Linear Algebra Homework 3

Deadline:11/12 24:00

0. Lecture notes from 10/27 to 11/5, please upload as a separate pdf file to the corresponding assignment.

1. Let A (m×n) , B (n×k) be two matrices. Show that the columns of AB are linear combinations of the columns of A. Then show that C(AB) $\subseteq$ C(A).

2. (a) Suppose column j of B is a combination of previous columns of B. Show that column j of AB is the same combination of previous columns of AB. Then AB cannot have new pivot columns, so $rank(AB) \leq rank(B)$.

   (b) Find A_1 and A_2 so that rank(A_1B) = 1 and rank(A_2B)=0 for B $= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

3. Consider A $= \begin{bmatrix} 2 & 8 & 1 & 0 & 7 \\ -3 & -12 & 0 & 2 & 2 \\ 5 & 20 & -2 & -1 & 0 \end{bmatrix}$ Find $N(A)$ the null space of A. Please also identify pivot variables and free variables.

4. Which of the following subsets of R^3 are actually subspaces?

   (a) The plane of vectors (b_1,b_2,b_3) with $b_1 = b_2$

   (b) The plane of vectors with $b_1 = 1$.

   (c) The vectors with $b_1b_2b_3 = 0$.

   (d) All linear combinations of v = (1,4,0) and w = (2,2,2).

   (e) All vectors that satisfy $b_1 + b_2 + b_3 = 0$.

   (f) All vectors with $b_1 \leq b_2 \leq b_3$

5. The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a "vector" in the space M of all 2 by 2 matrices. Write down the zero vector in this space, the vector $\frac{1}{2}$ A, and the vector -A. What matrices are in the smallest subspace containing A?

6. Suppose you know that the 3 by 4 matrix A has the vector s = (2, 3, 1, 0) as the only special solution to $Ax = 0$.

   (a) What is the *rank* of A and the complete solution to $Ax = 0$?

   (b) What is the exact row reduced echelon form R of A?

   (c) How do you know that $Ax = b$ can be solved for all b?