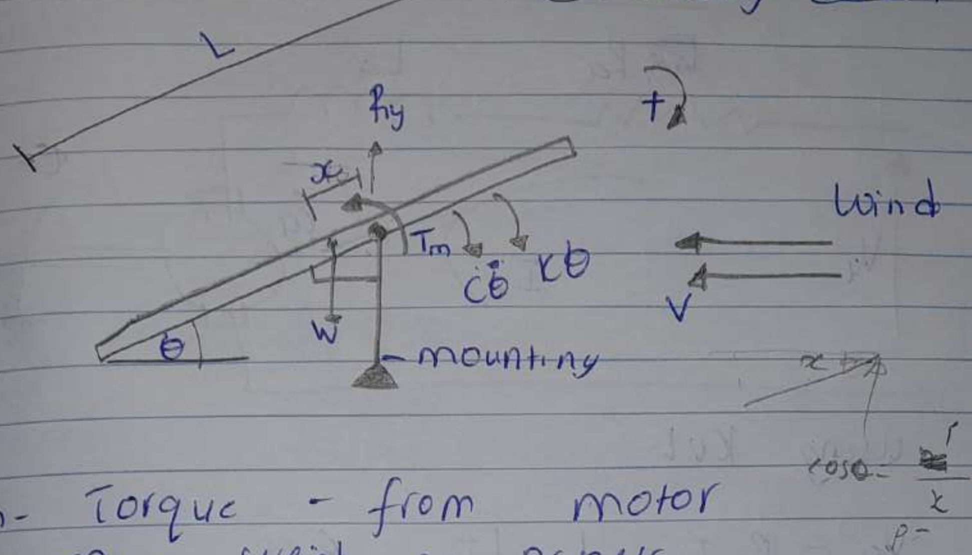
14 January 2024



T_m - Torque - from motor
$W = mg$ - weigh of panels
x - distance from $c.g$ to pivot point
R_y - Reactio force from pivot point
c - damping
k - Stiffness

Equation of motion

Taking moments about $c.g$ $\sum M_{cg} = J\ddot{\theta}$

$$J\ddot{\theta} = T_m - c\dot{\theta} - k\theta \pm R_y x \pm F_w x$$

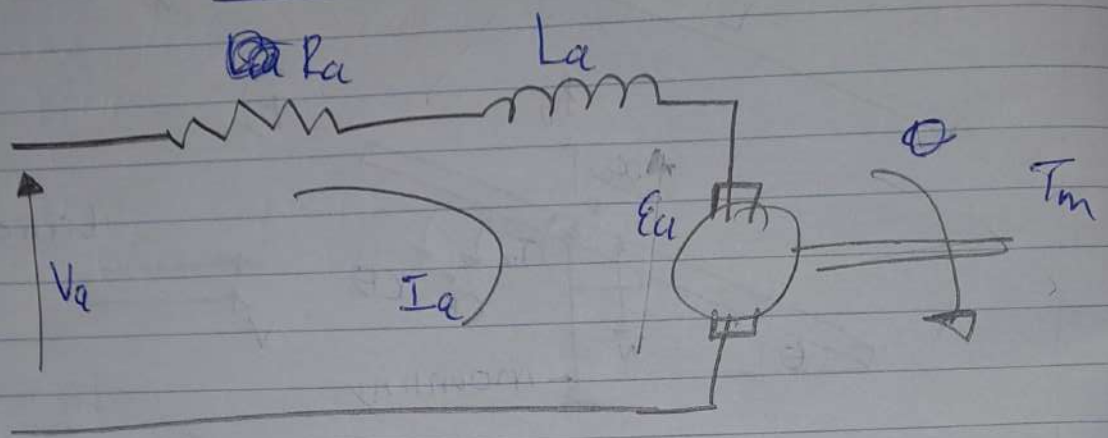
where F_w - wind force

$F_w = q_h C_f A$
$q_h = 0{,}5 \rho V^2$
C_f - Ranges ~~fro~~ between 1,2 - 1,8
A - projected area
ρ - density of air

$$J\ddot{\theta} = T_m - c\dot{\theta} - k\theta \pm R_y \underline{x} + \underbrace{\tfrac{1}{2}\rho V^2 C_f A x}_{}$$

non-linear

motor



using KVL:

$$V_a = R_a I_a + L_a \frac{dI_a}{dt} + \varepsilon_a$$

$\varepsilon_a = k_e \dot{\beta}$ — $\dot{\beta}$ - motor angular velocity ~~k_e - motor torque constant~~

$T_m = k_t \dot{\beta}$ — electrical

$\boxed{T_m = k_t I_a}$ — - Mechanical

or

$T_m = k_v V_a^2$

k_t - motor torque constant
k_e - Back EMF constant
ε_a - Back EMF

update
~~complete~~ non-linear

$$J\ddot{\theta} = k_t I_a - C\dot{\theta} - k\theta \mp R_y x \mp \frac{1}{2}\rho V^2 C_f A x$$

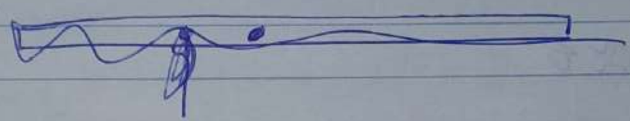
$$\ddot{\theta} = \frac{k_t I_a}{J} - \frac{C\dot{\theta}}{J} - \frac{k\theta}{J} + \frac{R_y x}{J} \mp \frac{\rho V^2 C_f A x}{2J}$$

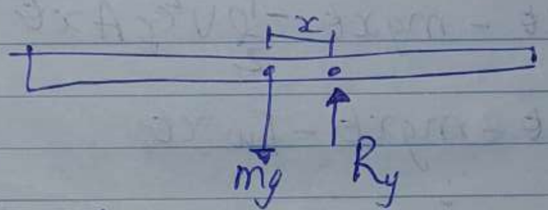
but x is not always ~~perd~~ perpendicular to forces

~~but~~ $\cancel{J = J_{cg} + mx^2}$
~~parallel axix theorem~~
~~not need~~

$$\cancel{J\ddot{\theta} = T_m - C\dot{\theta} - K\theta + R_y x +}$$

$$J\ddot{\theta} = T_m - C\dot{\theta} - K\theta - R_y x\cos\theta - \frac{\rho V^2 C_f A x\cos\theta}{2}$$





$\Sigma F_y = 0$ ↑+

$R_y - mg = 0$

$R_y = mg$

$$\boxed{J\ddot{\theta} = T_m - C\dot{\theta} - K\theta - mgx\cos\theta - \frac{\rho V^2 C_f A x\cos\theta}{2}}$$

$\boxed{T_m = K_v V_a^2}$ K_v - motor voltage constant

$$\boxed{V_a = R_a I_a + L_a \frac{dI_a}{dt} + \varepsilon_a}$$

$\boxed{\varepsilon_a = K_e \dot{\beta}}$

$\boxed{\dot{\beta} = T_r \dot{\theta}}$ T_r - mechanical transmission ratio
gear ratio

The $\cos\theta$ and V^2 are non-linear terms

Using Taylors expansion, neglecting higher order terms

~~$J\ddot{\theta} = T_m$~~

Small angle perturbation

$\cos\theta \approx \theta$

$$J\ddot{\theta} = T_m - c\dot{\theta} - k\theta - mgx\theta - \frac{1}{2}\rho V^2 C_f A x\theta$$

$$J\ddot{\theta} = T_m - c\dot{\theta} - k\theta - mgx\theta - F_w x\theta$$

$$J\ddot{\theta} = k_t I_a - c\dot{\theta} - k\theta - mgx\theta - F_w x\theta$$

Small pertubation

$$\ddot{\theta} - \ddot{\theta}_0 = \Delta\ddot{\theta}$$
$$I_a - I_{a0} = \Delta I_a$$
$$\dot{\theta} - \dot{\theta}_0 = \Delta\dot{\theta}$$
$$\theta - \theta_0 = \Delta\theta$$
$$V - V_0 = \Delta V$$

$$f = J\ddot{\theta} - k_t I_a + c\dot{\theta} + k\theta - mgx\theta - F_w x\theta$$

<u>initial conditions</u> - Equilibrium

$$f_0 = J\ddot{\theta}_0 - k_t I_{a0} + c\dot{\theta}_0 + k\theta_0 + mgx\theta_0 + \frac{\rho V_0^2 C_f A x\theta_0}{2} = 0$$

$$\left.\frac{df}{d\ddot{\theta}}\right|_0 \Delta\ddot{\theta} = J\Delta\ddot{\theta}$$

$$\left.\frac{df}{dI_a}\right|_0 \Delta I_a = -k_t \Delta I_a$$

$$\left.\frac{df}{d\dot{\theta}}\right|_0 \Delta\dot{\theta} = c\Delta\dot{\theta}$$

$$\left.\frac{df}{d\theta}\right|_0 \Delta\theta = \left(k + mgx + \frac{\rho V_0^2 C_f A x}{2}\right)\Delta\theta$$

$$\left.\frac{df}{dV}\right|_0 \Delta V = \cancel{2}\frac{\rho V_0 C_f A x \theta_0}{\cancel{2}}\Delta V$$

$$f = f_0 + \left.\frac{df}{d\ddot{\theta}}\right|_0 \Delta\ddot{\theta} + \left.\frac{df}{dI_a}\right|_0 \Delta I_a + \left.\frac{df}{d\dot{\theta}}\right|_0 \Delta\dot{\theta} + \left.\frac{df}{d\theta}\right|_0 \Delta\theta + \left.\frac{df}{dV}\right|_0 \Delta V$$

$$f = \overset{0}{\cancel{f_0}} - k_t\Delta I_a + c\Delta\dot{\theta} + \left(k + mgx + \tfrac{1}{2}\rho V_0^2 C_f A x\right)\Delta\theta + \rho V_0 C_f A x\theta_0 \Delta V + J\Delta\ddot{\theta} = 0$$

<u>linearized</u>

$$J\Delta\ddot{\theta} = k_t\Delta I_a - c\Delta\dot{\theta} - \left(k + mgx + \frac{\rho V_0^2 C_f A x}{2}\right)\Delta\theta - \rho V_0 C_f A x\theta_0 \Delta V$$

if V_a - voltage is the input
and the θ is the output

then the linearized equation
would be:

$$J\Delta\ddot{\theta} = 2K_v V_{a0}\Delta V_a - C\Delta\dot{\theta} - \left(k + mgx + \frac{\rho V_0 C_f A x}{2}\right)\Delta\theta$$
$$- \rho V_0 C_f A x \theta_0 \Delta V$$

find $\frac{\theta(s)}{V_a(s)} = G(s)$ transfer function

take Laplace - zero initial cond

$$Js^2\theta(s) = 2K_v V_{a0} V_a(s) - Cs\theta(s) - \left(k + mgx + \frac{\rho V_0 C_f A x}{2}\right)\theta(s) - \rho V_0 C_f A x \theta_0 V(s)$$

$$\left(Js^2 + Cs + k + mgx + \frac{\rho V_0^2 C_f A x}{2}\right)\theta(s) = 2K_v V_{a0} V_a(s)$$
$$- \rho V_0 C_f A x \theta_0 V(s)$$

$$\frac{\theta(s)}{V_a(s)} = \frac{2K_v V_{a0} - \cancel{\frac{\rho V_0 C_f A x \theta_0 V(s)}{V_a(s)}}^{\,0}}{Js^2 + Cs + k + mgx + \frac{\rho V_0^2 C_f A x}{2}}$$

we can rearrange this or we can
neglect the $\frac{2\rho V_0 C_f A x \theta_0 V(s)}{V_a(s)}$ $\theta_0 = 0$

$$\frac{\theta(s)}{V_a(s)} = \frac{2K_v V_{a0}}{Js^2 + Cs + k + mgx + \frac{\rho V_0^2 C_f A x}{2}}$$

the characteristic equation will be

$$\boxed{Js^2 + Cs + k + mgx + \frac{\rho V_0^2 C_f A x}{2} = 0}$$

J - inertia
C - damping
k - stiffness
mg - weight
x - distance from C.g to mounting point
ρ - density of air
V_0 - wind speed
C_f - net force constant (1,2 - 1,8)
A - exposed area

so the stability of the panel is independent of the voltage input ??

The stability of the system is determined by the influence of
* mass
* damping
* stiffness
* gravitational forces
* wind force.

The voltage input affects the response of the system (how the θ changes to change in voltages but it does not directly determine the stability of the System.

* Stability is determined by the inherent Characteristics described by the Characterstics equation.

* Voltage inputs influences the behaivor of the System not Stability.

Surface area of the panels

$S = 1762 \times 1134 \ mm^2$

$S = 1,99 \ m^2$

Motor to be used

Sun Tracer OG+ Solar motor Sutable for Country house,

Wind resistance 130km/h,

12Volts

SM3SPMOG+

The motor uses 20wh through the day

Torque · 2,5 Nm
Current : 0,3 A
Voltage : 24 V

LDR - servo literature needed on
this
<u>missing information</u>

K_t =
K_v =
K_e =
R =
V_a =
N =
C =
L =