

# HW4

$$\begin{aligned} 1. V_{OV} &= V_{GS} - V_t \\ \Rightarrow 0.2 &= V_{GS} - 0.5 \\ \Rightarrow V_{GS} &= 0.7V \end{aligned}$$

$$V_{DS} = 1V$$

$V_{DS} > V_{GS} - V_{th}$

$$\begin{aligned} I_D &= \frac{1}{2} \cdot k_n (V_{GS} - V_{OV})^2 \\ &= \frac{10 \times 10^{-3} \times (0.2)^2}{2} \\ &= 0.2mA \\ I_D &= \frac{V_{DD} - V_{DS}}{R_D} \\ \Rightarrow 0.2 \times 10^{-3} &= \frac{5 - 1}{R_D} \\ \Rightarrow R_D &= 20k\Omega \end{aligned}$$

$$\begin{aligned} g_m &= \frac{2I_D}{V_{OV}} \\ &= \frac{2(0.2 \times 10^{-3})}{0.2} \\ &= 2 \times 10^{-3} \text{ A/V} \\ A_V &= -g_m \cdot R_D \\ &= -(2 \times 10^{-3}) \cdot 20 \times 10^3 \\ &= -40 \text{ V/V} \end{aligned}$$

$$\begin{aligned} (b) \frac{1}{2} k_n \cdot (V_{GS} - V_t)^2 &= \frac{V_{DD} - (V_{GS} - V_t)}{R_D} \\ \Rightarrow \frac{1}{2} \times 10^{-3} \times V_{OV}^2 &= \frac{5 - V_{OV}}{20 \times 10^3} \\ \Rightarrow V_{OV} &= 0.219 = V_{DS} \\ \Rightarrow V_{GS} &= 0.219 + V_t \\ &= 0.719 \end{aligned}$$

$$B(0.719, 0.219)$$

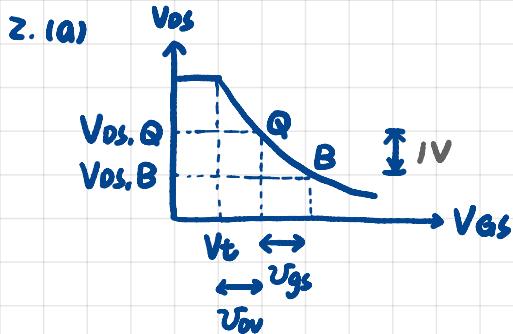
$$V_O > V_{GS} - V_t$$

$V_O + V_0 > V_{GS} - V_t$

$$\begin{aligned} 1 + V_0 &> 0.2 \\ V_0 &> 0.2 - 1 \\ V_0 &> 0.8V \end{aligned}$$

the negative swing of the output should be less than 0.8V, so the maximum allowable negative swing is about -0.19V

$$\begin{aligned} A_V &= \frac{V_O}{V_S} \\ \Rightarrow -40 &= \frac{V_O}{-0.19V} \\ \Rightarrow V_O &\approx 19.5mV \end{aligned}$$



$$\begin{aligned} ① V_{DS} + 0.5 &\leq V_{DD} \\ \Rightarrow V_{DS} &\leq 4.5 \end{aligned}$$

$$\begin{aligned} ② V_{DS} - 0.5 &\geq V_{OV} \\ \Rightarrow V_{DS} &\geq 0.7 \end{aligned}$$

$$\begin{aligned} V_{DS,Q} &= V_{DS,B} + 0.5V \\ V_{OV,B} &= \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = V_{DS,g} \quad ① \end{aligned}$$

$$V_{DS,B} = V_{OV} + \hat{V}_{GS} = 0.2 + \frac{2.5}{k_n R_D} \quad ②$$

$$\hat{V}_{GS} = \frac{0.5}{|A_V|} = \frac{0.5}{k_n R_D V_{OV}} = \frac{0.5}{k_n R_D (0.2)} = \frac{2.5}{k_n R_D}$$

$$①, ② \Rightarrow \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} = 0.2 + \frac{2.5}{k_n R_D}$$

$$\Rightarrow \sqrt{2k_n R_D (5) + 1} = 0.2k_n R_D + 3.5$$

$$\Rightarrow k_n R_D = 213.684$$

$$① \Rightarrow V_{DS,B} = \frac{\sqrt{2(213.684)(5) + 1} - 1}{213.684} = 0.212V$$

$$\therefore V_{DS,Q} = 0.212 + 0.5 = 0.712V$$

$$\begin{aligned} (b) A_V &= -k_n R_D V_{OV} \\ &= -213.684 \times 0.2 \\ &= -42.7368 \end{aligned}$$

$$|A_V| = \frac{\hat{V}_{DS}}{\hat{V}_{GS}} \Rightarrow 42.7368 = \frac{0.5V}{\hat{V}_{GS}} \therefore \hat{V}_{GS} = 0.0119V$$

$$A_V = -g_m R_D = -\frac{2I_D R_D}{V_{OV}} = -\frac{2(V_{DD} - V_{DS})}{V_{OV}} = -43V/V = \frac{0.5}{V_{GS}}$$

$$(c) V_D = I_D R_D = V_{DD} - V_{DS}$$

$$\Rightarrow R_D = \frac{5 - 0.712}{100 \times 10^{-6}} = 42.88k\Omega$$

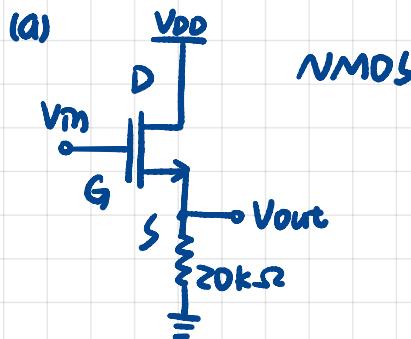
$$(d) A_V = -k_n' \left( \frac{W}{L} \right) V_{DD} \cdot R_D$$

$$\Rightarrow -42.7368 = -200 \times 10^{-6} A_V \cdot \left( \frac{W}{L} \right) \times 0.2V = 42.88 k\Omega$$

$$\Rightarrow \frac{W}{L} = 24.9 \#$$

3. 脫離 cut-off:  $V_{GS} - V_{th} > 0$

脫離 sat.:  $V_{DS} < V_{DD}$



$$\textcircled{1} \quad V_{in} = 0$$

$$V_{GS} = V_{in} - V_{out}$$

$$V_{GS} - V_{th} > 0$$

$$0.7V$$

$$V_{in} - V_{out} - V_{th} > 0$$

$$0.7$$

$$\textcircled{2} \quad \text{sat.} \quad I_D = \frac{1}{2} \times 200 \times 10^{-6} \times 5 \times (V_{GS} - V_{th})^2$$

$$= 5 \times 10^{-4} \times (V_{in} - V_{out} - 0.7)^2 \Rightarrow \frac{V_{out}}{20k} = 5 \times 10^{-4} (1.1 - V_{out})^2$$

$$\frac{V_{out} - 0}{20k} = I_D \Rightarrow V_{out} = 0.815 \text{ or } 1.485$$

檢查有沒有符合  $V_{GS} - V_{th} > 0$

$$V_{in} - V_{out} > 0.7$$

$\therefore 1.485 \text{ 不合}$

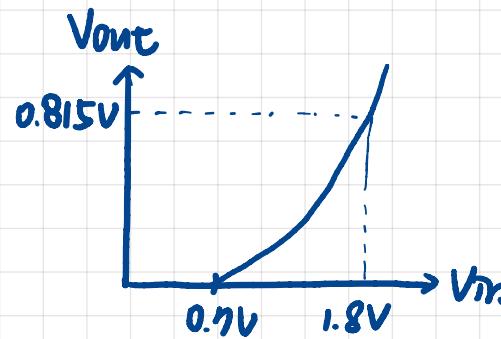
③ 要跑到 linear  $\Rightarrow V_{DS} < V_{DD}$

$$\frac{V_{DD} - V_{out}}{1.8} > \frac{V_{GS} - V_{th}}{1.8}$$

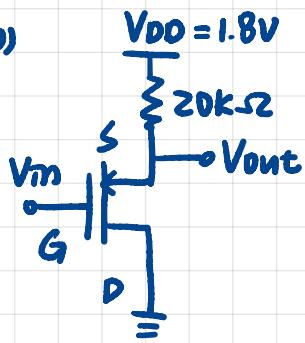
$$\frac{V_{in} - V_{out} - V_{th}}{1.8} > 0.7$$

$\Rightarrow \because V_{in}$  就算最大  $3 \quad V_{DS} > V_{DD}$

$\therefore$  不會進到 linear



(b)



PMOS

③ sat.

$$\begin{aligned} I_D &= \frac{1}{2} \times 200 \times 10^{-6} \times 5 \times (V_{out} - V_m - 0.7)^2 \\ \frac{1.8 - V_{out}}{20k} &= I_D \end{aligned}$$

$$\Rightarrow \frac{1.8 - V_{out}}{20k} = 5 \times 10^{-4} (V_{out} - 2.5)^2$$

$$\Rightarrow 1.8 - V_{out} = 10 (V_{out} - 2.5)^2$$

$$\textcircled{1} V_{SG} > |V_{th}|$$

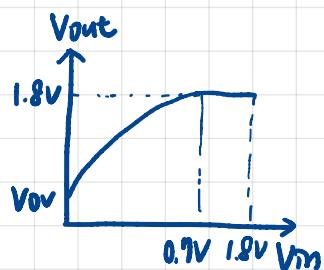
$$V_{out} - V_m > 0.7$$

③ linear  $\Rightarrow |V_{SD}| < |V_{ov}|$ 

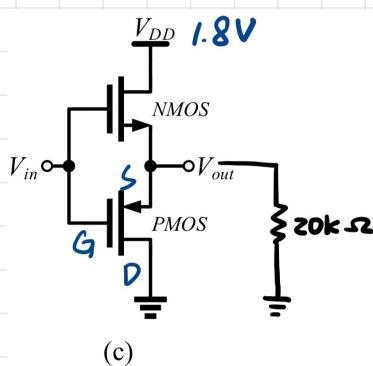
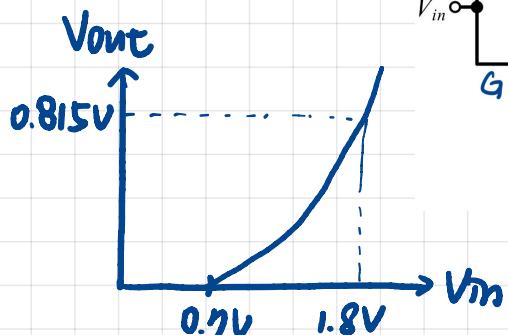
$$V_{out} - 0 = V_{SG} - V_{th}$$

$$V_{SG} > V_{SG} - V_m - V_{th}$$

$$\because V_m > 0 \therefore |V_{SD}| > |V_{ov}| \text{ (always)}$$

 $\therefore$  不會進到 linear(c)  $\because$  PMOS 不會通

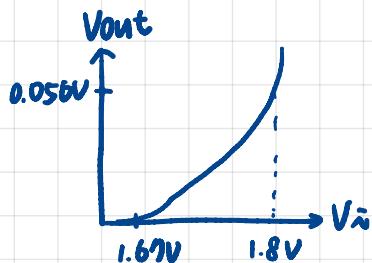
.. 司密因不計

 $\Rightarrow$  則 (c) 和 (a) 相同

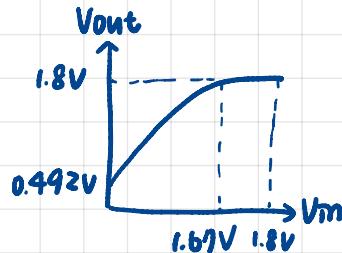
$$4. V_{th} = 0.7 + 0.4(\sqrt{1.7} - \sqrt{0.7})V^{\frac{1}{2}} \\ \approx 1.67$$

$$(a) I_D = \frac{1}{2} \cdot \frac{100}{200} \times 10^{-6} \times 5 \times (V_{GS} - V_{th})^2 \\ = 10^{-4} \times 5 \times (1.8 - V_{out} - 1.67)^2 \\ = \frac{V_{out} - 0}{20 \times 10^3}$$

$$\Rightarrow V_{out} = 0.804 \text{ or } 0.056$$



$$(b) I_D(1.8 - V_{out} - 1.67)^2 = 1.8 - V_S \\ \Rightarrow V_S = 0.492 \text{ or } -0.332$$



(c) ~~(b)~~(a) \*

$$5. (a) I_D = \frac{1}{2} \mu_0 \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{th})^2 \\ = \frac{1}{2} \cdot 5 \times 10^{-3} \times (0.6 - 0.4)^2 \\ = \frac{1}{2} \cdot 5 \times 10^{-3} \times 0.04 \\ = 10^{-4} A *$$

$$1.8 - 10^{-4} \times 10 \times 10^3 = V_{DS}$$

$$\Rightarrow V_{DS} = 0.8 V *$$

$$(b) g_m = k_n(V_{GS} - V_{th}) \\ = 5 \times 10^{-3} (0.6 - 0.4) \\ = 5 \times 10^{-3} \times 0.2 \\ = 10^{-3} *$$

$$g_m = \frac{2I_D}{V_{DS}}$$

$$(c) A_V = -g_m R_o \\ = -10^{-3} \cdot 10 \times 10^3 \\ = -10 V/V *$$

$$(d) r_o = \frac{1}{\lambda I_D} \\ = \frac{1}{0.1 \cdot 10^{-4}} = 10^5 \Omega *$$

$$A_V = -g_m (r_o || R_o) \\ = -10^{-3} \cdot \frac{10^5 \times 10^4}{10^5 + 10^4} \\ = -9.09 V/V *$$

## 6. NMOS

$$(a) A_v = g_m R_o = 10^4 / v$$

$$\Rightarrow g_m \cdot 20 \times 10^3 = 10$$

$$\Rightarrow g_m = 5 \times 10^{-4} A/V_s$$

$$(b) V_D = 0.2V$$

$$V_{DD} = V_D + I_D \cdot R_D$$

$$\Rightarrow 1.8 = 0.2 + I_D \cdot 20 \times 10^3$$

$$\Rightarrow I_D = 8 \times 10^{-5} A$$

$$g_m = \frac{2I_D}{V_{DD}}$$

$$\Rightarrow V_{DD} = \frac{16 \times 10^{-5}}{5 \times 10^{-4}} = 3.2 \times 10^{-1} V_s$$

$$(c) g_m = \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot V_{DD}$$

$$\Rightarrow 5 \times 10^{-4} = 200 \times 10^{-6} \cdot \frac{W}{L} \cdot 3.2 \times 10^{-1}$$

$$\Rightarrow \frac{W}{L} = 7.81 \mu$$

$$(d) V_{th} = 0.4V$$

$$I_{DS} = \frac{1}{2} \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{th})^2$$

$$\Rightarrow 8 \times 10^{-5} = \frac{1}{2} \times \frac{5 \times 10^{-4}}{3.2 \times 10^{-1}} \times (V_{GS} - 0.4)^2$$

$$\Rightarrow 16 \times 10^{-5} = \frac{5 \times 10^{-4}}{32 \times 10^{-2}} \times 10^{-2} \times (V_{GS} - 0.4)^2$$

$$\Rightarrow \left[ 16 \times \frac{32}{5} \times 10^{-3} \right]^{\frac{1}{2}} + 0.4 = V_{GS}$$

$$\Rightarrow V_{GS} = 0.72 V_s$$