Fração continuada dos números naturais

Fração continuada de primos

In[95]:=
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For[i = 0, i < 20, i++;
 Print["n=", i, "; pi(n)=", PrimePi[i], "; p_n=",
  Prime[i], "; pi(p_n)=", PrimePi[Prime[i]], "; ", Sqrt[Prime[i]], "=",
  ContinuedFraction[Sqrt[Prime[i]], 15], ";"]]
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n=1; pi(n)=0; p_n=2; pi(p_n)=1; $\sqrt{2}$ ={1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2};

n=2; pi(n)=1; p_n=3; pi(p_n)=2; $\sqrt{3}$ ={1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2};

n=3; pi(n)=2; p_n=5; pi(p_n)=3; $\sqrt{5}$ ={2, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4};

n=4; pi(n)=2; p_n=7; pi(p_n)=4; $\sqrt{7}$ ={2, 1, 1, 1, 4, 1, 1, 1, 4, 1, 1, 1, 4, 1, 1};

n=5; pi(n)=3; p_n=11; pi(p_n)=5; $\sqrt{11}$ ={3, 3, 6, 3, 6, 3, 6, 3, 6, 3, 6, 3, 6, 3, 6};

n=6; pi(n)=3; p_n=13; pi(p_n)=6; $\sqrt{13}$ ={3, 1, 1, 1, 1, 6, 1, 1, 1, 1, 6, 1, 1, 1, 1};

n=7; pi(n)=4; p_n=17; pi(p_n)=7; $\sqrt{17}$ ={4, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8};

n=8; pi(n)=4; p_n=19; pi(p_n)=8; $\sqrt{19}$ ={4, 2, 1, 3, 1, 2, 8, 2, 1, 3, 1, 2, 8, 2, 1};

n=9; pi(n)=4; p_n=23; pi(p_n)=9; $\sqrt{23}$ ={4, 1, 3, 1, 8, 1, 3, 1, 8, 1, 3, 1, 8, 1, 3};

n=10; pi(n)=4; p_n=29; pi(p_n)=10; $\sqrt{29}$ ={5, 2, 1, 1, 2, 10, 2, 1, 1, 2, 10, 2, 1, 1, 2};

n=11; pi(n)=5; p_n=31; pi(p_n)=11; $\sqrt{31}$ ={5, 1, 1, 3, 5, 3, 1, 1, 10, 1, 1, 3, 5, 3, 1};

n=12; pi(n)=5; p_n=37; pi(p_n)=12; $\sqrt{37}$ ={6, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12};

n=13; pi(n)=6; p_n=41; pi(p_n)=13; $\sqrt{41}$ ={6, 2, 2, 12, 2, 2, 12, 2, 2, 12, 2, 2, 12, 2, 2};

n=14; pi(n)=6; p_n=43; pi(p_n)=14; $\sqrt{43}$ ={6, 1, 1, 3, 1, 5, 1, 3, 1, 1, 12, 1, 1, 3, 1};

n=15; pi(n)=6; p_n=47; pi(p_n)=15; $\sqrt{47}$ ={6, 1, 5, 1, 12, 1, 5, 1, 12, 1, 5, 1, 12, 1, 5};

n=16; pi(n)=6; p_n=53; pi(p_n)=16; $\sqrt{53}$ ={7, 3, 1, 1, 3, 14, 3, 1, 1, 3, 14, 3, 1, 1, 3};

n=17; pi(n)=7; p_n=59; pi(p_n)=17; $\sqrt{59}$ ={7, 1, 2, 7, 2, 1, 14, 1, 2, 7, 2, 1, 14, 1, 2};

n=18; pi(n)=7; p_n=61; pi(p_n)=18; $\sqrt{61}$ ={7, 1, 4, 3, 1, 2, 2, 1, 3, 4, 1, 14, 1, 4, 3};

n=19; pi(n)=8; p_n=67; pi(p_n)=19; $\sqrt{67}$ ={8, 5, 2, 1, 1, 7, 1, 1, 2, 5, 16, 5, 2, 1, 1};

n=20; pi(n)=8; p_n=71; pi(p_n)=20; $\sqrt{71}$ ={8, 2, 2, 1, 7, 1, 2, 2, 16, 2, 2, 1, 7, 1, 2};

Fração continuada de naturais não-primos não-quadrados-perfeitos

In[96]:=
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For[i = 4, i ≤ Prime[20], i++;
 If[¬ PrimeQ[i] ⋀ Sqrt[i] != Floor[Sqrt[i]], Print["n=", i, "; pi(n)=",
   PrimePi[i], "; ", Sqrt[i], "=", ContinuedFraction[Sqrt[i], 15]], False]]
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n=6; pi(n)=3; $\sqrt{6}$ ={2, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4}

n=8; pi(n)=4; $2\sqrt{2}$ ={2, 1, 4, 1, 4, 1, 4, 1, 4, 1, 4, 1, 4, 1, 4}

n=10; pi(n)=4; $\sqrt{10}$ ={3, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6}

n=12; pi(n)=5; $2\sqrt{3}$ ={3, 2, 6, 2, 6, 2, 6, 2, 6, 2, 6, 2, 6, 2, 6}

n=14; pi(n)=6; $\sqrt{14}$ ={3, 1, 2, 1, 6, 1, 2, 1, 6, 1, 2, 1, 6, 1, 2}

n=15; pi(n)=6; $\sqrt{15}$ ={3, 1, 6, 1, 6, 1, 6, 1, 6, 1, 6, 1, 6, 1, 6}

n=18; pi(n)=7; $3\sqrt{2}$ ={4, 4, 8, 4, 8, 4, 8, 4, 8, 4, 8, 4, 8, 4, 8}

n=20; pi(n)=8; $2\sqrt{5}$ ={4, 2, 8, 2, 8, 2, 8, 2, 8, 2, 8, 2, 8, 2, 8}

n=21; pi(n)=8; $\sqrt{21}$ ={4, 1, 1, 2, 1, 1, 8, 1, 1, 2, 1, 1, 8, 1, 1}

n=22; pi(n)=8; $\sqrt{22}$ ={4, 1, 2, 4, 2, 1, 8, 1, 2, 4, 2, 1, 8, 1, 2}

n=24; pi(n)=9; $2\sqrt{6}$ ={4, 1, 8, 1, 8, 1, 8, 1, 8, 1, 8, 1, 8, 1, 8}

n=26; pi(n)=9; $\sqrt{26}$ ={5, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10}

n=27; pi(n)=9; $3\sqrt{3}$ ={5, 5, 10, 5, 10, 5, 10, 5, 10, 5, 10, 5, 10, 5, 10}

n=28; pi(n)=9; $2\sqrt{7}$ ={5, 3, 2, 3, 10, 3, 2, 3, 10, 3, 2, 3, 10, 3, 2}

n=30; pi(n)=10; $\sqrt{30}$ ={5, 2, 10, 2, 10, 2, 10, 2, 10, 2, 10, 2, 10, 2, 10}

n=32; pi(n)=11; $4\sqrt{2}$ ={5, 1, 1, 1, 10, 1, 1, 1, 10, 1, 1, 1, 10, 1, 1}

n=33; pi(n)=11; $\sqrt{33}$ ={5, 1, 2, 1, 10, 1, 2, 1, 10, 1, 2, 1, 10, 1, 2}

n=34; pi(n)=11; $\sqrt{34}$ ={5, 1, 4, 1, 10, 1, 4, 1, 10, 1, 4, 1, 10, 1, 4}

n=35; pi(n)=11; $\sqrt{35}$ ={5, 1, 10, 1, 10, 1, 10, 1, 10, 1, 10, 1, 10, 1, 10}

n=38; pi(n)=12; $\sqrt{38}$ ={6, 6, 12, 6, 12, 6, 12, 6, 12, 6, 12, 6, 12, 6, 12}

n=39; pi(n)=12; $\sqrt{39}$ ={6, 4, 12, 4, 12, 4, 12, 4, 12, 4, 12, 4, 12, 4, 12}

n=40; pi(n)=12; $2\sqrt{10}$ ={6, 3, 12, 3, 12, 3, 12, 3, 12, 3, 12, 3, 12, 3, 12}

n=42; pi(n)=13; $\sqrt{42}$ ={6, 2, 12, 2, 12, 2, 12, 2, 12, 2, 12, 2, 12, 2, 12}

n=44; pi(n)=14; $2\sqrt{11}$ ={6, 1, 1, 1, 2, 1, 1, 1, 12, 1, 1, 1, 2, 1, 1}

n=45; pi(n)=14; $3\sqrt{5}$ ={6, 1, 2, 2, 2, 1, 12, 1, 2, 2, 2, 1, 12, 1, 2}

n=46; pi(n)=14; $\sqrt{46}$ ={6, 1, 3, 1, 1, 2, 6, 2, 1, 1, 3, 1, 12, 1, 3}

n=48; pi(n)=15; $4\sqrt{3}$ ={6, 1, 12, 1, 12, 1, 12, 1, 12, 1, 12, 1, 12, 1, 12}

n=50; pi(n)=15; $5\sqrt{2}$ ={7, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14}

n=51; pi(n)=15; $\sqrt{51}$ ={7, 7, 14, 7, 14, 7, 14, 7, 14, 7, 14, 7, 14, 7, 14}

n=52; pi(n)=15; $2\sqrt{13}$ ={7, 4, 1, 2, 1, 4, 14, 4, 1, 2, 1, 4, 14, 4, 1}

n=54; pi(n)=16; $3\sqrt{6}$ ={7, 2, 1, 6, 1, 2, 14, 2, 1, 6, 1, 2, 14, 2, 1}

n=55; pi(n)=16; $\sqrt{55}$ ={7, 2, 2, 2, 14, 2, 2, 2, 14, 2, 2, 2, 14, 2, 2}

n=56; pi(n)=16; $2\sqrt{14}$ ={7, 2, 14, 2, 14, 2, 14, 2, 14, 2, 14, 2, 14, 2, 14}

n=57; pi(n)=16; $\sqrt{57}$ ={7, 1, 1, 4, 1, 1, 14, 1, 1, 4, 1, 1, 14, 1, 1}

n=58; pi(n)=16; $\sqrt{58}$ ={7, 1, 1, 1, 1, 1, 1, 14, 1, 1, 1, 1, 1, 1, 14}

n=60; pi(n)=17; $2\sqrt{15}$ ={7, 1, 2, 1, 14, 1, 2, 1, 14, 1, 2, 1, 14, 1, 2}

n=62; pi(n)=18; $\sqrt{62}$ ={7, 1, 6, 1, 14, 1, 6, 1, 14, 1, 6, 1, 14, 1, 6}

n=63; pi(n)=18; $3\sqrt{7}$ ={7, 1, 14, 1, 14, 1, 14, 1, 14, 1, 14, 1, 14, 1, 14}

n=65; pi(n)=18; $\sqrt{65}$ ={8, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16}

n=66; pi(n)=18; $\sqrt{66}$ ={8, 8, 16, 8, 16, 8, 16, 8, 16, 8, 16, 8, 16, 8, 16}

n=68; pi(n)=19; $2\sqrt{17}$ ={8, 4, 16, 4, 16, 4, 16, 4, 16, 4, 16, 4, 16, 4, 16}

n=69; pi(n)=19; $\sqrt{69}$ ={8, 3, 3, 1, 4, 1, 3, 3, 16, 3, 3, 1, 4, 1, 3}

n=70; pi(n)=19; $\sqrt{70}$ ={8, 2, 1, 2, 1, 2, 16, 2, 1, 2, 1, 2, 16, 2, 1}

n=72; pi(n)=20; $6\sqrt{2}$ ={8, 2, 16, 2, 16, 2, 16, 2, 16, 2, 16, 2, 16, 2, 16}

Fração continuada de quadrados perfeitos

In[97]:=
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For[i = 0, i < 20, i++;
 Print["n=", i, "; pi(n)=", PrimePi[i],
  "; p_n=", Prime[i], "; pi(p_n)=", PrimePi[Prime[i]],
  "; n-pi(n)=", i - PrimePi[i], "; p_n-pi(p_n)=", Prime[i] - PrimePi[Prime[i]]]]
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n=1; pi(n)=0; p_n=2; pi(p_n)=1; n-pi(n)=1; p_n-pi(p_n)=1

n=2; pi(n)=1; p_n=3; pi(p_n)=2; n-pi(n)=1; p_n-pi(p_n)=1

n=3; pi(n)=2; p_n=5; pi(p_n)=3; n-pi(n)=1; p_n-pi(p_n)=2

n=4; pi(n)=2; p_n=7; pi(p_n)=4; n-pi(n)=2; p_n-pi(p_n)=3

n=5; pi(n)=3; p_n=11; pi(p_n)=5; n-pi(n)=2; p_n-pi(p_n)=6

n=6; pi(n)=3; p_n=13; pi(p_n)=6; n-pi(n)=3; p_n-pi(p_n)=7

n=7; pi(n)=4; p_n=17; pi(p_n)=7; n-pi(n)=3; p_n-pi(p_n)=10

n=8; pi(n)=4; p_n=19; pi(p_n)=8; n-pi(n)=4; p_n-pi(p_n)=11

n=9; pi(n)=4; p_n=23; pi(p_n)=9; n-pi(n)=5; p_n-pi(p_n)=14

n=10; pi(n)=4; p_n=29; pi(p_n)=10; n-pi(n)=6; p_n-pi(p_n)=19

n=11; pi(n)=5; p_n=31; pi(p_n)=11; n-pi(n)=6; p_n-pi(p_n)=20

n=12; pi(n)=5; p_n=37; pi(p_n)=12; n-pi(n)=7; p_n-pi(p_n)=25

n=13; pi(n)=6; p_n=41; pi(p_n)=13; n-pi(n)=7; p_n-pi(p_n)=28

n=14; pi(n)=6; p_n=43; pi(p_n)=14; n-pi(n)=8; p_n-pi(p_n)=29

n=15; pi(n)=6; p_n=47; pi(p_n)=15; n-pi(n)=9; p_n-pi(p_n)=32

n=16; pi(n)=6; p_n=53; pi(p_n)=16; n-pi(n)=10; p_n-pi(p_n)=37

n=17; pi(n)=7; p_n=59; pi(p_n)=17; n-pi(n)=10; p_n-pi(p_n)=42

n=18; pi(n)=7; p_n=61; pi(p_n)=18; n-pi(n)=11; p_n-pi(p_n)=43

n=19; pi(n)=8; p_n=67; pi(p_n)=19; n-pi(n)=11; p_n-pi(p_n)=48

n=20; pi(n)=8; p_n=71; pi(p_n)=20; n-pi(n)=12; p_n-pi(p_n)=51