



Square root convergents

Problem 57

It is possible to show that the square root of two can be expressed as an infinite continued fraction.

$$\sqrt{2} = 1 + 1/(2 + 1/(2 + 1/(2 + \dots))) = 1.414213\dots$$

By expanding this for the first four iterations, we get:

$$1 + 1/2 = 3/2 = 1.5$$

$$1 + 1/(2 + 1/2) = 7/5 = 1.4$$

$$1 + 1/(2 + 1/(2 + 1/2)) = 17/12 = 1.41666\dots$$

$$1 + 1/(2 + 1/(2 + 1/(2 + 1/2))) = 41/29 = 1.41379\dots$$

The next three expansions are 99/70, 239/169, and 577/408, but the eighth expansion, 1393/985, is the first example where the number of digits in the numerator exceeds the number of digits in the denominator.

In the first one-thousand expansions, how many fractions contain a numerator with more digits than denominator?