FINANCIAL ANALYTICS IN R

The Time Value of Money

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Instructor

Intuition

An Example

Suppose that:

- You owe me $100
- I can invest any cash I have with a 10% annual growth rate

Then I am ambivalent between getting paid certain values at certain points in time:

Time (Years)	Computation	Value ($)
0		100
1	$100 \times (1 + 0.1)$	110
2	$110 \times (1 + 0.1) = 100 \times (1 + 0.1)$^2	121

Definitions

Present value (PV): The value of a cashflow *as if I were receiving it today*

Future value (FV): The stated value of a cashflow *at the point I'm given it*

Time periods (n): The amount of time in the future that I receive the future value

Discount rate (r): The interest rate at which I can invest cash that I have

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0		100
1	$100 \times (1 + 0.1)$	110
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Time Value of Money Equation

Present value (PV): The value of a cashflow *as if I were receiving it today*
Future value (FV): The stated value of a cashflow *at the point I'm given it*
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Time	Computation	Value
0		100
1	$100 \times (1 + 0.1)$	110
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...	...	...
n	$PV \times (1 + r)^n$	FV

Time Value of Money Equation

Future Value: $\mathrm{FV} = \mathrm{PV} * (1 + \mathrm{r})^{\mathrm{n}}$

Present Value: $\mathrm{PV} = \mathrm{FV}/(1 + \mathrm{r})^{\mathrm{n}}$

In R

```
fv <- pv * (1 + r)^n
pv <- fv / (1 + r)^n
# alternative: pv <- fv * (1 + r) ^ (-1 * n)
```

In `tidyverse`

```
library(dplyr)
mutate(data, pv = fv / (1 + r)^n)
```

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Let's practice!

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Using Different Discount Rates

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A rate conversion example

Start Date: January 1
Initial investment: $100
Monthly Rate of Return: 1.00%

Date	t	Calculation	Future Value
February 1	1	$100 * (1 + 0.01)$	101
March 1	2	$101 * (1 + 0.01) = 100 * (1 + 0.01)^2$	102.1
April 1	3	$102.1 * (1 + 0.01) = 100 * (1 + 0.01)^3$	103.03
...	...	...	...
January 1	12	$100 * (1 + 0.01)^{12}$	112.68

A rate conversion example (cont.)

Date	t	Calculation	Future Value
February 1	1	$100 * (1 + 0.01)$	101
March 1	2	$101 * (1 + 0.01) = 100 * (1 + 0.01)^2$	102.1
April 1	3	$102.1 * (1 + 0.01) = 100 * (1 + 0.01)^3$	103.03
...	...	...	...
January 1	12	$100 * (1 + 0.01)^{12}$	112.68

By extrapolation: $100 * (1 + \text{Monthly Rate})^{12} = 100 * (1 + \text{Annual Rate})$

Conversion Formula: $\text{Annual Rate} = [(1 + \text{Monthly Rate})^{12}] - 1$

The rate conversion formula

r1: Discount rate (growth rate) measured per some unit of time
r2: Discount rate (growth rate) measured per some other unit of time

```
r2 = [(1 + r1)^(# r1 units / 1 r2 unit) ] - 1
```

```
r_quart <- (1 + r_mth)^3 - 1
r_quart <- (1 + r_ann)^(1/4) - 1
```

Real versus Nominal Measures

Cost/Purchasing Power Today: $50

Cost Tomorrow: $70
--> *inflation; less purchasing power*

Cost Tomorrow: $30
--> *deflation; more purchasing power*

Real versus Nominal Formulas

r_real: Discount rate as measured in real dollars
r_nominal: Discount rate as measured in inflation-adjusted dollars
inflation_rate: Inflation rate

$$\text{r\_real} = (1 + \text{r\_nominal}) \; / \; (1 + \text{inflation\_rate}) - 1$$

$$\text{r\_nominal} = (1 + \text{r\_real}) \; * \; (1 + \text{inflation\_rate}) - 1$$

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Discounting Multiple Cashflows

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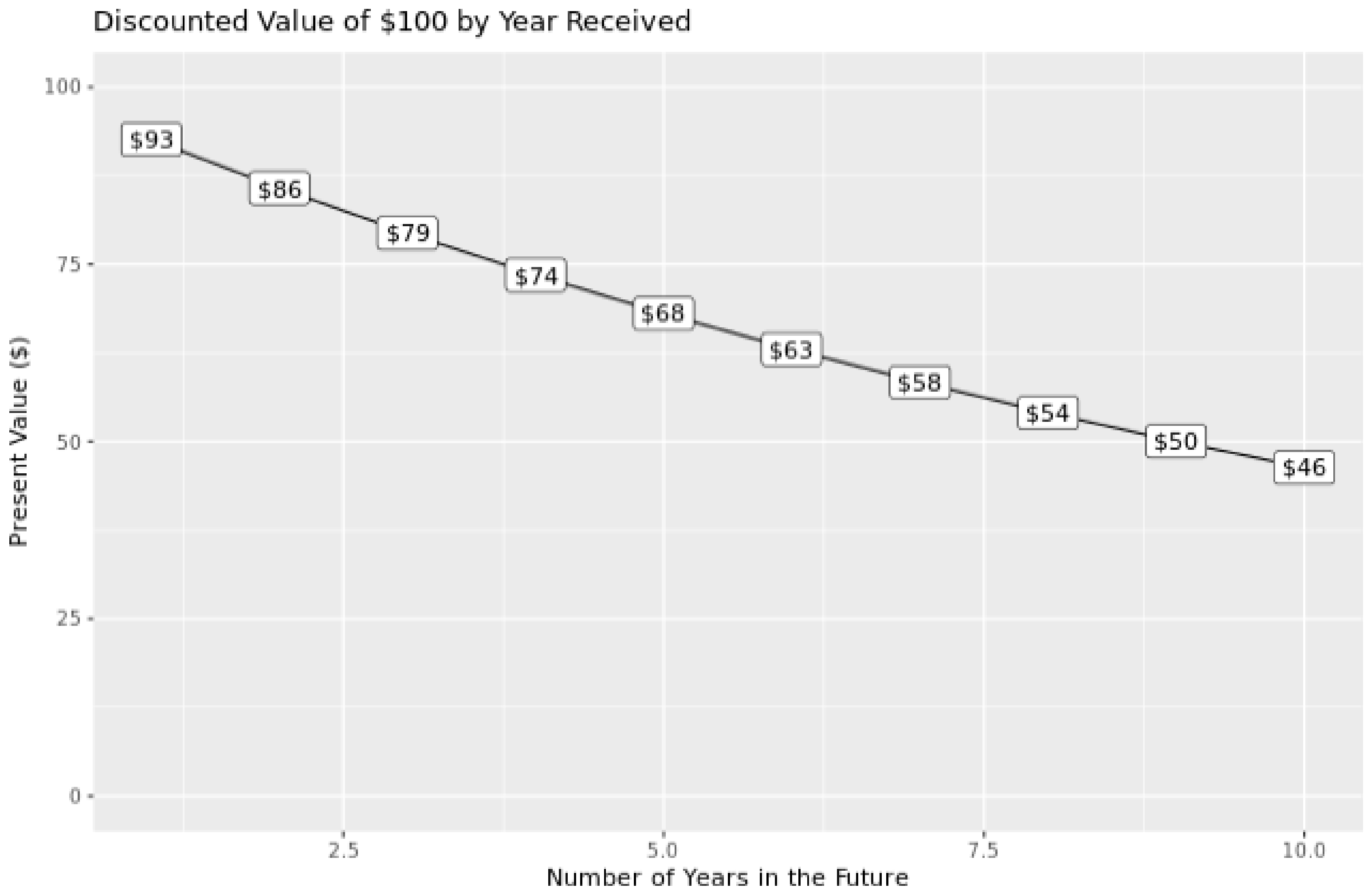
Streams of Cashflows (1)



Streams of Cashflows (2)

Month	1	2	3	4	5	6
Cash Received	100.00	200.00	300.00	400.00	500.00	500.00
Cash Spent	150.00	175.00	200.00	225.00	250.00	250.00
Net Cash	(50.00)	25.00	100.00	175.00	250.00	250.00

Stream of Cashflows (3)

Month	1	2	3	4	5	6
Cash Received	100.00	200.00	300.00	400.00	500.00	500.00
Cash Spent	150.00	175.00	200.00	225.00	250.00	250.00
Net Cash	(50.00)	25.00	100.00	175.00	250.00	250.00

```
pv <- calc_pv(fv = 100, r = 0.01, n = 3)
pv
> [1] 97.05901
```

```
cashflows <- c(0, -50, 25, 100, 175, 250, 250)
discounted_cashflows <- calc_pv(cashflows, r = 0.01, n = 0:6)
discounted_cashflows
> [1]   0.00000 -49.50495  24.50740  97.05901 168.17156 237.86642 235.51131
```

```
sum(discounted_cashflows)
> [1] 713.6108
```

Summarizing Multiple Cashflow Streams

option	time	cashflow
A	1	350
A	2	350
A	3	350
B	1	500
B	2	500

```
many_cashflows %>%
    group_by(option) %>%
    summarize( PV = sum(calc_pv(cashflow, 0.08, n = time))
```

option	PV
A	901.9839
B	891.6324

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