Linear Algebra Homework 5

Deadline:12/11 24:00

0. Lecture notes from 11/24 to 12/3, please upload as a separate pdf file to the corresponding assignment.

1. Write down three equations for the line $b = C + Dt$ to go through $b = 7$ at $t = -1, b = 7$ at $t = 1$, and $b = 21$ at $t = 2$. Find the least squares solution$\tilde{x} = (C, D)$ and draw the closest line.

2. Find the projection $p = A\tilde{x}$ in problem 1. This gives the three heights of the closest line. Show that the error vector is $e = (2, -6, 4)$. Why is $Pe = 0$?

3. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ with $\vec{a_1} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\vec{a_2} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\vec{a_3} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

   (a) perform Gram-schmidt to get orthonormal $\vec{q_1}, \vec{q_2}, \vec{q_3}$ from $\vec{a_1}, \vec{a_2}, \vec{a_3}$

   (b) QR decompose A

4. If Q has orthonormal columns, what is the least squares solution x to $Qx = b$?

5. Find the determinants of rotations and reflections:

$$Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ and } Q = \begin{bmatrix} 1 - 2\cos\theta^2 & -2\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 1 - 2\sin\theta^2 \end{bmatrix}$$

6. By applying row operations to produce an upper triangular U, compute $\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$ and $\det \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

7. Find the cofactor matrix C and multiply A times C^T. Compare AC^T with A^{-1}:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

8. Solve the linear equations by Cramer's Rule $x_j = \det B_j / \det A$:

$$\begin{aligned} 2x_1 + 5x_2 &= 1 \\ x_1 + 4x_2 &= 2 \end{aligned}$$

9. Find the eigenvalues of A and B and AB and BA:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

   (a) Are the eigenvalues of AB equal to eigenvalues of A times eigenvalues of B?

   (b) Are the eigenvalues of AB equal to the eigenvalues of BA?