Quiz 3

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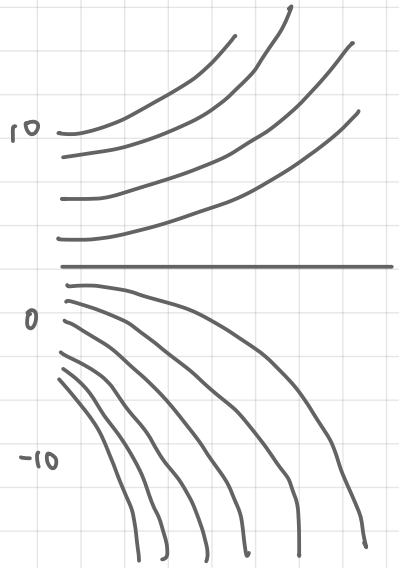


1. $\frac{dy}{dt} = -y+5$, $y(0)=y_0$

$\frac{dy}{-y+5} = dt$

$\Rightarrow \left(\frac{1}{-y+5}\right)dy = dt$

$\Rightarrow \int \left(\frac{1}{-y+5}\right)dy = \int 1\,dt$

$\Rightarrow \ln|-y+5| = t+C$

$e^{\ln|-y+5|} = e^{t+C}$

$\Rightarrow |y-5| = e^{t+C}$

$\Rightarrow y-5 = Ce^t$

$\Rightarrow y = 5+Ce^t$

$\Rightarrow y(t) = 5-5e^t \quad (\because y(0)=0)$ #

2. $2t^2y'' + 3ty' - y = 0$, $t>0$

① $y_1(t) = t^{\frac{1}{2}} = \sqrt{t} \longrightarrow y' = \frac{1}{2\sqrt{t}} \longrightarrow y'' = \frac{-1}{4\cdot t^{3/2}}$

$2t^2\cdot\frac{-1}{4\cdot\frac{t^3}{2}} + 3t\cdot\frac{1}{2\sqrt{t}} - \sqrt{t}$

$= \frac{\sqrt{t}}{2} + \frac{\sqrt{t}}{2} - \sqrt{t}$

$= 0$ #

② $y_2(t) = t^{-1} = \frac{1}{t} \longrightarrow y' = -t^{-2} \longrightarrow y'' = 2t^{-3}$

$2t^2\cdot 2t^{-3} + 3t\cdot(-t^{-2}) - t^{-1}$

$= t^{-1} - t^{-1}$

$= 0$ #

3. $y'+2y=0$

$y = e^{rt}$

$\Rightarrow y' = re^{rt}$

$re^{rt} + 2e^{rt} = 0$

$\Rightarrow (r+2)e^{rt} = 0$

$\Rightarrow r = -2$ #

$y'+2y=0$

$\int 2\,dt = 2t \longrightarrow \mu(t) = e^{2t}$

$e^{2t}y' + e^{2t}\cdot 2y = 0$

$\Rightarrow (e^{2t}\cdot y)' = 0$

$\Rightarrow \int (e^{2t}\cdot y)'\,dt = 0$

$\Rightarrow e^{2t}y + A = 0$

$\Rightarrow y = \frac{-A}{e^{2t}}$ #

4. $t \to \infty$

(1) $y' - 2y = t^2 e^{2t}$

$\int 2\,dt = -2t \longrightarrow \mu(t) = e^{2t}$

$e^{-2t}y' - e^{-2t}y = t^2$

$\int (e^{-2t}y)'\,dt = \int t^2\,dt$

$e^{-2t}y + C = \frac{1}{3}t^3$

$\Rightarrow y = e^{2t}(\frac{1}{3}t^3 + C)$ #

(2) $2y' + y = 3t^2$

$\Rightarrow y' + \frac{1}{2}y = \frac{3}{2}t^2$

$\int \frac{1}{2}\,dt = \frac{1}{2}t \longrightarrow \mu(t) = e^{\frac{1}{2}t}$

$e^{\frac{1}{2}t}y' + \frac{1}{2}\cdot e^{\frac{1}{2}t}y = \frac{3}{2}t^2 \cdot e^{\frac{1}{2}t}$

$\Rightarrow (e^{\frac{1}{2}t}y)' = \frac{3}{2}t^2 \cdot e^{\frac{1}{2}t}$

$\Rightarrow e^{\frac{1}{2}t}y + C = \frac{3}{2}\int t^2 \cdot e^{\frac{1}{2}t}\,dt$

$\Rightarrow y(t) = 3t^2 - 12t + 24 + Ce^{-\frac{t}{2}}$ #

5. $y' = \frac{2x}{1+2y}$. $y(2) = 0$

$(1+2y)\frac{dy}{dx} = 2x$

$\Rightarrow \int (1+2y)\,dy = \int 2x\,dx$

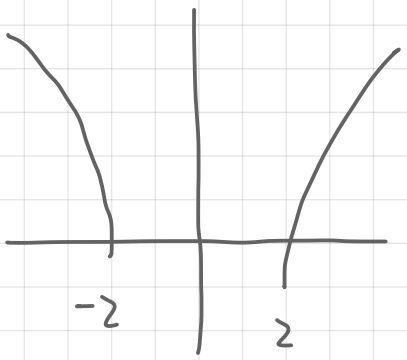
$\Rightarrow y + y^2 + C = x^2$

$\Rightarrow y^2 + y + (C - x^2) = 0$

$\Rightarrow y = \frac{-1 \pm \sqrt{4x^2 - 15}}{2}$

$\because y(2) = 0$

$\therefore y = \frac{-1 + \sqrt{4x^2 - 15}}{2}$



$\Rightarrow$ right curve from $\frac{1}{2}\sqrt{15}$ to ∞ #