



Word Meaning and Similarity

Word Senses and Word Relations



Reminder: lemma and wordform

- **A lemma or citation form**
 - Same stem, part of speech, rough semantics
- **A wordform**
 - The “inflected” word as it appears in text

Wordform	Lemma
banks	bank
sung	sing
duermes	dormir



Lemmas have senses

- One lemma “bank” can have many meanings:

Sense 1: • ...a **bank**¹ can hold the investments in a custodial account...

Sense 2: • “...as agriculture burgeons on the east **bank**² the river will shrink even more”

- **Sense (or word sense)**
 - A discrete representation
of an aspect of a word’s meaning.
- The lemma **bank** here has two senses



Homonymy

Homonyms: words that share a form but have unrelated, distinct meanings:

- **bank₁**: financial institution, **bank₂**: sloping land
- **bat₁**: club for hitting a ball, **bat₂**: nocturnal flying mammal

1. Homographs (bank/bank, bat/bat)

2. Homophones:

1. **Write** and **right**
2. **Piece** and **peace**



Homonymy causes problems for NLP applications

- Information retrieval
 - “bat care”
- Machine Translation
 - bat: **murciélagos** (animal) or **bate** (for baseball)
- Text-to-Speech
 - bass (stringed instrument) vs. bass (fish)



Polysemy

- 1. The **bank** was constructed in 1875 out of local red brick.
- 2. I withdrew the money from the **bank**
- Are those the same sense?
 - Sense 2: “A financial institution”
 - Sense 1: “The building belonging to a financial institution”
- A **polysemous** word has **related** meanings
 - Most non-rare words have multiple meanings



Metonymy or Systematic Polysemy:

A systematic relationship between senses

- Lots of types of polysemy are systematic
 - School, university, hospital
 - All can mean the institution or the building.
- A systematic relationship:
 - Building ↔ Organization
- Other such kinds of systematic polysemy:

Author (Jane Austen wrote Emma)

↔ Works of Author (I love Jane Austen)

Tree (Plums have beautiful blossoms)

↔ Fruit (I ate a preserved plum)



How do we know when a word has more than one sense?

- The “zeugma” test: Two senses of serve?
 - Which flights **serve** breakfast?
 - Does Lufthansa **serve** Philadelphia?
 - ?Does Lufthansa serve breakfast and San Jose?
- Since this conjunction sounds weird,
 - we say that these are two different senses of “serve”



Synonyms

- Word that have the same meaning in some or all contexts.
 - filbert / hazelnut
 - couch / sofa
 - big / large
 - automobile / car
 - vomit / throw up
 - Water / H₂O
- Two lexemes are synonyms
 - if they can be substituted for each other in all situations
 - If so they have the same **propositional meaning**



Synonyms

- But there are few (or no) examples of perfect synonymy.
 - Even if many aspects of meaning are identical
 - Still may not preserve the acceptability based on notions of politeness, slang, register, genre, etc.
- Example:
 - Water/H₂O
 - Big/large
 - Brave/courageous



Synonymy is a relation between senses rather than words

- Consider the words *big* and *large*
- Are they synonyms?
 - How **big** is that plane?
 - Would I be flying on a **large** or small plane?
- How about here:
 - Miss Nelson became a kind of **big** sister to Benjamin.
 - ?Miss Nelson became a kind of **large** sister to Benjamin.
- Why?
 - *big* has a sense that means being older, or grown up
 - *large* lacks this sense



Antonyms

- Senses that are opposites with respect to one feature of meaning
- Otherwise, they are very similar!
dark/light short/long fast/slow rise/fall
hot/cold up/down in/out
- More formally: antonyms can
 - define a binary opposition
or be at opposite ends of a scale
 - long/short, fast/slow
 - Be **reversives**:
 - rise/fall, up/down



Hyponymy and Hypernymy

- One sense is a **hyponym** of another if the first sense is more specific, denoting a subclass of the other
 - *car* is a hyponym of *vehicle*
 - *mango* is a hyponym of *fruit*
- Conversely **hypernym/superordinate** (“hyper is super”)
 - *vehicle* is a **hypernym** of *car*
 - *fruit* is a hypernym of *mango*

Superordinate/hyper	vehicle	fruit	furniture
Subordinate/hyponym	car	mango	chair



Hyponymy more formally

- Extensional:
 - The class denoted by the superordinate extensionally includes the class denoted by the hyponym
- Entailment:
 - A sense A is a hyponym of sense B if *being an A* entails *being a B*
- Hyponymy is usually transitive
 - (A hypo B and B hypo C entails A hypo C)
- Another name: the **IS-A hierarchy**
 - A **IS-A** B (or A **ISA** B)
 - B **subsumes** A



Hyponyms and Instances

- WordNet has both **classes** and **instances**.
- An **instance** is an individual, a proper noun that is a unique entity
 - San Francisco is an **instance** of city
 - But city is a class
 - city is a **hyponym** of municipality...location...



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Word Senses and Word Relations

[illegible]

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Applications of Thesauri and Ontologies

- Information Extraction
- Information Retrieval
- Question Answering
- Bioinformatics and Medical Informatics
- Machine Translation



WordNet 3.0

- A hierarchically organized lexical database
- On-line thesaurus + aspects of a dictionary
 - Some other languages available or under development
 - (Arabic, Finnish, German, Portuguese...)

Category	Unique Strings
Noun	117,798
Verb	11,529
Adjective	22,479
Adverb	4,481



Senses of “bass” in Wordnet

Noun

- **S: (n) bass** (the lowest part of the musical range)
- **S: (n) bass, bass part** (the lowest part in polyphonic music)
- **S: (n) bass, basso** (an adult male singer with the lowest voice)
- **S: (n) sea bass, bass** (the lean flesh of a saltwater fish of the family Serranidae)
- **S: (n) freshwater bass, bass** (any of various North American freshwater fish with lean flesh (especially of the genus Micropterus))
- **S: (n) bass, bass voice, basso** (the lowest adult male singing voice)
- **S: (n) bass** (the member with the lowest range of a family of musical instruments)
- **S: (n) bass** (nontechnical name for any of numerous edible marine and freshwater spiny-finned fishes)

Adjective

- **S: (adj) bass, deep** (having or denoting a low vocal or instrumental range) *"a deep voice"; "a bass voice is lower than a baritone voice"; "a bass clarinet"*



How is “sense” defined in WordNet?

- The **synset (synonym set)**, the set of near-synonyms, instantiates a sense or concept, with a **gloss**
- Example: **chump** as a noun with the **gloss**:
“a person who is gullible and easy to take advantage of”
- This sense of “chump” is shared by 9 words:
chump¹, fool², gull¹, mark⁹, patsy¹, fall guy¹,
sucker¹, soft touch¹, mug²
- Each of **these** senses have this same gloss
 - (Not **every** sense; sense 2 of gull is the aquatic bird)



WordNet Hypernym Hierarchy for “bass”

- [S: \(n\) bass](#), [basso](#) (an adult male singer with the lowest voice)
 - [direct hypernym](#) / [inherited hypernym](#) / [sister term](#)
 - [S: \(n\) singer](#), [vocalist](#), [vocalizer](#), [vocaliser](#) (a person who sings)
 - [S: \(n\) musician](#), [instrumentalist](#), [player](#) (someone who plays a musical instrument (as a profession))
 - [S: \(n\) performer](#), [performing artist](#) (an entertainer who performs a dramatic or musical work for an audience)
 - [S: \(n\) entertainer](#) (a person who tries to please or amuse)
 - [S: \(n\) person](#), [individual](#), [someone](#), [somebody](#), [mortal](#), [soul](#) (a human being) *"there was too much for one person to do"*
 - [S: \(n\) organism](#), [being](#) (a living thing that has (or can develop) the ability to act or function independently)
 - [S: \(n\) living thing](#), [animate thing](#) (a living (or once living) entity)
 - [S: \(n\) whole](#), [unit](#) (an assemblage of parts that is regarded as a single entity) *"how big is that part compared to the whole?"*; *"the team is a unit"*
 - [S: \(n\) object](#), [physical object](#) (a tangible and visible entity; an entity that can cast a shadow) *"it was full of rackets, balls and other objects"*
 - [S: \(n\) physical entity](#) (an entity that has physical existence)
 - [S: \(n\) entity](#) (that which is perceived or known or inferred to have its own distinct existence (living or nonliving))



WordNet Noun Relations

Relation	Also called	Definition	Example
Hypernym	Superordinate	From concepts to superordinates	<i>breakfast</i> ¹ → <i>meal</i> ¹
Hyponym	Subordinate	From concepts to subtypes	<i>meal</i> ¹ → <i>lunch</i> ¹
Member Meronym	Has-Member	From groups to their members	<i>faculty</i> ² → <i>professor</i> ¹
Has-Instance		From concepts to instances of the concept	<i>composer</i> ¹ → <i>Bach</i> ¹
Instance		From instances to their concepts	<i>Austen</i> ¹ → <i>author</i> ¹
Member Holonym	Member-Of	From members to their groups	<i>copilot</i> ¹ → <i>crew</i> ¹
Part Meronym	Has-Part	From wholes to parts	<i>table</i> ² → <i>leg</i> ³
Part Holonym	Part-Of	From parts to wholes	<i>course</i> ⁷ → <i>meal</i> ¹
Antonym		Opposites	<i>leader</i> ¹ → <i>follower</i> ¹

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WordNet 3.0

- Where it is:
 - <http://wordnetweb.princeton.edu/perl/webwn>
- Libraries
 - Python: WordNet from NLTK
 - <http://www.nltk.org/Home>
 - Java:
 - JWNL, extJWNL on sourceforge



MeSH: Medical Subject Headings thesaurus from the National Library of Medicine

- **MeSH (Medical Subject Headings)**
 - 177,000 entry terms that correspond to 26,142 biomedical “headings”

- **Hemoglobins**

Entry Terms: Eryhem, Ferrous Hemoglobin, Hemoglobin

Synset

Definition: The oxygen-carrying proteins of ERYTHROCYTES. They are found in all vertebrates and some invertebrates. The number of globin subunits in the hemoglobin quaternary structure differs between species. Structures range from monomeric to a variety of multimeric arrangements



The MeSH Hierarchy

1. + **Anatomy [A]**
2. + **Organisms [B]**
3. + **Diseases [C]**
4. - **Chemicals and Drugs [D]**
 - [Inorganic Chemicals \[D01\]](#) +
 - [Organic Chemicals \[D02\]](#) +
 - [Heterocyclic Compounds \[D03\]](#) +
 - [Polycyclic Compounds \[D04\]](#) +
 - [Macromolecular Substances \[D05\]](#) +
 - [Hormones, Hormone Substitutes, and](#)
 - [Enzymes and Coenzymes \[D08\]](#) +
 - [Carbohydrates \[D09\]](#) +
 - [Lipids \[D10\]](#) +
 - [Amino Acids, Peptides, and Proteins](#)
 - [Nucleic Acids, Nucleotides, and Nucl](#)
 - [Complex Mixtures \[D20\]](#) +
 - [Biological Factors \[D23\]](#) +
 - [Biomedical and Dental Materials \[D25\]](#) +
 - [Pharmaceutical Preparations \[D26\]](#) +

[Amino Acids, Peptides, and Proteins \[D12\]](#)

[Proteins \[D12.776\]](#)

[Blood Proteins \[D12.776.124\]](#)

[Acute-Phase Proteins \[D12.776.124.050\]](#) +

[Anion Exchange Protein 1, Erythrocyte \[D12.776.124.078\]](#)

[Ankyrins \[D12.776.124.080\]](#)

[beta 2-Glycoprotein I \[D12.776.124.117\]](#)

[Blood Coagulation Factors \[D12.776.124.125\]](#) +

[Cholesterol Ester Transfer Proteins \[D12.776.124.197\]](#)

[Fibrin \[D12.776.124.270\]](#) +

[Glycophorin \[D12.776.124.300\]](#)

[Hemocyanin \[D12.776.124.337\]](#)

▶ [Hemoglobins \[D12.776.124.400\]](#)

[Carboxyhemoglobin \[D12.776.124.400.141\]](#)

[Erythrocyte proteins \[D12.776.124.400.220\]](#)



Uses of the MeSH Ontology

- Provide synonyms (“entry terms”)
 - E.g., glucose and dextrose
- Provide hypernyms (from the hierarchy)
 - E.g., glucose ISA monosaccharide
- Indexing in MEDLINE/PubMED database
 - NLM’s bibliographic database:
 - 20 million journal articles
 - Each article hand-assigned 10-20 MeSH terms

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Word Meaning and Similarity

Word Similarity:
Thesaurus Methods



Word Similarity

- **Synonymy**: a binary relation
 - Two words are either synonymous or not
- **Similarity (or distance)**: a looser metric
 - Two words are more similar if they share more features of meaning
- Similarity is properly a relation between **senses**
 - The word “bank” is not similar to the word “slope”
 - Bank¹ is similar to fund³
 - Bank² is similar to slope⁵
- But we’ll compute similarity over both words and senses

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Why word similarity

- Information retrieval
- Question answering
- Machine translation
- Natural language generation
- Language modeling
- Automatic essay grading
- Plagiarism detection
- Document clustering



Word similarity and word relatedness

- We often distinguish **word similarity** from **word relatedness**
 - **Similar words**: near-synonyms
 - **Related words**: can be related any way
 - car, bicycle: **similar**
 - car, gasoline: **related**, not similar

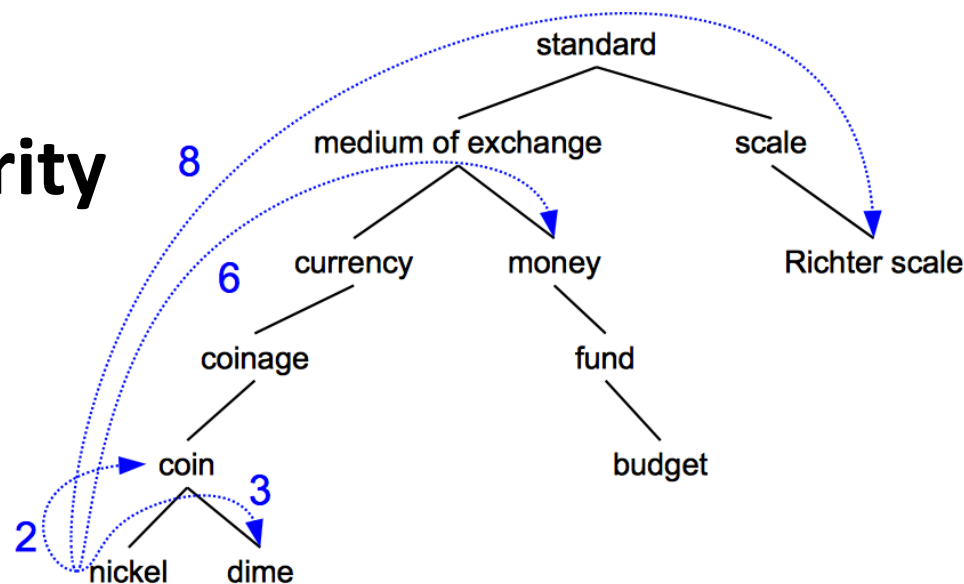


Two classes of similarity algorithms

- Thesaurus-based algorithms
 - Are words “nearby” in hypernym hierarchy?
 - Do words have similar glosses (definitions)?
- Distributional algorithms
 - Do words have similar distributional contexts?



Path based similarity



- Two concepts (senses/synsets) are similar if they are near each other in the thesaurus hierarchy
 - =have a short path between them
 - concepts have path 1 to themselves



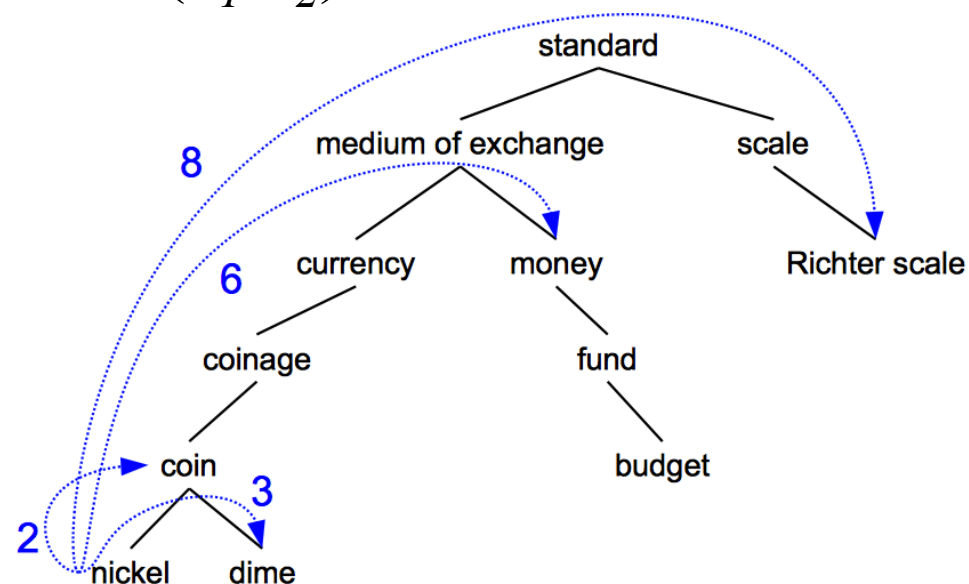
Refinements to path-based similarity

- $\text{pathlen}(c_1, c_2) = 1 + \text{number of edges in the shortest path in the hypernym graph between sense nodes } c_1 \text{ and } c_2$
- ranges from 0 to 1 (identity)
- $\text{simpath}(c_1, c_2) = \frac{1}{\text{pathlen}(c_1, c_2)}$
- $\text{wordsim}(w_1, w_2) = \max_{c_1 \in \text{senses}(w_1), c_2 \in \text{senses}(w_2)} \text{sim}(c_1, c_2)$



Example: path-based similarity

$$\text{simpath}(c_1, c_2) = 1/\text{pathlen}(c_1, c_2)$$



$$\text{simpath}(\textit{nickel}, \textit{coin}) = 1/2 = .5$$

$$\text{simpath}(\textit{fund}, \textit{budget}) = 1/2 = .5$$

$$\text{simpath}(\textit{nickel}, \textit{currency}) = 1/4 = .25$$

$$\text{simpath}(\textit{nickel}, \textit{money}) = 1/6 = .17$$

$$\text{simpath}(\textit{coinage}, \textit{Richter scale}) = 1/6 = .17$$



Problem with basic path-based similarity

- Assumes each link represents a uniform distance
 - But *nickel* to *money* seems to us to be closer than *nickel* to *standard*
 - Nodes high in the hierarchy are very abstract
- We instead want a metric that
 - Represents the cost of each edge independently
 - Words connected only through abstract nodes
 - are less similar



Information content similarity metrics

Resnik 1995. Using information content to evaluate semantic similarity in a taxonomy. IJCAI

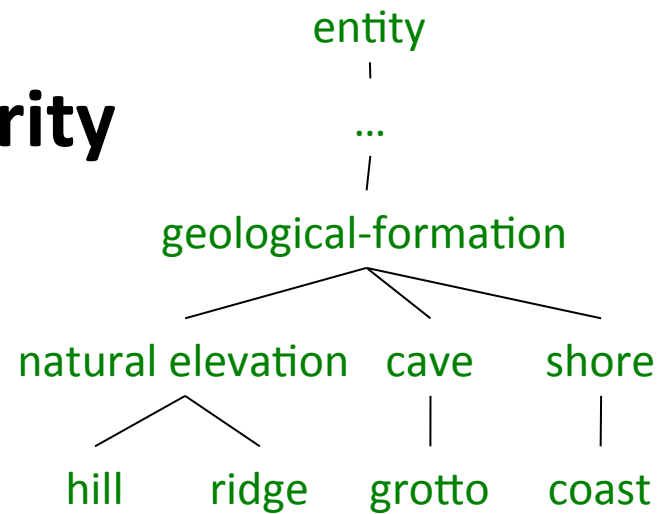
- Let's define $P(c)$ as:
 - The probability that a randomly selected word in a corpus is an instance of concept c
 - Formally: there is a distinct random variable, ranging over words, associated with each concept in the hierarchy
 - for a given concept, each observed noun is either
 - a member of that concept with probability $P(c)$
 - not a member of that concept with probability $1-P(c)$
 - All words are members of the root node (Entity)
 - $P(\text{root})=1$
 - The lower a node in hierarchy, the lower its probability



Information content similarity

Train by counting in a corpus

- Each instance of `hill` counts toward frequency of *natural elevation*, *geological formation*, *entity*, etc
- Let $\text{words}(c)$ be the set of all words that are children of node c
 - $\text{words}(\text{"geo-formation"}) = \{\text{hill}, \text{ridge}, \text{grotto}, \text{coast}, \text{cave}, \text{shore}, \text{natural elevation}\}$
 - $\text{words}(\text{"natural elevation"}) = \{\text{hill}, \text{ridge}\}$



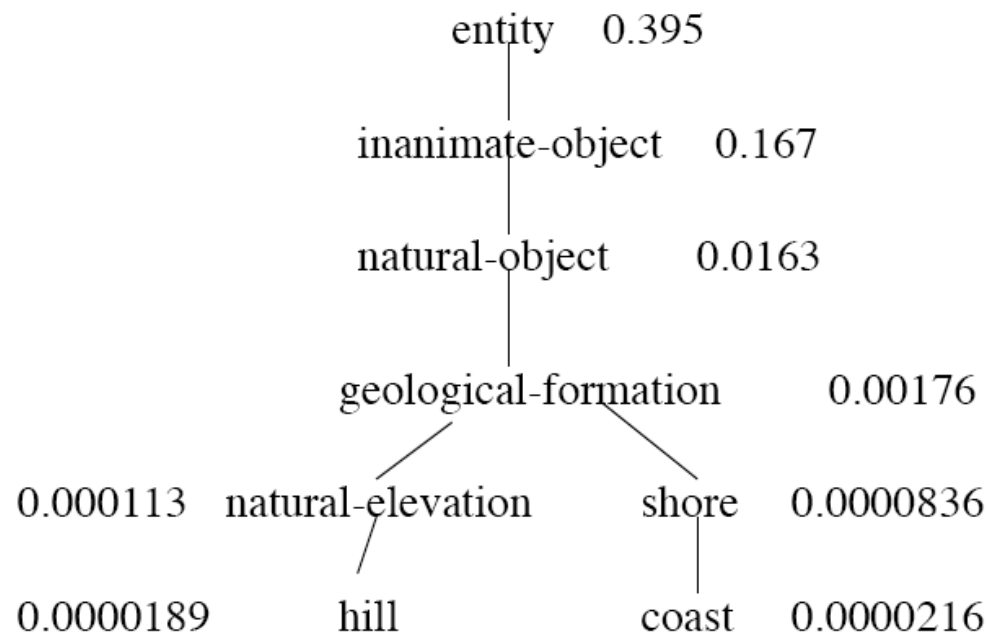
$$P(c) = \frac{\sum_{w \in \text{words}(c)} \text{count}(w)}{N}$$



Information content similarity

- WordNet hierarchy augmented with probabilities $P(c)$

D. Lin. 1998. An Information-Theoretic Definition of Similarity. ICML 1998



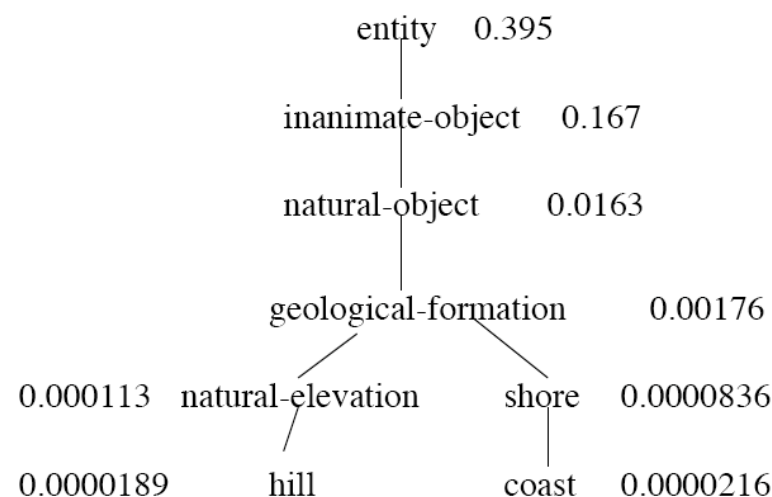


Information content: definitions

- Information content:
 $IC(c) = -\log P(c)$
- Most informative subsumer
(Lowest common subsumer)

$$LCS(c_1, c_2) =$$

The most informative (lowest)
node in the hierarchy
subsuming both c_1 and c_2



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Using information content for similarity: the Resnik method

Philip Resnik. 1995. Using Information Content to Evaluate Semantic Similarity in a Taxonomy. IJCAI 1995.
Philip Resnik. 1999. Semantic Similarity in a Taxonomy: An Information-Based Measure and its Application to Problems of Ambiguity in Natural Language. JAIR 11, 95-130.

- The similarity between two words is related to their common information
- The more two words have in common, the more similar they are
- Resnik: measure common information as:
 - The information content of the most informative (lowest) subsumer (MIS/LCS) of the two nodes
 - $\text{sim}_{\text{resnik}}(c_1, c_2) = -\log P(\text{LCS}(c_1, c_2))$



Dekang Lin method

Dekang Lin. 1998. An Information-Theoretic Definition of Similarity. ICML

- Intuition: Similarity between A and B is not just what they have in common
- The more **differences** between A and B, the less similar they are:
 - Commonality: the more A and B have in common, the more similar they are
 - Difference: the more differences between A and B, the less similar
- Commonality: $IC(\text{common}(A,B))$
- Difference: $IC(\text{description}(A,B)) - IC(\text{common}(A,B))$



Dekang Lin similarity theorem

- The similarity between A and B is measured by the ratio between the amount of information needed to state the commonality of A and B and the information needed to fully describe what A and B are

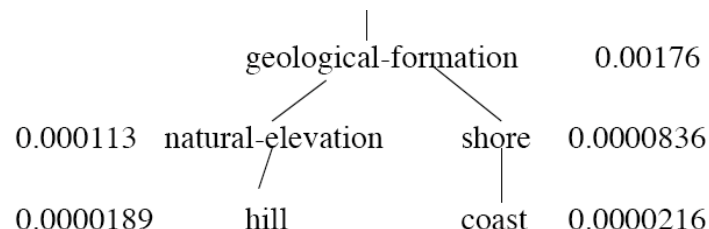
$$sim_{Lin}(A, B) \propto \frac{IC(common(A, B))}{IC(description(A, B))}$$

- Lin (altering Resnik) defines $IC(common(A, B))$ as 2 x information of the LCS

$$sim_{Lin}(c_1, c_2) = \frac{2 \log P(LCS(c_1, c_2))}{\log P(c_1) + \log P(c_2)}$$



Lin similarity function



$$sim_{Lin}(A, B) = \frac{2 \log P(LCS(c_1, c_2))}{\log P(c_1) + \log P(c_2)}$$

$$sim_{Lin}(\text{hill}, \text{coast}) = \frac{2 \log P(\text{geological-formation})}{\log P(\text{hill}) + \log P(\text{coast})}$$

$$= \frac{2 \ln 0.00176}{\ln 0.0000189 + \ln 0.0000216}$$

$$= .59$$



The (extended) Lesk Algorithm

- A thesaurus-based measure that looks at **glosses**
- Two concepts are similar if their glosses contain similar words
 - **Drawing paper**: **paper** that is **specially prepared** for use in drafting
 - **Decal**: the art of transferring designs from **specially prepared paper** to a wood or glass or metal surface
- For each n -word phrase that's in both glosses
 - Add a score of n^2
 - **Paper** and **specially prepared** for $1 + 2^2 = 5$
 - Compute overlap also for other relations
 - glosses of hypernyms and hyponyms



Summary: thesaurus-based similarity

$$\text{sim}_{\text{path}}(c_1, c_2) = \frac{1}{\text{pathlen}(c_1, c_2)}$$

$$\text{sim}_{\text{resnik}}(c_1, c_2) = -\log P(LCS(c_1, c_2)) \quad \text{sim}_{\text{lin}}(c_1, c_2) = \frac{2 \log P(LCS(c_1, c_2))}{\log P(c_1) + \log P(c_2)}$$

$$\text{sim}_{\text{jiangconrath}}(c_1, c_2) = \frac{1}{\log P(c_1) + \log P(c_2) - 2 \log P(LCS(c_1, c_2))}$$

$$\text{sim}_{\text{eLesk}}(c_1, c_2) = \sum_{r, q \in \text{RELS}} \text{overlap}(\text{gloss}(r(c_1)), \text{gloss}(q(c_2)))$$



Libraries for computing thesaurus-based similarity

- NLTK
 - [http://nltk.github.com/api/nltk.corpus.reader.html?highlight=similarity - nltk.corpus.reader.WordNetCorpusReader.res_similarity](http://nltk.github.com/api/nltk.corpus.reader.html?highlight=similarity-nltk.corpus.reader.WordNetCorpusReader.res_similarity)
- WordNet::Similarity
 - <http://wn-similarity.sourceforge.net/>
 - Web-based interface:
 - <http://marimba.d.umn.edu/cgi-bin/similarity/similarity.cgi>



Evaluating similarity

- Intrinsic Evaluation:
 - Correlation between algorithm and human word similarity ratings
- Extrinsic (task-based, end-to-end) Evaluation:
 - Malapropism (spelling error) detection
 - WSD
 - Essay grading
 - Taking TOEFL multiple-choice vocabulary tests

Levied is closest in meaning to:

imposed, believed, requested, correlated



Word Similarity: Distributional Similarity (I)



Problems with thesaurus-based meaning

- We don't have a thesaurus for every language
- Even if we do, they have problems with **recall**
 - Many words are missing
 - Most (if not all) phrases are missing
 - Some connections between senses are missing
 - Thesauri work less well for verbs, adjectives
 - Adjectives and verbs have less structured hyponymy relations



Distributional models of meaning

- Also called vector-space models of meaning
- Offer much higher recall than hand-built thesauri
 - Although they tend to have lower precision
- Zellig Harris (1954): “**oculist** and **eye-doctor** ... occur in almost the same environments....
If A and B have almost identical environments we say that they are synonyms.
- Firth (1957): “You shall know a word by the company it keeps!”



Intuition of distributional word similarity

- Nida example:

A bottle of **tesgüino** is on the table
Everybody likes **tesgüino**
Tesgüino makes you drunk
We make **tesgüino** out of corn.

- From context words humans can guess **tesgüino** means
 - an alcoholic beverage like **beer**
- Intuition for algorithm:
 - Two words are similar if they have similar word contexts.



Reminder: Term-document matrix

- Each cell: count of term t in a document d : $tf_{t,d}$:
 - Each document is a **count vector** in \mathbb{N}^v : a column below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0



Reminder: Term-document matrix

- Two documents are similar if their vectors are similar

	As You Like It	Twelfth Night	Julius Caesar	Henry V
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The words in a term-document matrix

- Each word is a **count vector** in \mathbb{N}^D : a row below

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The Term-Context matrix

- Instead of using entire documents, use smaller contexts
 - Paragraph
 - Window of 10 words
- A word is now defined by a vector over counts of context words



Sample contexts: 20 words (Brown corpus)

- equal amount of sugar, a sliced lemon, a tablespoonful of **apricot** preserve or jam, a pinch each of clove and nutmeg,
 - on board for their enjoyment. Cautiously she sampled her first **pineapple** and another fruit whose taste she likened to that of
 - of a recursive type well suited to programming on the **digital** computer. In finding the optimal R-stage policy from that of
 - substantially affect commerce, for the purpose of gathering data and **information** necessary for the
- 60 study authorized in the first section of this



Term-context matrix for word similarity

- Two **words** are similar in meaning if their context vectors are similar

	aardvark	computer	data	pinch	result	sugar	...
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	



Should we use raw counts?

- For the term-document matrix
 - We used **tf-idf** instead of raw term counts
- For the term-context matrix
 - **Positive Pointwise Mutual Information (PPMI)** is common



Pointwise Mutual Information

- **Pointwise mutual information:**

- Do events x and y co-occur more than if they were independent?

$$\text{PMI}(X, Y) = \log_2 \frac{P(x, y)}{P(x)P(y)}$$

- **PMI between two words:** (Church & Hanks 1989)

- Do words x and y co-occur more than if they were independent?

$$\text{PMI}(\text{word}_1, \text{word}_2) = \log_2 \frac{P(\text{word}_1, \text{word}_2)}{P(\text{word}_1)P(\text{word}_2)}$$

- **Positive PMI between two words** (Niwa & Nitta 1994)

- Replace all PMI values less than 0 with zero



Computing PPMI on a term-context matrix

- Matrix F with W rows (words) and C columns (contexts)
- f_{ij} is # of times w_i occurs in context c_j

	aardvark	computer	data	pinch	result	sugar
apricot	0	0	0	1	0	1
pineapple	0	0	0	1	0	1
digital	0	2	1	0	1	0
information	0	1	6	0	4	0

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad p_{i*} = \frac{\sum_{j=1}^C f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad p_{*j} = \frac{\sum_{i=1}^W f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}}$$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*} p_{*j}} \quad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}}$$

apricot
pineapple
digital
information

Count(w,context)

	computer	data	pinch	result	sugar
apricot	0	0	1	0	1
pineapple	0	0	1	0	1
digital	2	1	0	1	0
information	1	6	0	4	0

$$p(w=\text{information}, c=\text{data}) = 6/19 = .32$$

$$p(w=\text{information}) = 11/19 = .58$$

$$p(c=\text{data}) = 7/19 = .37$$

$$p(w_i) = \frac{\sum_{j=1}^C f_{ij}}{N}$$

$$p(c_j) = \frac{\sum_{i=1}^W f_{ij}}{N}$$

p(w,context)

p(w)

	computer	data	pinch	result	sugar	
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	



$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_i * p_j}$$

	p(w,context)					p(w)
	computer	data	pinch	result	sugar	
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

- $pmi(\text{information}, \text{data}) = \log_2 (.32 / (.37 * .58)) = .58$

(.57 using full precision)

	PPMI(w,context)				
	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-



Weighing PMI

- PMI is biased toward infrequent events
- Various weighting schemes help alleviate this
 - See Turney and Pantel (2010)
- Add-one smoothing can also help



	Add-2 Smoothed Count(w,context)				
	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

	p(w,context) [add-2]					p(w)
	computer	data	pinch	result	sugar	
apricot	0.03	0.03	0.05	0.03	0.05	0.20
pineapple	0.03	0.03	0.05	0.03	0.05	0.20
digital	0.07	0.05	0.03	0.05	0.03	0.24
information	0.05	0.14	0.03	0.10	0.03	0.36
p(context)	0.19	0.25	0.17	0.22	0.17	



PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

PPMI(w,context) [add-2]

	computer	data	pinch	result	sugar
apricot	0.00	0.00	0.56	0.00	0.56
pineapple	0.00	0.00	0.56	0.00	0.56
digital	0.62	0.00	0.00	0.00	0.00
information	0.00	0.58	0.00	0.37	0.00



Word Meaning and Similarity

Word Similarity: Distributional Similarity (I)

[illegible]



Using syntax to define a word's context

- Zellig Harris (1968)
 - “The meaning of entities, and the meaning of grammatical relations among them, is related to the restriction of combinations of these entities relative to other entities”
- Two words are similar if they have similar parse contexts
- **Duty** and **responsibility** (Chris Callison-Burch's example)

Modified by adjectives	additional, administrative, assumed, collective, congressional, constitutional ...
Objects of verbs	assert, assign, assume, attend to, avoid, become, breach ...



Co-occurrence vectors based on syntactic dependencies

Dekang Lin, 1998 “Automatic Retrieval and Clustering of Similar Words”

- The contexts C are different dependency relations
 - Subject-of- “absorb”
 - Prepositional-object of “inside”
- Counts for the word cell:

	subj-of, absorb	subj-of, adapt	subj-of, behave	...	pobj-of, inside	pobj-of, into	...	nmod-of, abnormality	nmod-of, anemia	nmod-of, architecture	...	obj-of, attack	obj-of, call	obj-of, come from	obj-of, decorate	...	nmod, bacteria	nmod, body	nmod, bone marrow
cell	1	1	1		16	30		3	8	1		6	11	3	2		3	2	2



PMI applied to dependency relations

Hindle, Don. 1990. Noun Classification from Predicate-Argument Structure. ACL

Object of “drink”	Count	PMI
tea	2	11.8
liquid	2	10.5
wine	2	9.3
anything	3	5.2
it	3	1.3

- “Drink it” more common than “drink wine”
- But “wine” is a better “drinkable” thing than “it”



Reminder: cosine for computing similarity

Dot product

Unit vectors

$$\cos(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\vec{v}}{|\vec{v}|} \cdot \frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

v_i is the PPMI value for word v in context i

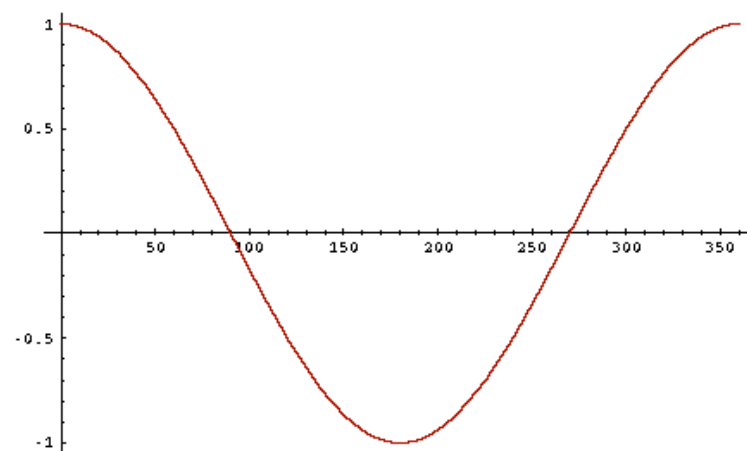
w_i is the PPMI value for word w in context i .

$\cos(\vec{v}, \vec{w})$ is the cosine similarity of \vec{v} and \vec{w}



Cosine as a similarity metric

- -1: vectors point in opposite directions
 - +1: vectors point in same directions
 - 0: vectors are orthogonal
-
- Raw frequency or PPMI are non-negative, so cosine range 0-1





$$\cos(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

	large	data	computer
apricot	1	0	0
digital	0	1	2
information	1	6	1

Which pair of words is more similar?

$$\text{cosine}(\text{apricot}, \text{information}) = \frac{1+0+0}{\sqrt{1+0+0} \sqrt{1+36+1}} = \frac{1}{\sqrt{38}} = .16$$

$$\text{cosine}(\text{digital}, \text{information}) = \frac{0+6+2}{\sqrt{0+1+4} \sqrt{1+36+1}} = \frac{8}{\sqrt{38} \sqrt{5}} = .58$$

$$\text{cosine}(\text{apricot}, \text{digital}) = \frac{0+0+0}{\sqrt{1+0+0} \sqrt{0+1+4}} = 0$$



Other possible similarity measures

$$\text{sim}_{\text{cosine}}(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^N v_i \times w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

$$\text{sim}_{\text{Jaccard}}(\vec{v}, \vec{w}) = \frac{\sum_{i=1}^N \min(v_i, w_i)}{\sum_{i=1}^N \max(v_i, w_i)}$$

$$\text{sim}_{\text{Dice}}(\vec{v}, \vec{w}) = \frac{2 \times \sum_{i=1}^N \min(v_i, w_i)}{\sum_{i=1}^N (v_i + w_i)}$$

$$\text{sim}_{\text{JS}}(\vec{v} || \vec{w}) = D(\vec{v} | \frac{\vec{v} + \vec{w}}{2}) + D(\vec{w} | \frac{\vec{v} + \vec{w}}{2})$$



Evaluating similarity (the same as for thesaurus-based)

- Intrinsic Evaluation:
 - Correlation between algorithm and human word similarity ratings
- Extrinsic (task-based, end-to-end) Evaluation:
 - Spelling error detection, WSD, essay grading
 - Taking TOEFL multiple-choice vocabulary tests

Levied is closest in meaning to which of these:
imposed, believed, requested, correlated

[illegible]