



Cyclical figurate numbers

Problem 61

Triangle, square, pentagonal, hexagonal, heptagonal, and octagonal numbers are all figurate (polygonal) numbers and are generated by the following formulae:

Triangle $P_{3,n}=n(n+1)/2$ 1, 3, 6, 10, 15, ... Square $P_{4,n}=n^2$ 1, 4, 9, 16, 25, ... Pentagonal $P_{5,n}=n(3n-1)/2$ 1, 5, 12, 22, 35, ... Hexagonal $P_{6,n}=n(2n-1)$ 1, 6, 15, 28, 45, ... Heptagonal $P_{7,n}=n(5n-3)/2$ 1, 7, 18, 34, 55, ... Octagonal $P_{8,n}=n(3n-2)$ 1, 8, 21, 40, 65, ...

The ordered set of three 4-digit numbers: 8128, 2882, 8281, has three interesting properties.

- 1. The set is cyclic, in that the last two digits of each number is the first two digits of the next number (including the last number with the first).
- 2. Each polygonal type: triangle ($P_{3,127}$ =8128), square ($P_{4,91}$ =8281), and pentagonal ($P_{5,44}$ =2882), is represented by a different number in the set.
- 3. This is the only set of 4-digit numbers with this property.

Find the sum of the only ordered set of six cyclic 4-digit numbers for which each polygonal type: triangle, square, pentagonal, hexagonal, heptagonal, and octagonal, is represented by a different number in the set.