000
result = mc_integral(T, num_samples)
print("The Monte Carlo estimate of the integral is:", result)

# %%

```