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Assignment 3

1. Calculate by Matlab (q1.m)

(a) Divided-difference table:

x_i	$F[x_i]$	$F[x_i,x_{i+1}]$	$F[x_i,x_{i+1},x_{i+2}]$	$F[x_i,\ldots,x_{i+3}]$	$F[x_i,\ldots,x_{i+4}]$
-0.2	1.2300	2.2200	-11.8833	-103.5833	73.6111
0.3	2.3400	-8.4750	-1.5250	-81.5000	0
0.7	-1.0500	-7.5600	14.7750	0	0
-0.3	6.5100	-16.4250	0	0	0
0.1	-0.0600	0	0	0	0

(b) Interpolated value at x = 0.4: 2.7860

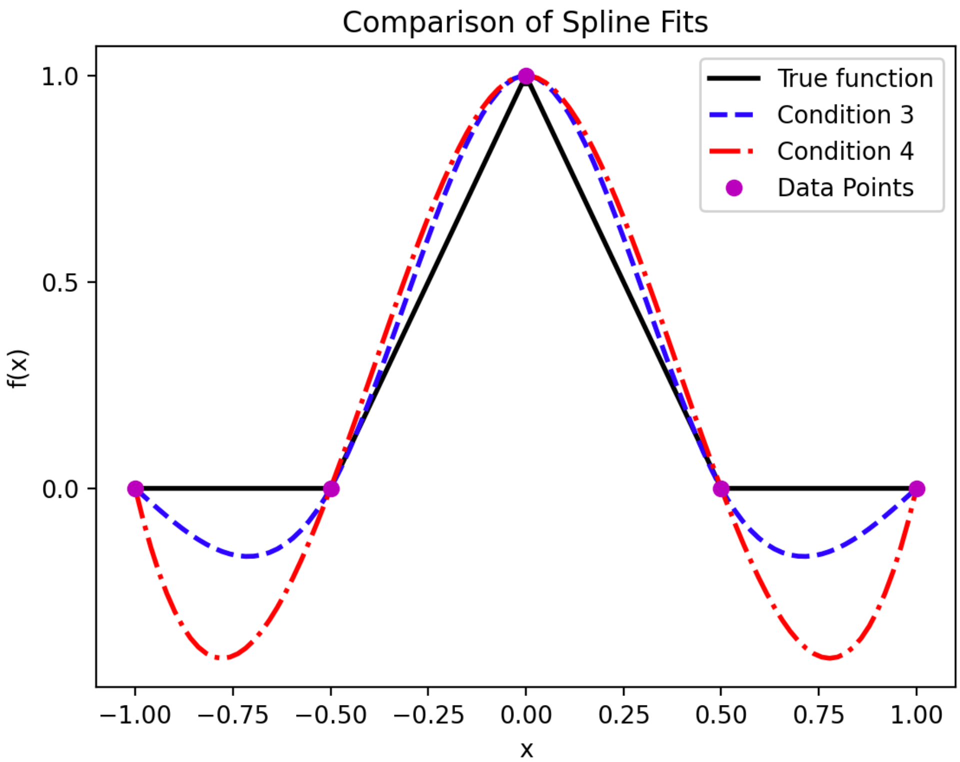
(c) Three points: 0.10 0.30 0.70

(1) Divided-difference table:

x_i	$F[x_i]$	$F[x_i,x_{i+1}]$	$F[x_i,x_{i+1},x_{i+2}]$
0.1	-0.0600	12.0000	-34.1250
0.3	2.3400	-8.4750	0
0.7	-1.0500	0	0

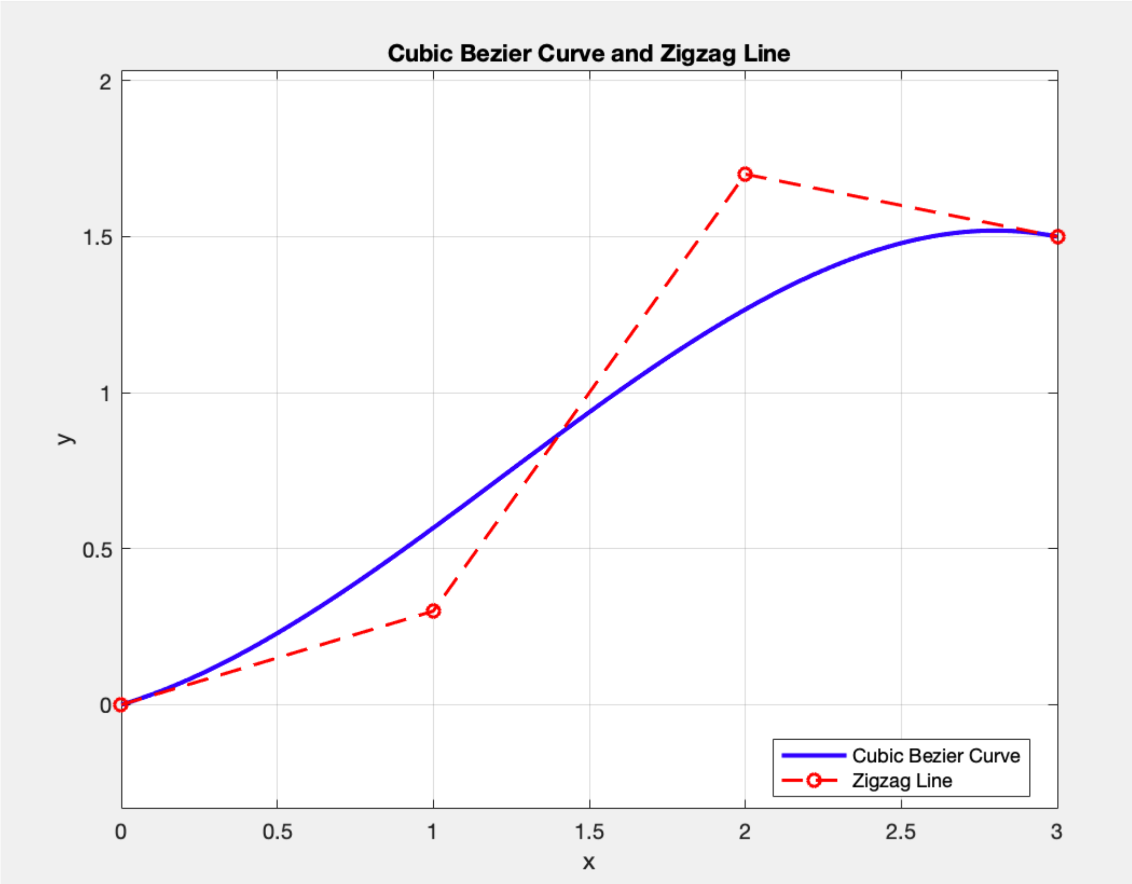
(2) Interpolated value at x = 0.4: 2.51625

2. Calculate by Python (q2.py) -> Condition 3 gives a best fit.



3. Calculate by Matlab (q3.m)

(a)



(b) (0,0), (1,0.3), (2,1.7), (3,1.5)

① 過四點的3次方程式

$P(u) = au^3 + bu^2 + cu$

$$\begin{cases} P(1) = a + b + c = 0.3 \\ P(2) = 8a + 4b + 2c = 1.7 \\ P(3) = 27a + 9b + 3c = 1.5 \end{cases} \Rightarrow \begin{cases} a = -\frac{9}{20} \\ b = \frac{19}{10} \\ c = \frac{-23}{20} \end{cases} \Rightarrow P(u) = -\frac{9}{20}u^3 + \frac{19}{10}u^2 - \frac{23}{20}u$$

$$\Rightarrow P'(u) = -\frac{27}{20}u^2 + \frac{19}{5}u - \frac{23}{20}$$

② 設新的 control point 為 $(1, \alpha), (2, \beta)$

$$\begin{cases} P'(0) = -\frac{23}{20} = 3(P_1 - P_0) = 3\alpha \Rightarrow \alpha = -\frac{23}{60} \\ P'(1) = -\frac{27}{20} + \frac{19}{5} - \frac{23}{20} = \frac{13}{10} = 3(P_3 - P_2) = 3(1.5 - \beta) \Rightarrow \beta = \frac{22}{30} \end{cases}$$

new control points are $(1, -\frac{23}{60}), (2, \frac{22}{30})$ #

4. Calculate by Python (q4.py)

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cassidy@cassidydeMacBook-Air hw03 % python3 q4.py
The estimated value of z(2.8, 0.54) is 4.9712
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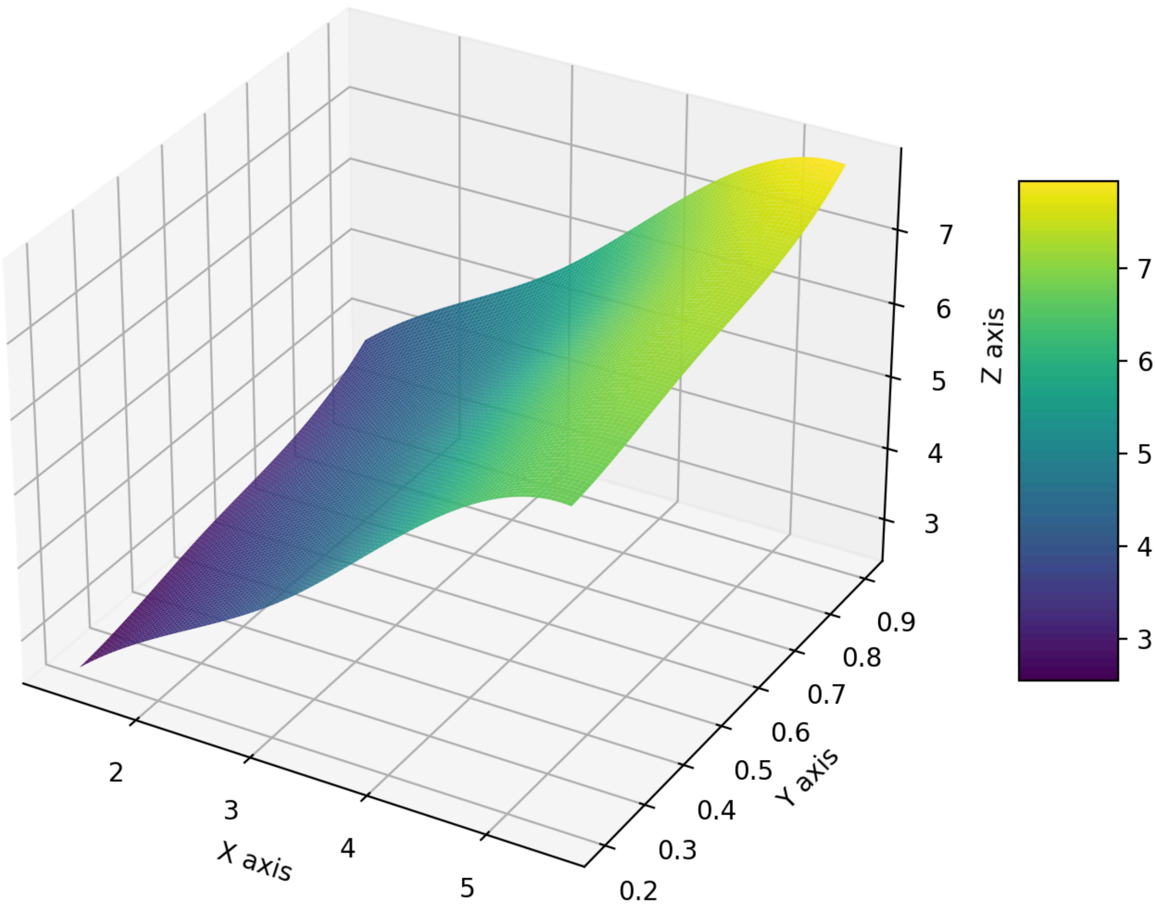

5. Calculate by Matlab (q5.m)

(a) normal equations

$$\begin{cases} a\left(\sum_{k=1}^{N} x_k^2\right) + b\left(\sum_{k=1}^{N} x_k y_k\right) + c\left(\sum_{k=1}^{N} x_k\right) = \sum_{k=1}^{N} z_k x_k \\ a\left(\sum_{k=1}^{N} x_k y_k\right) + b\left(\sum_{k=1}^{N} y_k^2\right) + c\left(\sum_{k=1}^{N} y_k\right) = \sum_{k=1}^{N} z_k y_k \\ a\left(\sum_{k=1}^{N} x_k\right) + b\left(\sum_{k=1}^{N} y_k\right) + Nc = \sum_{k=1}^{N} z_k \end{cases}$$

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(b) z = 1.59609x + -0.70238y + 0.22067.
    -> (a, b, c) = (1.59609, -0.70238, 0.22067)
(c) Sum of squares of deviations: 0.3194
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6. Calculate by Matlab (q6.m)

Chebyshev series -> $0.765198 \cdot T_0 - 0.229807 T_2 + 0.004953 T_4$

Power series -> $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

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Coefficients of Chebyshev:
  a0 = 0.765198, a1 = -0.000000, a2 = -0.229807, a3 = 0.000000, a4 = 0.004953
Max error Power: 1.1239
Max error Chebyshev: 0.93241
```

7. Calculate by Matlab (q7.m)

$a_0 = 0$

$a_n = \frac{2}{3}\int_{-1}^{2} (x^2-1)\cdot\cos\left(\frac{2n\pi x}{3}\right)dx$

$b_n = \frac{2}{3}\int_{-1}^{2} (x^2-1)\cdot\sin\left(\frac{2n\pi x}{3}\right)dx$

```
Display a0~a10, b1~b10:

a0  =    0.000
a1  = -1.28294, b1  = -0.31226
a2  =  0.29951, b2  =  0.43616
a3  =  0.10132, b3  = -0.31831
a4  = -0.23524, b4  =  0.07001
a5  =  0.14716, b5  =  0.12708
a6  =  0.02533, b6  = -0.15915
a7  = -0.12745, b7  =  0.05209
a8  =  0.09625, b8  =  0.07202
a9  =  0.01126, b9  = -0.10610
a10 = -0.08726, b10 =  0.03985
```