Linear Algebra Homework 4

Deadline:11/26 24:00

0. Lecture notes from 11/10 to 11/19, please upload as a separate pdf file to the corresponding assignment.

1. Find the largest possible number of independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

2. Find a basis for each of these subspaces of R^4:

   (a) All vectors whose components are equal.

   (b) All vectors whose components add to zero.

   (c) All vectors that are perpendicular to (1, 1, 0, 0) and (1, 0, 1, 1).

   (d) The column space and the nullspace of I (4 by 4).

3. Choose $x = (x_1, x_2, x_3, x_4)$ in R^4. It has 24 rearrangements like (x_2, x_1, x_3, x_4) and (x_4, x_3, x_1, x_2). Those 24 vectors, including x itself, span a subspace S. Find specific vectors x so that the dimension of S is: (a) zero, (b) one, (c) three, (d) four.

4. Find bases and dimensions for the four subspaces associated with A and B:
$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \text{and} B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{bmatrix}$$

5. Suppose the m by n matrices A and B have the same four subspaces. If they are both in row reduced echelon form, prove that F must equal G:
$$A = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} I & G \\ 0 & 0 \end{bmatrix}$$

6. If a subspace S is contained in a subspace V, prove that $S^\perp$ contains $V^\perp$.

7. Construct a 3 by 3 matrix A with no zero entries whose columns are mutually perpendicular. Compute $A^T A$. Why is it a diagonal matrix?

8. Suppose S is spanned by the vectors (1, 2, 2, 3) and (1, 3, 3, 2). Find two vectors that span $S_\perp$. This is the same as solving $Ax = 0$ for which A?

9. Project b onto the column space of A by solving $A^T A\hat{x} = A^T b$ and $p = A\hat{x}$:

   (a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$

   Find $e = b - p$. It should be perpendicular to the columns of A.