110550143 洪巧芸

Assignment 2

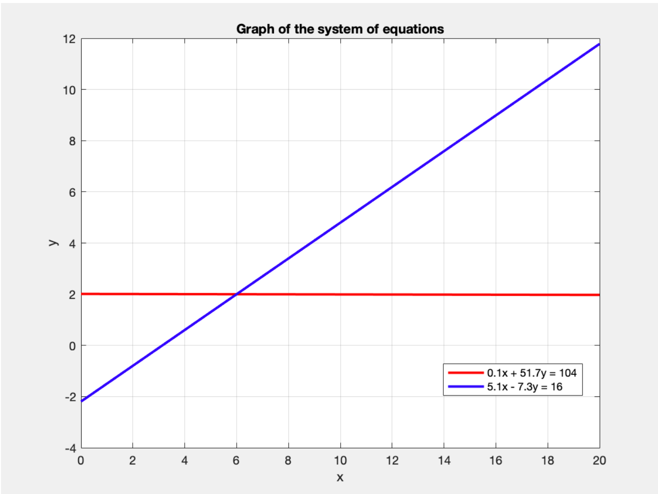
1. Calculate by Matlab (q1.m)

   -> no row interchanges

```
Solution:
    3.2099
    0.2346
    0.7160

Row interchanges: 0
```

2. Calculate by Matlab (q2.m)



(a) no row interchanges

$$\begin{bmatrix} 0.1 & 51.7 & 104 \\ 5.1 & -7.3 & 16 \end{bmatrix} \; \times -51$$

$51.7 \times 51 = 2636.7 \to 2640$

$-7.3 - 2640 = -2647.3 \to -2650$

$104 \times 51 = 5304 \to 5300$

$16 - 5300 = -5284 \to -5280$

$$\begin{bmatrix} 0.1 & 51.7 & 104 \\ 0 & -2650 & -5280 \end{bmatrix}$$

$-2650y = -5280 \Rightarrow y = 1.992\ldots \to y = 1.99$

$0.1x = 104 - \underbrace{51.7 \times 1.99}_{103} \Rightarrow x = 10$

$(x, y) = (10, 1.99)$

x誤差較大 #

(b) partial pivoting

$$\begin{bmatrix} 0.1 & 51.7 & 104 \\ 5.1 & -7.3 & 16 \end{bmatrix} \rightleftharpoons \begin{bmatrix} 5.1 & -7.3 & 16 \\ 0.1 & 51.7 & 104 \end{bmatrix} \; \div -51$$

$-7.3 / (-51) = 0.143$

$51.7 + 0.143 = 51.843 \to 51.8$

$16 / (-51) = -0.314$

$104 + (-0.314) = 103.686 \to 104$

$$\begin{bmatrix} 5.1 & -7.3 & 16 \\ 0 & 51.8 & 104 \end{bmatrix}$$

$51.8y = 104 \Rightarrow y = 2.007 \to y = 2.01$

$5.1x = 16 + \underbrace{7.3 \times 2.01}_{14.7} \Rightarrow x = 6.019\ldots \to x = 6.02$

$(x, y) = (6.02, 2.01)$

x,y誤差一樣 #

(c) Scaled partial pivoting

$$\begin{bmatrix} 0.1 & 51.7 & 104 \\ 5.1 & -7.3 & 16 \end{bmatrix} \begin{array}{c} \xrightarrow{\div 104} \\ \xrightarrow{\div 16} \end{array} \begin{bmatrix} 0.000962 & 0.497 & 1 \\ 0.319 & -0.456 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.319 & -0.456 & 1 \\ 0.000962 & 0.497 & 1 \end{bmatrix} \times \frac{0.000962}{0.319} = 0.003$$

$$\begin{bmatrix} 0.319 & -0.456 & 1 \\ 0 & 0.498 & 0.997 \end{bmatrix} \quad \begin{array}{l} -0.456 \times 0.003 = -0.01368 \to -0.00137 \\ 0.497 - (-0.00137) = 0.49837 \to 0.498 \\ 1 - 0.003 = 0.997 \end{array}$$

$0.498y = 0.997 \Rightarrow y = 2.002 \to y = 2.00$ $\quad (x,y) = (5.99, 2.00)$

$0.319x = \underbrace{1 + 0.456 \times 2}_{1.91} \Rightarrow x = 5.987 \to x = 5.99$ 和(a),(b)均不同 #

3. Calculate by Matlab (q3.m)

```
STEP(1)
L:
    1.0000         0         0         0
    1.0000    1.0000         0         0
    0.5000         0    1.0000         0
    0.5000         0         0    1.0000

U:
    1.0000   -0.5000    1.5000    1.0000
         0    1.5000   -1.5000    1.0000
         0    0.7500   -1.7500    0.5000
         0    1.7500    1.2500   -1.0000

STEP(2)
L:
    1.0000         0         0         0
    1.0000    1.0000         0         0
    0.5000    0.5000    1.0000         0
    0.5000    1.1667         0    1.0000

U:
    1.0000   -0.5000    1.5000    1.0000
         0    1.5000   -1.5000    1.0000
         0         0   -1.0000         0
         0         0    3.0000   -2.1667
```

```
STEP(3)
L:
    1.0000         0         0         0
    1.0000    1.0000         0         0
    0.5000    0.5000    1.0000         0
    0.5000    1.1667   -3.0000    1.0000

U:
    1.0000   -0.5000    1.5000    1.0000
         0    1.5000   -1.5000    1.0000
         0         0   -1.0000         0
         0         0         0   -2.1667

Matrix L:
    2.0000         0         0         0
    2.0000    2.0000         0         0
    1.0000    1.0000    2.0000         0
    1.0000    2.3333   -6.0000    2.0000

Matrix U:
    1.0000   -0.5000    1.5000    1.0000
         0    1.5000   -1.5000    1.0000
         0         0   -1.0000         0
         0         0         0   -2.1667
```

4. Calculate by Matlab (q4and5.m)

5. Calculate by Matlab (q4and5.m)

```
4. Jacobi method
  (a) Solution vector x:
       -0.1433
       -1.3746
       0.7199
  (b) Number of iterations: 32

5. Gauss-Seidel method
  (a) Solution vector x:
       -0.1433
       -1.3746
       0.7199
  (b) Number of iterations: 14
       -> 18 fewer iterations
```

6. Calculate by Matlab (q6.m)

```
PART(a)
(1) For initial guess x0 = [1, 1]:
    Converges to: [1, 1] (num(iter)=1)
(2) For initial guess x0 = [1, -1]:
    Alternates between: [1, -1], [-1, 1] (num(iter)=1000, 999)
(3) For initial guess x0 = [-1, 1]:
    Alternates between: [-1, 1], [1, -1] (num(iter)=1000, 999)
(4) For initial guess x0 = [2, 5]:
    Alternates between: [2, 5], [5, 2] (num(iter)=1000, 999)
(5) For initial guess x0 = [5, 2]:
    Alternates between: [5, 2], [2, 5] (num(iter)=1000, 999)

PART(b)
(1) For initial guess x0 = [1, 1]:
    Converges to: [1, 1] (num(iter)=1)
(3) For initial guess x0 = [1, -1]:
    Converges to: [-1, -1] (num(iter)=2)
(3) For initial guess x0 = [-1, 1]:
    Converges to: [1, 1] (num(iter)=2)
(4) For initial guess x0 = [2, 5]:
    Converges to: [5, 5] (num(iter)=2)
(5) For initial guess x0 = [5, 2]:
    Converges to: [2, 2] (num(iter)=2)

PART(c-a) 都 converges to [0,0]
(1) For initial guess x0 = [1, 1]:
    Converges to: [0.00199, 0.00199] (num(iter)=1241)
(1) For initial guess x0 = [1, -1]:
    Converges to: [0.00000, -0.00000] (num(iter)=2436)
(1) For initial guess x0 = [-1, 1]:
    Converges to: [-0.00000, 0.00000] (num(iter)=2436)
(1) For initial guess x0 = [2, 5]:
    Converges to: [0.00001, 0.00002] (num(iter)=2518)
(1) For initial guess x0 = [5, 2]:
    Converges to: [0.00002, 0.00001] (num(iter)=2518)

PART(c-b) 都 converges to [0,0]
(1) For initial guess x0 = [1, 1]:
    Converges to: [0.00099, 0.00098] (num(iter)=691)
(1) For initial guess x0 = [1, -1]:
    Converges to: [-0.00099, -0.00098] (num(iter)=691)
(1) For initial guess x0 = [-1, 1]:
    Converges to: [0.00099, 0.00098] (num(iter)=691)
(1) For initial guess x0 = [2, 5]:
    Converges to: [0.00099, 0.00099] (num(iter)=851)
(1) For initial guess x0 = [5, 2]:
    Converges to: [0.00099, 0.00098] (num(iter)=760)
```