2022 Differential Equation Quiz 3 (Written quiz)

1 Solve each of the following initial value problems and plot the solutions for several values of y_0

(1)
$$\frac{dy}{dt} = -y + 5$$
 , $y(0) = y_0(20points)$

2 Verify that each given function is a solution of the differential equation. (20 points)

$$2t^2y'' + 3ty' - y = 0$$
, $t > 0$; $y_1(t) = t^{1/2}$, $y_2(t) = t^{-1}$

3 Determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$. Write the general solution along with variable A as the arbitrary constant. (20 points)

$$y' + 2y = 0$$

4 Find the general solution of the given differential equations, and use it to determine how solutions behave as $t \to \infty$. Use integrating factor method. (5 points and 15 points respectively)

(1)
$$y' - 2y = t^2 e^{2t}$$

(2) $2y' + y = 3t^2$

5 Find the solution of the given initial value problem in explicit form and determine (at least approximately) the interval in which the solution is defined. (20 points)

$$y' = \frac{2x}{1+2y}, \quad y(2) = 0$$