Linear Algebra Homework 2

Deadline:10/29 24:00

0. Lecture notes from 10/13 to 10/22, please upload as a separate pdf file to the corresponding assignment.

1. What three elimination matrices E_{21}, E_{31}, E_{32} put A into its upper triangular form $E_{32}E_{31}E_{21}A = U$? Multiply by $E_{32}^{-1}, E_{31}^{-1}, E_{21}^{-1}$ to factor A into L times U:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}, L = E_{21}^{-1}, E_{31}^{-1}, E_{32}^{-1}$$

2. Compute L and U for the symmetric matrix A:

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

   Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

3. Suppose A is rectangular (m by n) and S is symmetric(m by m).
   (a) Transpose $A^T SA$ to show its symmetry. What shape is this matrix?
   (b) Show why $A^T A$ has no negative numbers on its diagonal.

4. *Tridiagonal matrices* have zero entries except on the main diagonal and the two adjacent diagonals. Factor these into A = LU and A = LDL^T:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}$$

5. Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{F}\}$ where $\mathbb{F}$ is a field. Define addition of elements of V coordinatewise and for $c \in \mathbb{F}$ and $(a_1, a_2) \in V$, define,

$$c(a_1, a_2) = (a_1, 0)$$

   Is V a vector space over $\mathbb{F}$ with these operations? Justify your answer.

6. The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a "vector" in the space M of all 2 by 2 matrices. Write down the zero vector in this space, the vector $\frac{1}{2}A$, and the vector $-A$. What matrices are in the smallest subspace containing A?