Homework Policy

SUBMISSION

- DO NOT accecpt any late submissions.
- Handwritten / Typed / Latex are all fine.
- ONLY accept PDF files.
- Upload your works to E3 before the deadline.
- Name your file as studentID_hwX_[answer/note].pdf
  eg. 310550987_hw1_answer.pdf
  eg. 310550420_hw1_note.pdf

SCORE

- Explanations or calculation processes are needed.
- Make sure your works are clear enough to read (Not too dark, blurry, sloppy, no strong reflection).
- If a problem is not answered, no point is obtained from that problem.

QUESTION

- If you have any questions, please send an email to TAs. We will answer your question as soon as possible.

Linear Algebra Homework 1

Deadline:10/15 24:00

0. Lecture notes from 9/22 to 10/8, please upload as a separate pdf file.

1. Reduce this system to upper triangular form by two row operations:

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 7y + 5z &= 20 \\ -2y + 2z &= 0 \end{aligned}$$

   Circle the pivots. Solve by back substitution for z, y, x.

2. Suppose elimination takes A to U without row exchanges.
   Then row j of U is a combination of which rows of A?
   If $Ax = 0$, is $Ux = 0$?
   If $Ax = b$, is $Ux = b$?
   If A starts out lower triangular, what is the upper triangular U?

3. $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}$

   multiply A times BC. Then multiply AB times C.

4. For which three numbers a will elimination fail to give three pivots?
   $A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$ is singular for three values of a.

5. Find A^{-1} and B^{-1} ($if\ they\ exist$) by elimination on $\begin{bmatrix} A & I \end{bmatrix}$ and $\begin{bmatrix} B & I \end{bmatrix}$:
   $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

6. Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?
   $A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix}$ and $E_{32}E_{31}E_{21}A = U$.
   Multiply those E's to get one matrix M that does elimination: $MA = U$.

7. For which three numbers c is this matrix not invertible, and why not?
   $A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$

8. What three matrices E_{21} and E_{12} and D^{-1}reduce $A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ to the identity matrix?
   Multiply $D^{-1}E_{12}E_{21}$ to find A^{-1}.

9. If A has row 1 + row 2 = row 3, show that A is not invertible:

   (a) Explain why $Ax = (1, 0, 0)$ cannot have a solution.

   (b) Which right sides (b_1, b_2, b_3) might allow a solution to $Ax = b$?

   (c) What happens to row 3 in elimination?