

2022 Differential Equation Quiz2(Python)

- The test is based on the practice. You are asked to graphs.
- Internet and book are allowed!

1) Plot Fig 1 (70 points)

- Restraints:
 - Contour and eyes.(50 points)
 - Use 1200 uniform random samples for the circle that forms the face(20 points)
Origin at $(x, y) = (0, 0)$ and radius = 5
 - Eyes are just constant line $y = 2$ with 2 intervals as domains.
 - Can only use `matplotlib.pyplot.plot` for plotting
 - Use Sympy symbols for define the smile equation, but still need to plot with `matplotlib.pyplot.plot`
 - To plot a sympy symbolic equation with `matplotlib.pyplot.plot`, you can use the function `sympy.utilities.lambdify`.
 - Your equation for smile need to be cubic. You can base it on your homeworks equation.
i.e. $(y = \alpha(-x^3 + x^2) - \beta)$ for some α, β



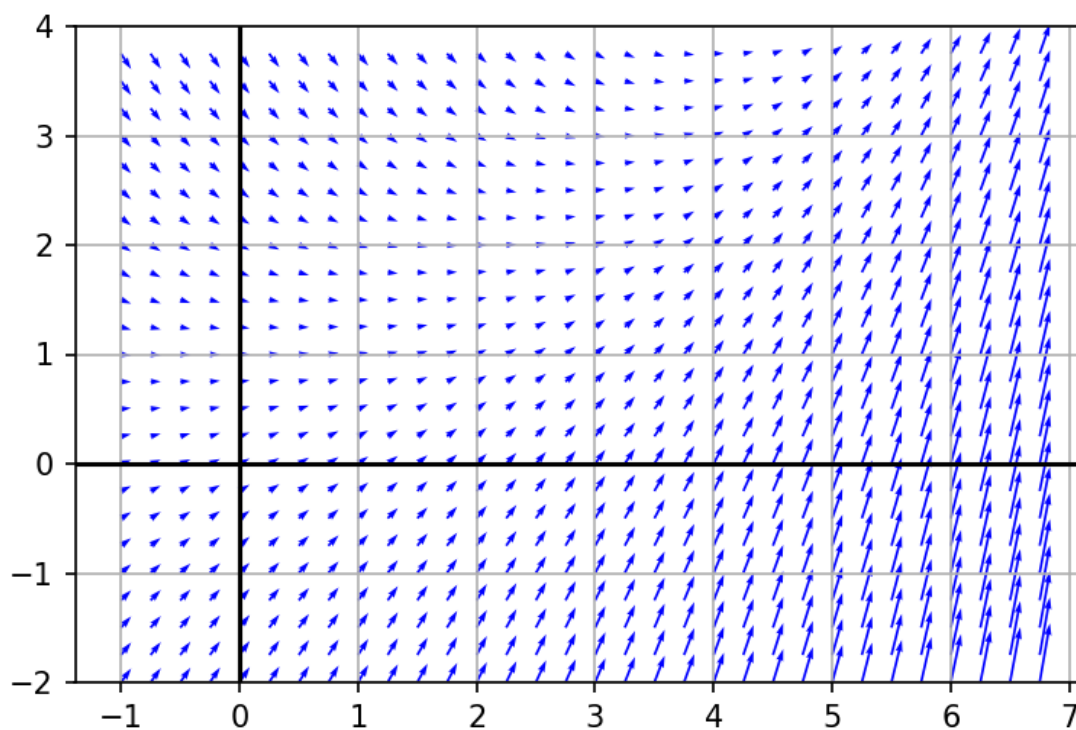
Fig. 1: Face

2) Plot a direction field for the equation $\frac{dy}{dt} + \frac{22y}{t} = \frac{1}{2}e^{t/3}$. See Figure 2a (30 points)

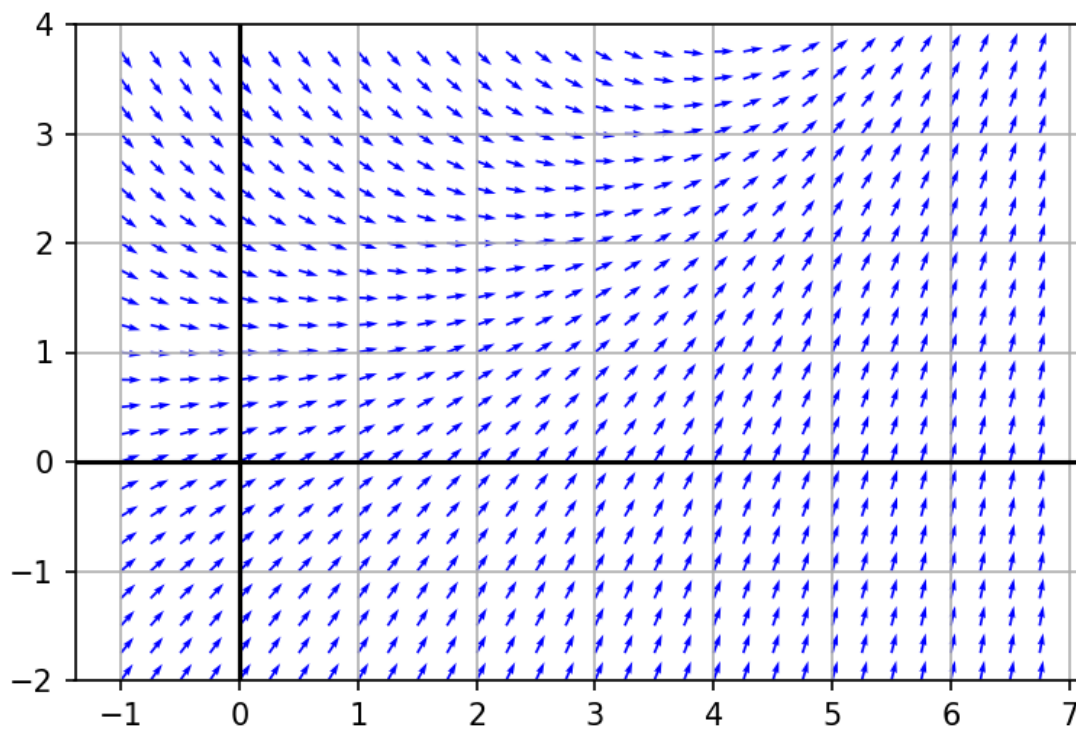
- Restraints:
 - Samples should be at 0.25 step interval. i.e.(15 points)

```
dt = 0.25
t = np.arange(-1,7,dt)
y = np.arange(-2,4,dt)
...
#meshgrid
...
#dydt = <your differential equation>
...
#quiver
```

- Use `meshgrid`
- Use `quiver`
- if you can normalize your arrows (See Figure 2b)(15 points)



(a) Unnormalized direction field



(b) Normalized direction field

Fig. 2: Direction fields for $\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}$