Modelo

$$y_{it} = \overset{1}{\alpha_{it}} + \overset{2}{\beta_{1,it}\, x_{1it}} + \overset{3}{\beta_{2,it}\, x_{2it}} + \overset{4}{\beta_{3,it}\, x_{3it}} + \varepsilon_{it}$$

$$i = \{1, \dots, p\} \qquad K = 4$$

Considere $p = 3$. Então:

$$y_{1t} = \alpha_{1t} + \beta_{1,1t}\, x_{11t} + \beta_{2,1t}\, x_{21t} + \beta_{3,1t}\, x_{31t} + \varepsilon_{1t}$$

$$y_{2t} = \alpha_{2t} + \beta_{1,2t}\, x_{12t} + \beta_{2,2t}\, x_{2,2t} + \beta_{3,2t}\, x_{3,2t} + \varepsilon_{2t}$$

$$y_{3t} = \alpha_{3t} + \beta_{1,3t}\, x_{1,3t} + \beta_{2,3t}\, x_{2,3t} \quad \beta_{3,3t}\, x_{3,3t} + \varepsilon_{3t}$$

$\iff$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\substack{y_t \\ (p \times 1)}} = \underbrace{\begin{bmatrix} 1 & x_{1,1t} & x_{2,1t} & x_{3,1t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_{1,2t} & x_{2,2t} & x_{3,2t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_{1,3t} & x_{2,3t} & x_{3,3t} \end{bmatrix}}_{\substack{X_t \\ (p \times K \cdot p)}} \underbrace{\begin{bmatrix} \alpha_{1t} \\ \beta_{1,1t} \\ \beta_{2,1t} \\ \beta_{3,1t} \\ \alpha_{2t} \\ \beta_{1,2t} \\ \beta_{2,2t} \\ \beta_{3,2t} \\ \alpha_{3t} \\ \beta_{1,3t} \\ \beta_{2,3t} \\ \beta_{3,3t} \end{bmatrix}}_{\substack{\gamma_t \\ (Kp \times 1)}} + \underset{(p \times 1)}{\varepsilon_t}$$

$$\iff \quad y_t = X_t \gamma_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H)$$

$$\gamma_{t+1} = \gamma_t + \eta_t, \qquad \eta_t \sim N(0, Q)$$

$$\gamma_1 \sim N(0, 10^7)$$

State space model

$$\underset{(p\times 1)}{y_t} = \underset{(p\times m)}{Z_t}\,\underset{(m\times 1)}{\alpha_t} + \underset{(p\times 1)}{\varepsilon_t} \qquad \varepsilon_t \sim N(0, \overset{(p\times p)}{H_t})$$

$$\underset{(m\times 1)}{\alpha_{t+1}} = \underset{(m\times m)}{T_t}\,\underset{(m\times 1)}{\alpha_t} + \underset{(m\times r)}{R_t}\,\underset{(r\times 1)}{\eta_t} \qquad \eta_t \sim N(0, \underset{(r\times r)}{Q_t})$$

$$Z_t = X_t \ ; \quad T = I_m \ ; \quad R = I_r$$

Defina
$$\alpha_{t|t} \equiv \mathbb{E}[\alpha_t \mid I_t]$$
$$P_{t|t} \equiv \mathbb{E}[(\alpha_t - \alpha_{t|t})(\alpha_t - \alpha_{t|t})']$$

1. Inicialização do filtro:

$$\alpha_{0|0} = 0$$
$$P_{0|0} = I\,10^7$$

2. Recursão

For $t \in \{1, \dots, mobs\}$

(i) Passo de propagação:
$$\alpha_{t|t-1} = T_t\,\alpha_{t|t}$$
$$P_{t|t-1} = T_t P_{t|t} T_t' + Q_t$$

(ii) Passo de previsão:
$$\hat{y}_t = Z_t\,\alpha_{t|t-1}$$
$$F_t = Z_t P_{t|t-1} Z_t' + R_t H_t R_t'$$
$$\hat{v}_t = y_t - \hat{y}_t$$

iii) Passo de correção:
$$K_t = P_{t|t-1} H_t' F_t^{-1}$$
$$x_{t|t} = x_{t|t-1} + K_t \hat{v}_t$$
$$P_{t|t} = P_{t|t-1} - K_t H_t P_{t|t-1}$$