

# Quiz 7

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1.  $4y'' - 4y' - 3y = 0$

$$4r^2 - 4r - 3 = 0$$

$$\Rightarrow r = \frac{4 \pm \sqrt{16 + 16 \cdot 3}}{8}$$

$$= \frac{4 \pm 8}{8}$$

$$= \frac{3}{2}, -\frac{1}{2}$$

$$\therefore y = C_1 e^{\frac{3}{2}t} + C_2 e^{-\frac{1}{2}t} \#$$

2.  $y'' - 2y' + 10y = 0$

$$r^2 - 2r + 10 = 0$$

$$\Rightarrow r = 1 \pm 3i$$

$$y = C_1 e^{at} \cos bt + C_2 e^{at} \sin bt,$$

$$\therefore a = 1, b = 3$$

$$\therefore y = C_1 e^t \cos 3t + C_2 e^t \sin 3t \#$$

3.  $r^2 + 9 = 0$

$$\Rightarrow r_1 = r_2 = \pm 3i$$

$$y_c = C_1 \cos 3t + C_2 \sin 3t$$

$$\text{Let } g = g_1 + g_2, \begin{cases} g_1 = b \\ g_2 = t^2 e^{3t} \end{cases}$$

$$\text{Let } \begin{cases} Y_1 = A \\ Y_2 = A e^{3t} + B t e^{3t} + C t^2 e^{3t} \end{cases}$$

$$Y_1: A'' + 9A = 0 \Rightarrow A = \frac{2}{3}$$

$$Y_2: (A e^{3t} + B t e^{3t} + C t^2 e^{3t})'' + 9(A e^{3t} + B t e^{3t} + C t^2 e^{3t}) = t^2 e^{3t}$$

$$\Rightarrow (3A e^{3t} + B e^{3t} + 3B t e^{3t} + 2C t e^{3t} + 3C t^2 e^{3t})' + 9A e^{3t} + 9B t e^{3t} + 9C t^2 e^{3t} = t^2 e^{3t}$$

$$\Rightarrow 18A + 6B + 9Bt + 2C + 12Ct + 18Ct^2 + 9Bt = t^2$$

$$\Rightarrow \begin{cases} 18A + 6B + 2C = 0 \\ 18A + 12C = 0 \\ 18C = 1 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{162} \\ B = -\frac{1}{27} \end{cases}$$

$$\therefore y = C_1 \cos 3t + C_2 \sin 3t + \frac{2}{3} + \frac{1}{162} e^{3t} - \frac{1}{27} t e^{3t} + \frac{1}{18} t^2 e^{3t} \#$$

4.  $r^2 + r + 4 = 0$

$$\Rightarrow r_{1,2} = -\frac{1}{2} \pm i \frac{\sqrt{15}}{2}$$

$$y_c = C_1 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{15}t}{2}\right) + C_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{15}t}{2}\right)$$

$$Y: A e^t + B e^{-t} (\because g(t) = e^t - e^{-t})$$

$$(A e^t + B e^{-t})'' + (A e^t + B e^{-t})' + 4(A e^t + B e^{-t}) = e^t - e^{-t}$$

$$\Rightarrow A e^t + B e^{-t} + A e^t - B e^{-t} + 4A e^t + 4B e^{-t} = e^t - e^{-t}$$

$$\Rightarrow 6A e^t + 4B e^{-t} = e^t - e^{-t}$$

$$\Rightarrow \begin{cases} 6A = 1 \\ 4B = -1 \end{cases} \Rightarrow \begin{cases} A = 1/6 \\ B = -1/4 \end{cases}$$

$$\Rightarrow Y = \frac{e^t}{6} - \frac{e^{-t}}{4}$$

$$\therefore y = C_1 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{15}t}{2}\right) + C_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{15}t}{2}\right) + \frac{e^t}{6} - \frac{e^{-t}}{4} \#$$

5. characteristic eq

$$r^2 + 3r = 0$$

$$r(r+3) = 0$$

$$r = 0 \text{ or } -3$$

$$\text{solution: } y = C_1 + C_2 e^{-3t}$$

$$y(0) = -2$$

$$= C_1 + C_2$$

$$y'(0) = 3$$

$$= -3C_2$$

$$\Rightarrow \begin{cases} C_1 + C_2 = -2 \\ C_1 - 3C_2 = 3 \end{cases} \Rightarrow y = -1 - e^{-3t} \#$$