tutorial_1_launchWindow

September 28, 2019

1 Fast Launch Window Grid Search

The lowest-order method is well suited for a preliminary evaluation of the launch window. Here, an Earth Mars transfer is shown as an example. Here, all data is created and processed directly in the notebook. The github code, e.g. gridSearch114.py, includes functionality for saving to file and subsequent loading for postprocessing.

Here, the jpl ephemeris included in Pykep are used, meaning that no kernel files need to be loaded.

The grid search is performed over a wide range of launch dates and times of flight. In order to not miss any solutions the ΔV is computed for numbers of revolution in [0, 5] for each spot in the launch window. The stepSize is set to 20 days for both departure date and time of flight. Running the grid search should take about 3min.

[1]:
```
import time

import numpy as np
import matplotlib as mlt
%matplotlib inline

from hodographicShaping_SI import hodographicShaping
from patchedTrajectoryUtils import ephemeris
from plottingUtilsLaunchWindow import plottingGridSearch
from plottingUtilsIndividualTrajectory import plotting
```

[2]:
```
# optimization setup
Nmin = 0
Nmax = 5
depDateMin = 7305.
depDateMax = 10226.
tofMin = 500.
tofMax = 2001.
depBody = 'earth'
arrBody = 'mars'
stepSize= 20
```

[3]:
```
# initialize arrays and counters
Ns = np.arange(Nmin, Nmax+1)
depDates = np.arange(depDateMin, depDateMax, stepSize)
tofs = np.arange(tofMin, tofMax, stepSize)
```

```python
deltaVs = np.zeros((len(tofs), len(depDates), len(Ns)))
numberOfTrajectories = np.prod(np.shape(deltaVs))
print('Grid search using the Hodographic shaping method')
print('Computing', numberOfTrajectories, 'trajectories')
nIndex = 0
tofIndex = 0
depIndex = 0
trajectoryCounter = 1
```

```
Grid search using the Hodographic shaping method
Computing 67032 trajectories
```

[4]:
```python
# run grid search
start = time.process_time()
for i in Ns:
    for j in depDates:
        for k in tofs:
            scStateDep, __, __ = ephemeris(depBody, j, mode='jpl')
            scStateArr, __, __ = ephemeris(arrBody, j+k, mode='jpl')
            transfer = hodographicShaping(scStateDep, scStateArr,
                        departureDate=k, tof=k, N=i,
                        departureBody = depBody,
                        arrivalBody = arrBody,
                        rShape =         'CPowPow2_scaled',
                        thetaShape =     'CPowPow2_scaled',
                        zShape =         'CosR5P3CosR5P3SinR5_scaled',
                        rShapeFree =     'PSin05PCos05_scaled',
                        thetaShapeFree = 'PSin05PCos05_scaled',
                        zShapeFree =     'P4CosR5P4SinR5_scaled')
            transfer.shapingRadial()
            transfer.shapingVertical()
            transfer.shapingTransverse()
            transfer.assembleThrust()
            transfer.evaluate(evalThrust=False)

            print(str(trajectoryCounter)+'/'+str(numberOfTrajectories)+'\t'+\
                str(int(np.floor(trajectoryCounter/numberOfTrajectories*100)))+\
                '%', end='\r')
            deltaVs[tofIndex, depIndex, nIndex] = transfer.deltaV
            tofIndex += 1
            trajectoryCounter += 1
        depIndex += 1
        tofIndex = 0
    nIndex += 1
    depIndex = 0

# print elapsed time
```

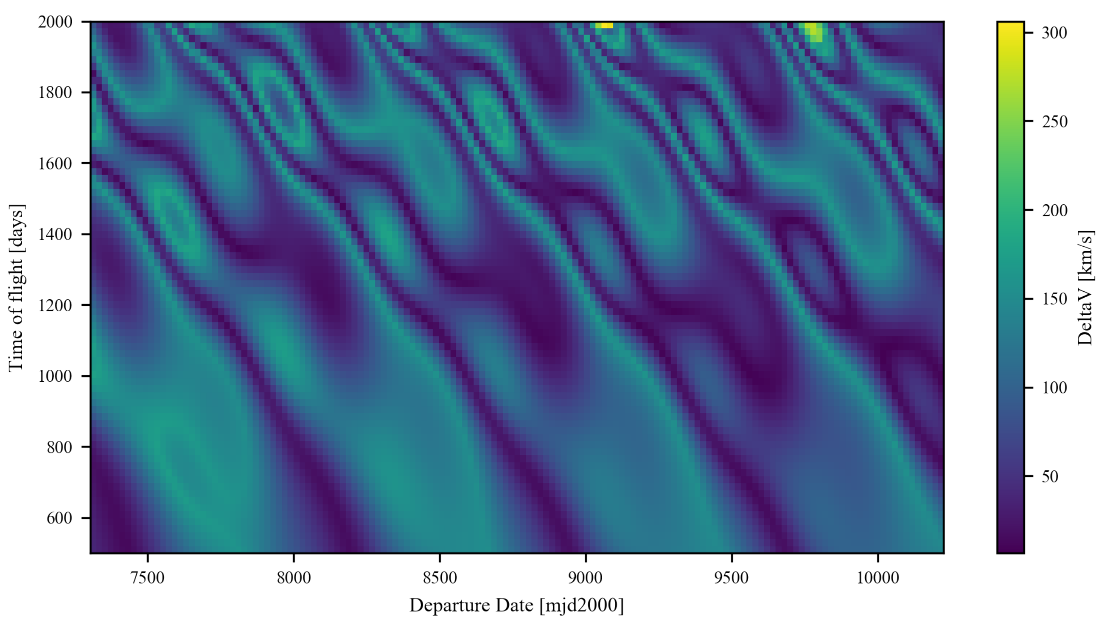
```
end = time.process_time()
print('Finished computation in ', round((end - start)/60, 2), ' min')
print('Computation time per trajectory was ',
        round((end - start)*1e3/numberOfTrajectories, 2), ' ms')

# combine results in a single dictionary
results = {}
results['deltaVs'] = deltaVs
results['depDates'] = depDates
results['Ns'] = Ns
results['tofs'] = tofs
```

```
Finished computation in  2.43  min
Computation time per trajectory was  2.17  ms
```

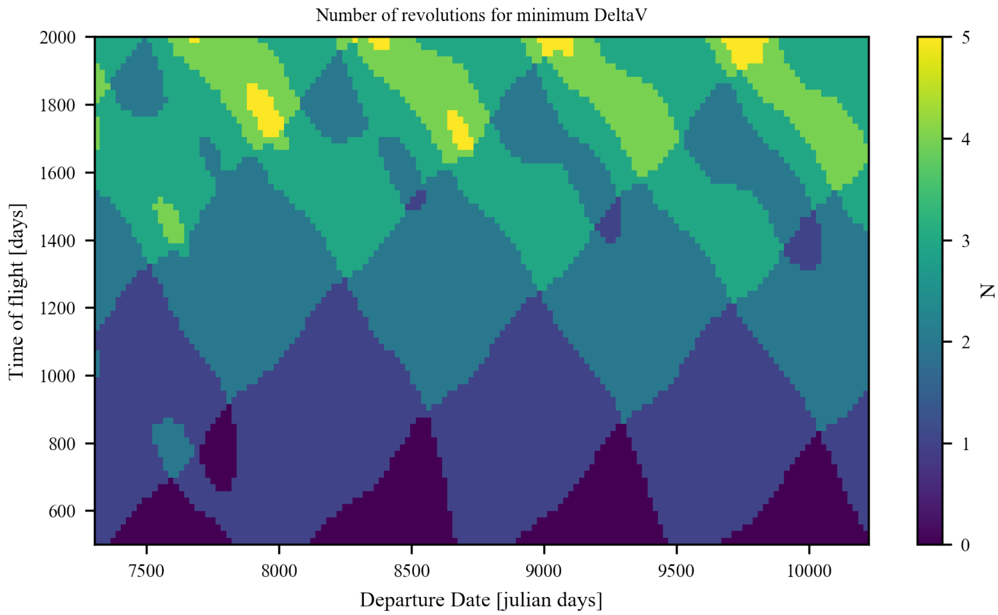
The results are visualized using the plotting functions in the plottingLaunchWindow class. For the overall result the lowest ΔV for each N is plotted.

[5]:
```
gridSearchVis = plottingGridSearch(results, save=False, folder=None)

# The best trajectory found for each point in the launch window
gridSearchVis.resultImshowBest()

# the corresponding number of revolutions
gridSearchVis.resultBestN()
```

```
Plotting best imshow
```



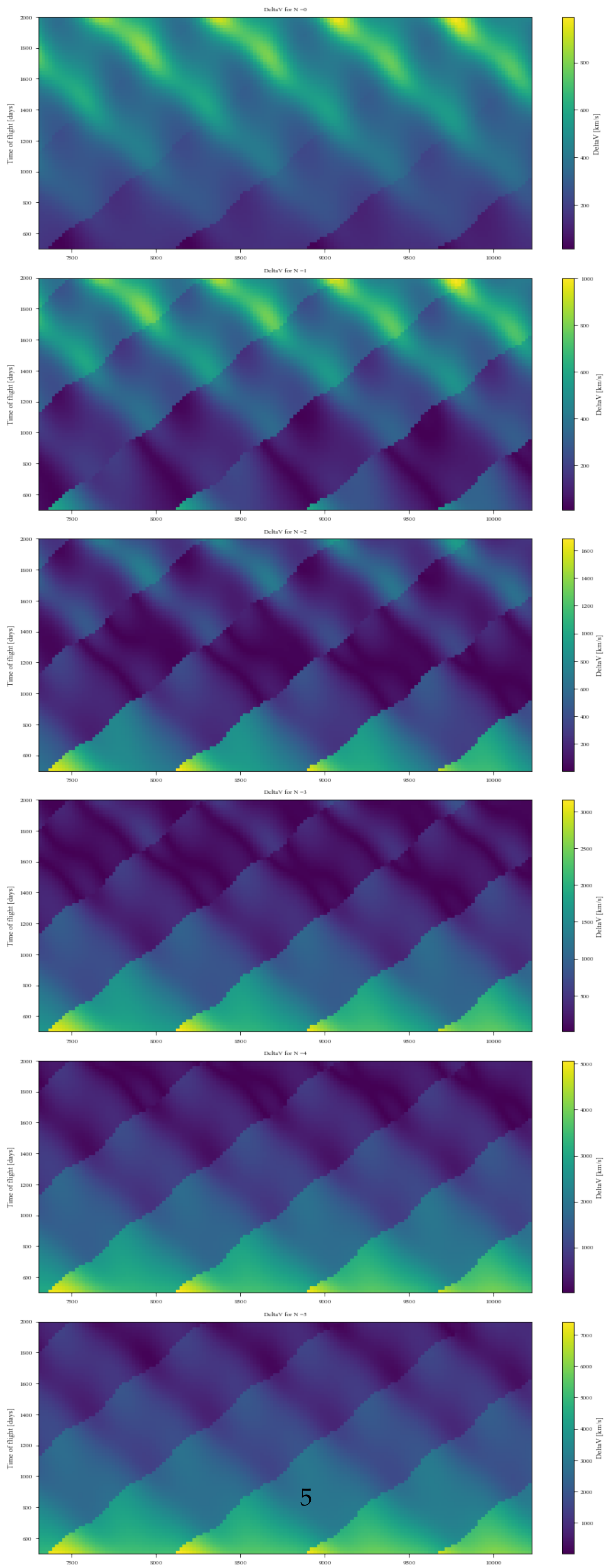
```
Plotting best Ns
```



The individual results for each N are shown here.

[6]:
```
gridSearchVis.resultContoursImshow()
```

```
Plotting contour (imshow)
```


DeltaV for N =0
Time of flight [days]
DeltaV [km/s]
DeltaV for N =1
Time of flight [days]
DeltaV [km/s]
DeltaV for N =2
Time of flight [days]
DeltaV [km/s]
DeltaV for N =3
Time of flight [days]
DeltaV [km/s]
DeltaV for N =4
Time of flight [days]
DeltaV [km/s]
DeltaV for N =5
Time of flight [days]
DeltaV [km/s]
Departure Date [julian days]

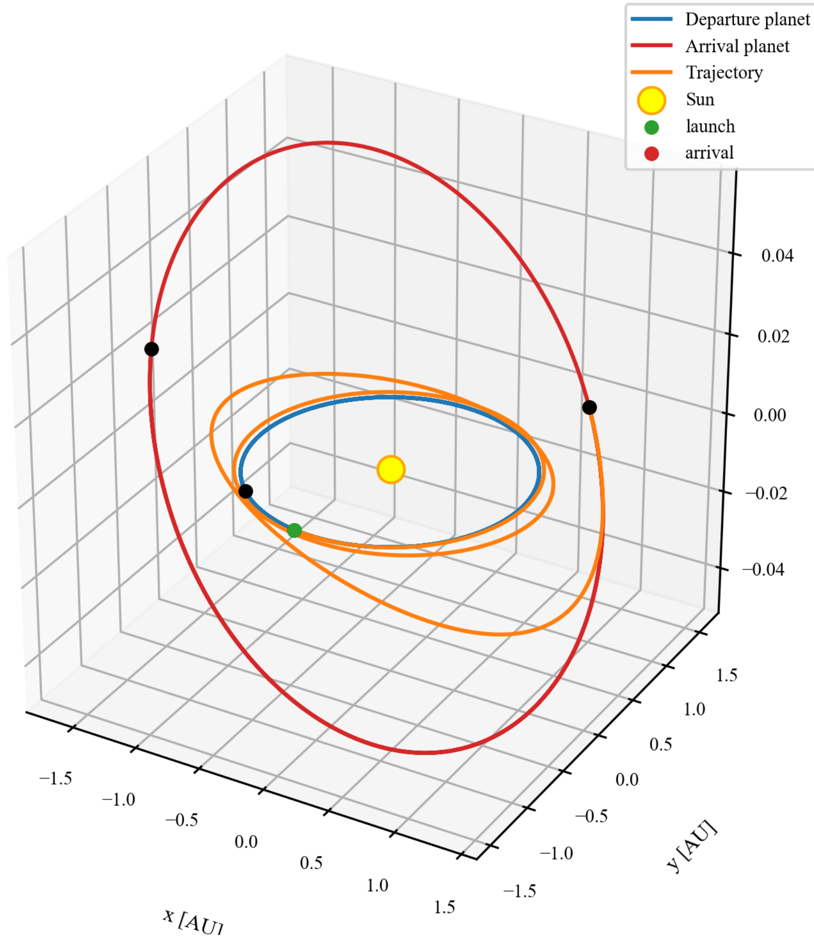
1.1 Retrieving the best trajectory

The trajectory with minimal ΔV can be retrieved and analysed separately.

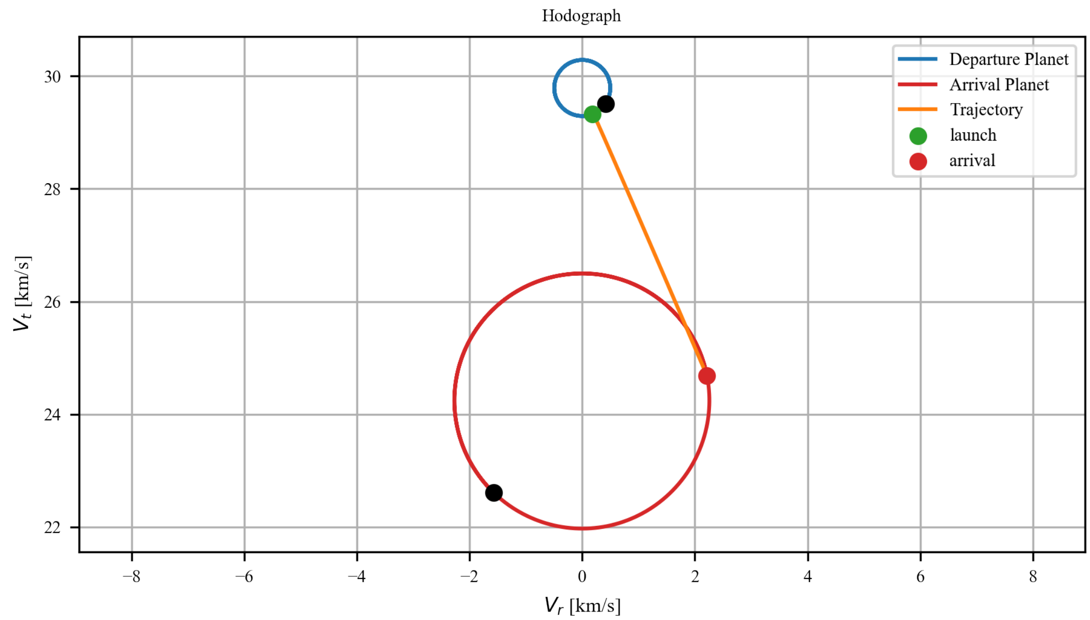
[7]:
```
# retrieve and recompute best trajectory
print('\nAnalyse the best trajectory')
minDeltaV = np.min(results['deltaVs'])
tofi, depi, ni = np.where(results['deltaVs'] == minDeltaV)
bestDepDate = int(results['depDates'][depi])
bestN = int(results['Ns'][ni])
bestTof = int(results['tofs'][tofi])
scStateDep, __, __ = ephemeris(depBody, bestDepDate, mode='jpl')
scStateArr, __, __ = ephemeris(arrBody, bestDepDate+bestTof, mode='jpl')
transfer = hodographicShaping(scStateDep, scStateArr,
            departureDate=bestDepDate,
            tof=bestTof,
            N=bestN,
            departureBody = depBody,
            arrivalBody = arrBody,
            rShape =         'CPowPow2_scaled',
            thetaShape =     'CPowPow2_scaled',
            zShape =         'CosR5P3CosR5P3SinR5_scaled',
            rShapeFree =     'PSin05PCos05_scaled',
            thetaShapeFree = 'PSin05PCos05_scaled',
            zShapeFree =     'P4CosR5P4SinR5_scaled')
transfer.shapingRadial()
transfer.shapingVertical()
transfer.shapingTransverse()
transfer.assembleThrust()
transfer.checkBoundaryConditions()
transfer.evaluate(evalThrust='Grid', nEvalPoints=1000)
transfer.status(printBC=False)

# plot best trajectory
visWiz = plotting(transfer, samples=1000, save=False, folder=None)
visWiz.trajectory3D(scaling=None)
visWiz.hodograph(twoDplot=True)
visWiz.thrust()
```

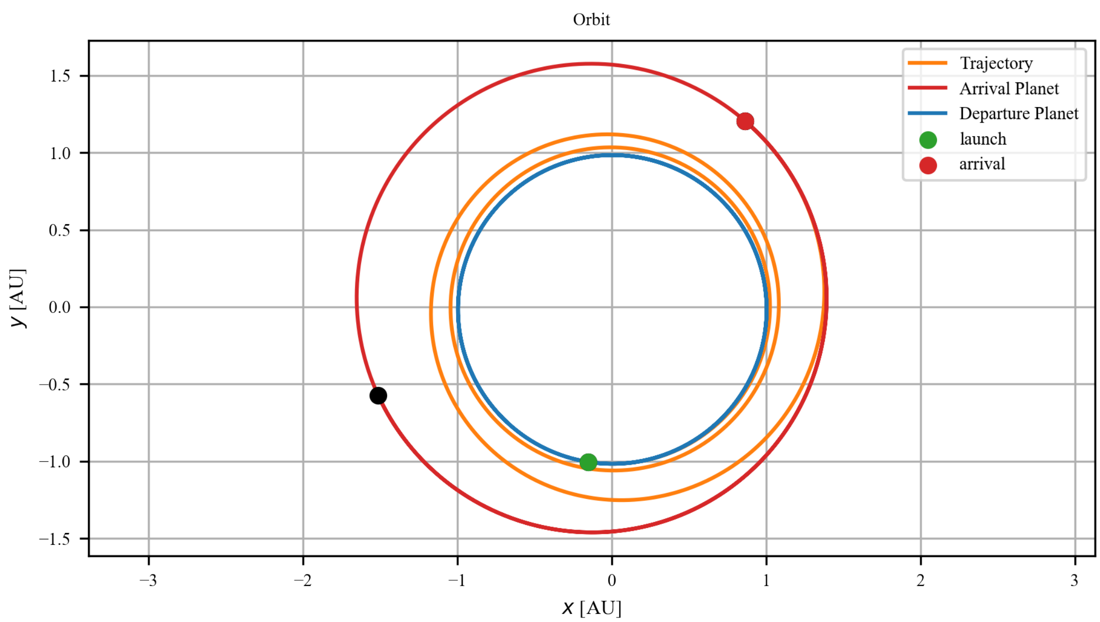
```
Analyse the best trajectory
###############################################################
Hodographic Shaping Problem: earth to mars

Settings
```

```
Departure state:  [ 1.52e+11 -1.72e+00  9.36e+06  1.83e+02  2.93e+04  2.84e-01]
Arrival state:    [2.22e+11 9.50e-01 6.15e+08 2.21e+03 2.47e+04 7.99e+02]
Departure date:  2027-Jun-13 00:00:00
Departure date:  10025 mjd2000
Arrival date:    2030-May-08 00:00:00
Time of Flight:  1060  days
Revolutions:     2
Transfer angle:  152.99  deg
Radial velocity:        CPowPow2_scaled
Traverse velocity:      CPowPow2_scaled
Axial velocity:         CosR5P3CosR5P3SinR5_scaled

Free part of shape (input)
Radial velocity free:   PSin05PCos05_scaled
Traverse velocity free: PSin05PCos05_scaled
Axial velocity free:    P4CosR5P4SinR5_scaled
Radial coefficients free:       [0 0]
Transverse coefficients free:   [0 0]
Vertical coefficients free:     [0 0]

Velocity functions
Radial coefficients:            [ 183.21 -587.52 2618.38]
Transverse coefficients:        [29326.3   1542.67 -6184.24]
Vertical coefficients:          [ 2.84e-01 -8.00e+02 -4.75e+01]
Position offsets (r0, theta0, z0):  [ 1.52e+11 -1.72e+00  9.36e+06]

Boundary condition check:
Velocity boundary conditions are satisfied!  Difference <  0.1  m/s
Position boundary conditions are satisfied!  Difference <  0.1 m and rad

Computation time
Computing this trajectory took 45.976 ms

Results
DeltaV:          6.43441  km/s
Max thrust:      0.0001438  m/s^2
################################################################

Begin plotting.
Sampling at 1000 points.
Done sampling planets.
Done sampling trajectory position.
Done sampling trajectory velocity.
Done sampling trajectory acceleration.
Plot 3D trajectory
```

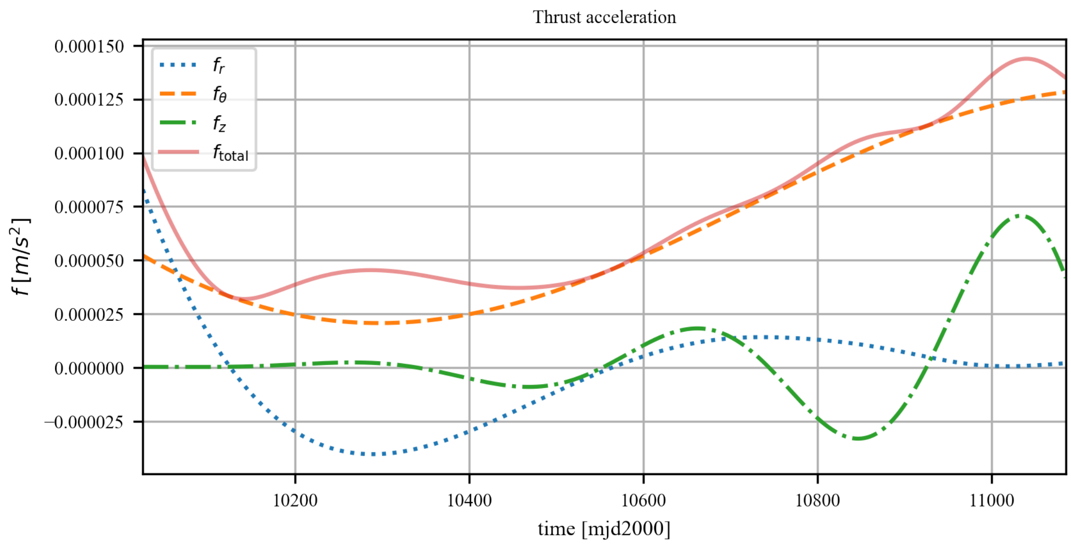



```
Plot hodograph
```


Hodograph
Departure Planet
Arrival Planet
Trajectory
launch
arrival
V_t [km/s]
V_r [km/s]
30
28
26
24
22
−8
−6
−4
−2
0
2
4
6
8


Orbit
Trajectory
Arrival Planet
Departure Planet
launch
arrival
y [AU]
x [AU]
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−3
−2
−1
0
1
2
3

Plot thrust



1.2 Evaluate an optimized launch window

Running each trajectory computation with free parameters and using an optimizer improves the results considerably. It also takes a lot longer. The step size was therefore reduced to 40 days. As this step size is fairly large, no adapted intitial guess is used and every optimization starts at 0. More information about initial guesses can be found in [1-3].

The progress of individual optimization runs can again be seen in the connected terminal window.
Running the following code overwrites the previous results and takes about 4 to 5 hours. I recommend looking at the results before running the code.

[1] D. Gondelach and R. Noomen, Hodographic-shaping method for low-thrust interplanetary trajectory design, Journal of Spacecraft and Rockets, 2015
[2] D. Gondelach, A Hodographic-Shaping Method for Low-Thrust Trajectory Design, MSc Thesis, TU Delft, 2012
[3] L. Stubbig, Investigating the use of neural network surrogate models in the evolutionary optimization of interplanetary low-thrust trajectories, MSc Thesis, TU Delft, 2019

[8]:
```
import pygmo as pg
from pygmoProblemsShaping import myProblemShapingSingle
```

[ ]:
```
# initialize arrays and counters
stepSize = 40
Ns = np.arange(Nmin, Nmax+1)
depDates = np.arange(depDateMin, depDateMax, stepSize)
tofs = np.arange(tofMin, tofMax, stepSize)
deltaVs = np.zeros((len(tofs), len(depDates), len(Ns)))
numberOfTrajectories = np.prod(np.shape(deltaVs))
print('Grid search using the Hodographic shaping method with 6 DoF')
print('Computing', numberOfTrajectories, 'trajectories')
```

```
nIndex = 0
tofIndex = 0
depIndex = 0
trajectoryCounter = 1

# optimization settings
nl = pg.nlopt('neldermead')
nl.maxeval = 2500
nl.xtol_abs = 1
algo = pg.algorithm(nl)
algo.set_verbosity(100)
startTotal = time.process_time()

# initializations
freeCs = np.zeros((len(tofs), len(depDates), len(Ns), 6))

for i in Ns:
    for j in depDates:
        for k in tofs:
            # counter
            print(str(trajectoryCounter)+'/'+str(numberOfTrajectories)+'\t'+\
                str(int(np.floor(trajectoryCounter/numberOfTrajectories*100)))+\
                '%', end='\r')

            # create problem instance
            scStateDep, __, __ = ephemeris(depBody, j, mode='jpl')
            scStateArr, __, __ = ephemeris(arrBody, j+k, mode='jpl')
            prob = pg.problem(myProblemShapingSingle(
                scStateDep, scStateArr,
                depDate=j, tof=k, N=i))
            pop = pg.population(prob, 1)

            # intial guess
            pop.set_x(0, [0, 0, 0, 0, 0, 0])
            initialGuess = pop.get_x()[pop.best_idx()]

            # run optimization
            start = time.process_time()
            pop = algo.evolve(pop)
            end = time.process_time()
            compTime = (end-start)

            # retrieve results
            bestTrajectory = pop.get_x()[pop.best_idx()]
            bestDeltaV = pop.get_f()[pop.best_idx()]
            nFeval = pop.problem.get_fevals()
            deltaVs[tofIndex, depIndex, nIndex] = bestDeltaV
```

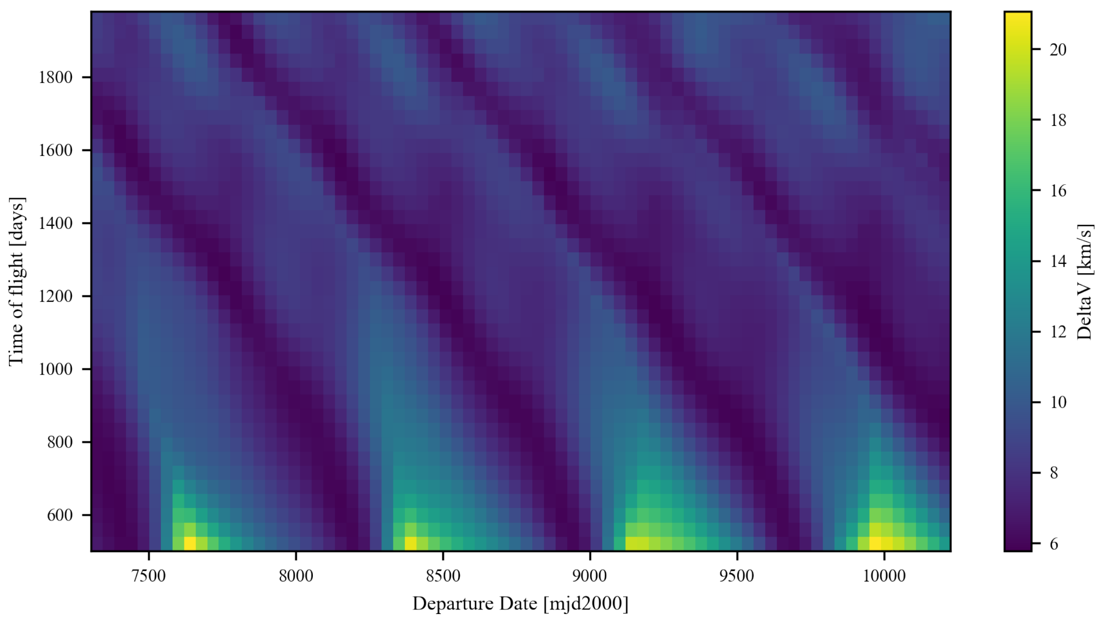
```
            freeCs[tofIndex, depIndex, nIndex, :] = bestTrajectory
            tofIndex += 1
            trajectoryCounter += 1

        depIndex += 1  # could be done with enumerate() in the for loop
        tofIndex = 0
    nIndex += 1
    depIndex = 0

# print elapsed time
endTotal = time.process_time()
print('Finished computation in ',\
      round((endTotal - startTotal)/60, 2), ' min')
print('Computation time per trajectory was ',
      round((endTotal - startTotal)/numberOfTrajectories, 2), ' s')

# combine results in a single dictionary
results = {}
results['deltaVs'] = deltaVs
results['depDates'] = depDates
results['Ns'] = Ns
results['tofs'] = tofs
results['freeCs'] = freeCs
```

```
Grid search using the Hodographic shaping method with 6 DoF
Computing 16872 trajectories
```

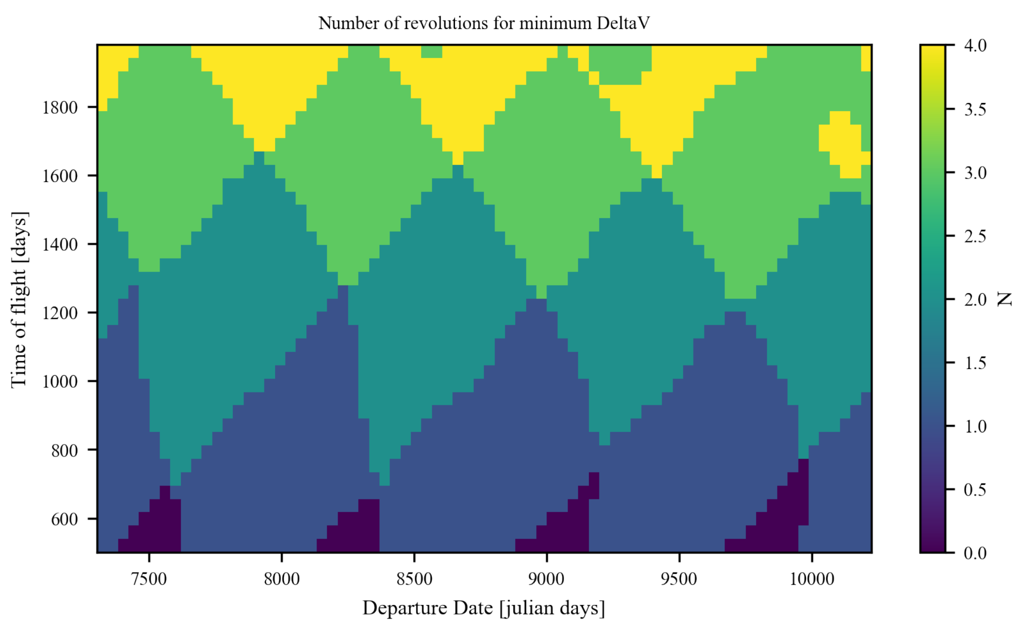
[12]:

```
gridSearchVis = plottingGridSearch(results, save=False, folder=None)

# The best trajectory found for each point in the launch window
gridSearchVis.resultImshowBest()

# the corresponding number of revolutions
gridSearchVis.resultBestN()

# the overview for each N
gridSearchVis.resultContoursImshow()
```

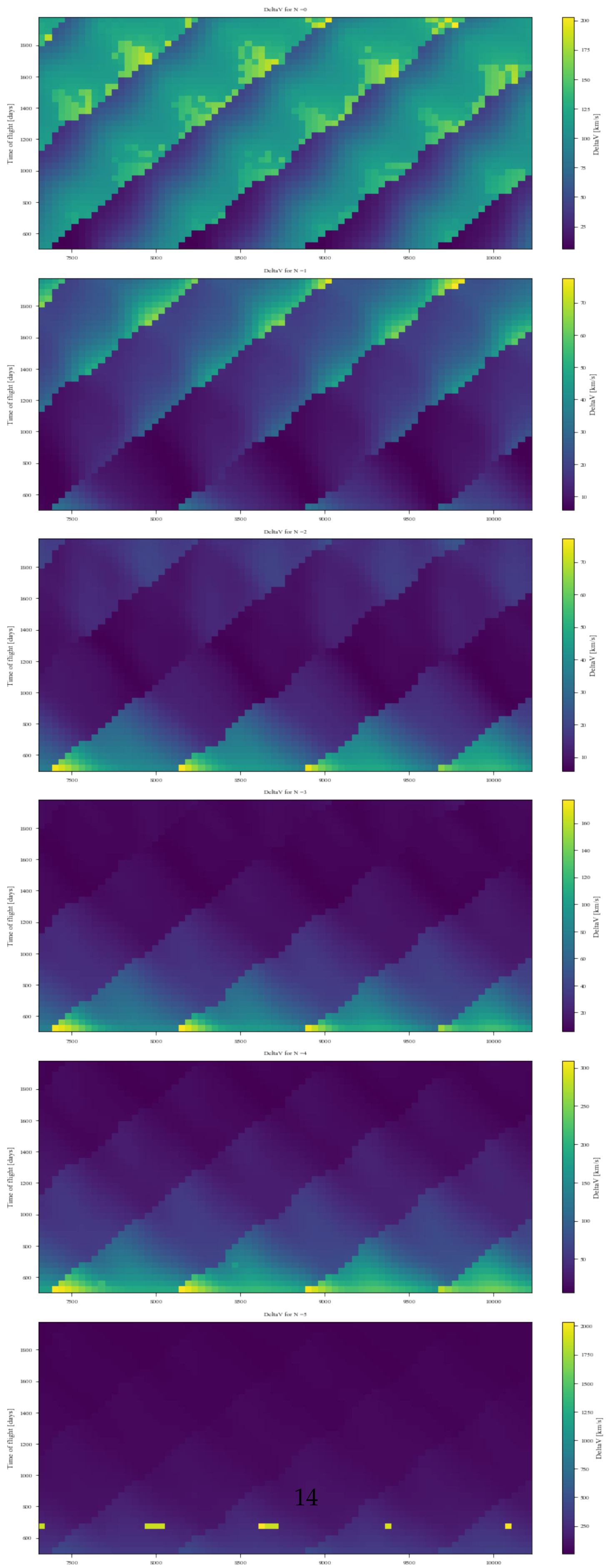
```
Plotting best imshow
```




```
Plotting best Ns
```




```
Plotting contour (imshow)
```



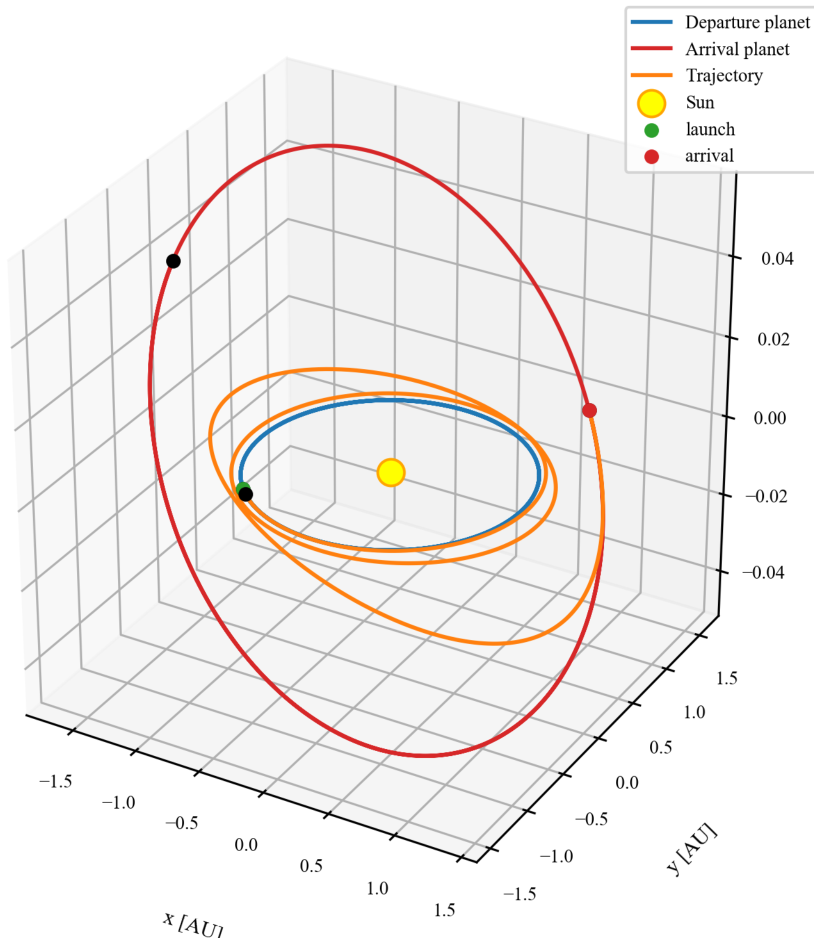
DeltaV for N =0
Time of flight [days]
DeltaV [km/s]
DeltaV for N =1
Time of flight [days]
DeltaV [km/s]
DeltaV for N =2
Time of flight [days]
DeltaV [km/s]
DeltaV for N =3
Time of flight [days]
DeltaV [km/s]
DeltaV for N =4
Time of flight [days]
DeltaV [km/s]
DeltaV for N =5
Time of flight [days]
DeltaV [km/s]
Departure Date [julian days]

The best trajectory is again retrieved and plotted, just as for the lowest-order search above.

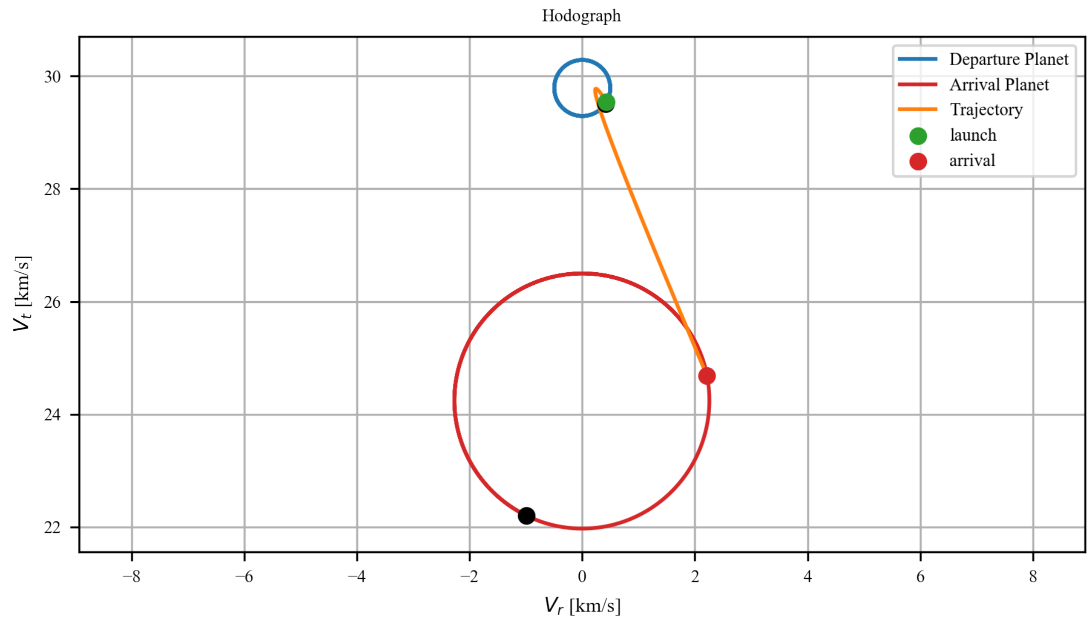
[11]:
```
# retrieve and recompute best trajectory
print('\nAnalyse the best trajectory')
minDeltaV = np.min(results['deltaVs'])
tofi, depi, ni = np.where(results['deltaVs'] == minDeltaV)
bestDepDate = int(results['depDates'][depi])
bestN = int(results['Ns'][ni])
bestTof = int(results['tofs'][tofi])
scStateDep, __, __ = ephemeris(depBody, bestDepDate, mode='jpl')
scStateArr, __, __ = ephemeris(arrBody, bestDepDate+bestTof, mode='jpl')
transfer = hodographicShaping(scStateDep, scStateArr,
            departureDate=bestDepDate,
            tof=bestTof,
            N=bestN,
            departureBody = depBody,
            arrivalBody = arrBody,
            rShape =         'CPowPow2_scaled',
            thetaShape =     'CPowPow2_scaled',
            zShape =         'CosR5P3CosR5P3SinR5_scaled',
            rShapeFree =     'PSin05PCos05_scaled',
            thetaShapeFree = 'PSin05PCos05_scaled',
            zShapeFree =     'P4CosR5P4SinR5_scaled')
transfer.shapingRadial()
transfer.shapingVertical()
transfer.shapingTransverse()
transfer.assembleThrust()
transfer.checkBoundaryConditions()
transfer.evaluate(evalThrust='Grid', nEvalPoints=1000)
transfer.status(printBC=False)

# plot best trajectory
vizzz = plotting(transfer, samples=1000, save=False, folder=None)
vizzz.trajectory3D(scaling=None)
vizzz.hodograph(twoDplot=True)
vizzz.thrust()
```

```
Analyse the best trajectory
###############################################################
Hodographic Shaping Problem: earth to mars

Settings
Departure state:  [ 1.51e+11 -2.39e+00  6.37e+06  4.31e+02  2.95e+04  1.36e+00]
Arrival state:    [2.22e+11 9.50e-01 6.15e+08 2.21e+03 2.47e+04 7.99e+02]
Departure date:  2027-May-04 00:00:00
```

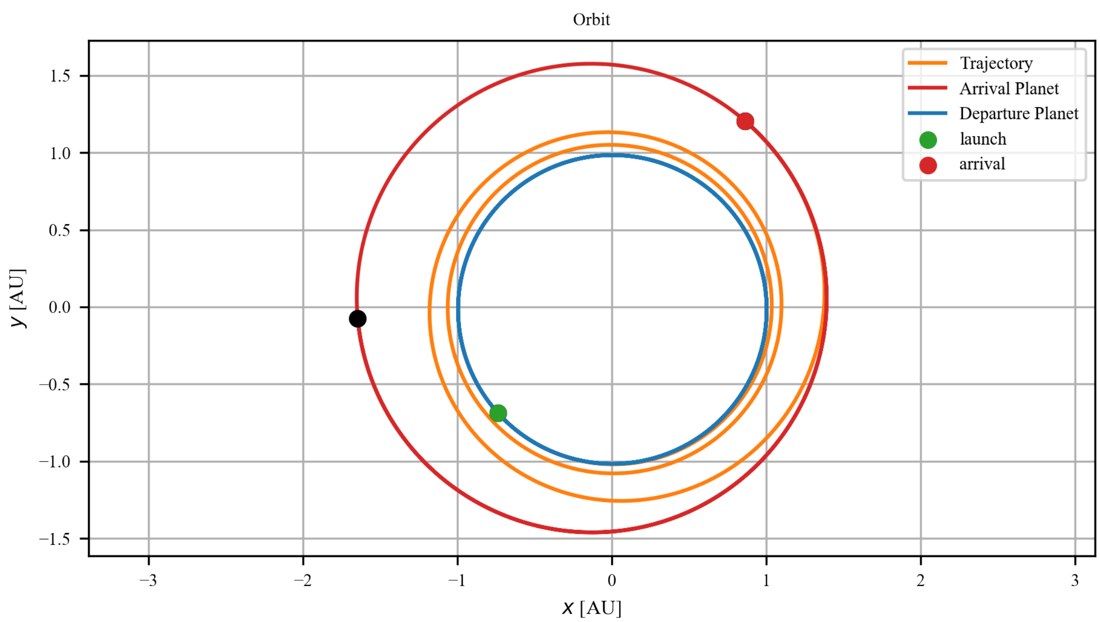
```
Departure date:  9985 mjd2000
Arrival date:    2030-May-08 00:00:00
Time of Flight:  1100  days
Revolutions:     2
Transfer angle:  191.46  deg
Radial velocity:         CPowPow2_scaled
Traverse velocity:       CPowPow2_scaled
Axial velocity:          CosR5P3CosR5P3SinR5_scaled

Free part of shape (input)
Radial velocity free:   PSin05PCos05_scaled
Traverse velocity free: PSin05PCos05_scaled
Axial velocity free:    P4CosR5P4SinR5_scaled
Radial coefficients free:        [0 0]
Transverse coefficients free:    [0 0]
Vertical coefficients free:      [0 0]

Velocity functions
Radial coefficients:             [  431.14 -1675.99  3458.91]
Transverse coefficients:         [29540.65  2664.23 -7520.15]
Vertical coefficients:           [   1.36 -800.67  -51.1 ]
Position offsets (r0, theta0, z0):  [ 1.51e+11 -2.39e+00  6.37e+06]

Boundary condition check:
Velocity boundary conditions are satisfied!  Difference <  0.1  m/s
Position boundary conditions are satisfied!  Difference <  0.1 m and rad

Computation time
Computing this trajectory took 14.691 ms

Results
DeltaV:          16.95092  km/s
Max thrust:      0.0002786  m/s^2
###############################################################

Begin plotting.
Sampling at 1000 points.
Done sampling planets.
Done sampling trajectory position.
Done sampling trajectory velocity.
Done sampling trajectory acceleration.
Plot 3D trajectory
```



Plot hodograph

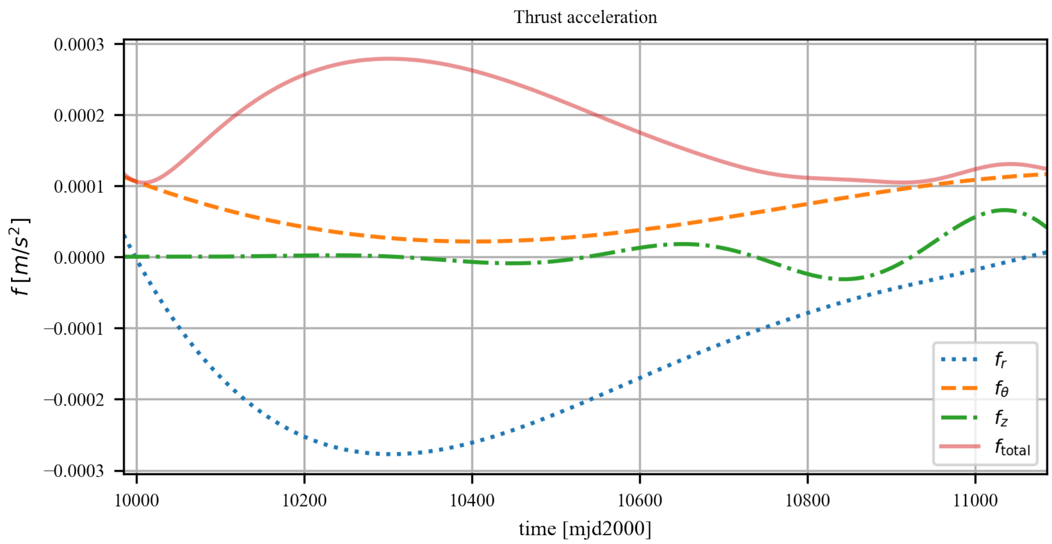


Hodograph
Departure Planet
Arrival Planet
Trajectory
launch
arrival
30
28
26
24
22
V_t [km/s]
−8
−6
−4
−2
0
2
4
6
8
V_r [km/s]



Orbit
Trajectory
Arrival Planet
Departure Planet
launch
arrival
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y [AU]
−3
−2
−1
0
1
2
3
x [AU]


```
Plot thrust
```


Thrust acceleration
0.0003
0.0002
0.0001
0.0000
−0.0001
−0.0002
−0.0003
$f\,[m/s^2]$
10000
10200
10400
10600
10800
11000
time [mjd2000]
f_r
f_θ
f_z
f_{total}

[ ]: