Universidad Autónoma de Guadalajara	**Analysis and Design of Computer Algorithms** **Master in Computer Science** **Universidad Autónoma de Guadalajara** **Second Examination**

1) Poe Travels with Clones of Barr (50 points)

Poe the Penguin is a mad scientist, and recently invented a machine that produces clones of Barr the Bear from South Pole dark matter. Poe has produced n clones of Barr the Bear (let us label them as $1, 2, \ldots, n$). Poe wishes to travel the universe with *exactly* k *clones* as his spaceship has $k+1$ seats (one for himself and k for the clones – Poe does not want to have empty seats!). The machine has a minor flaw in that some clones, if they are separated, would explode, and put an end to the universe. Luckily Poe knows which pairs of clones need to be kept together; let $E = \{(i_1, j_1), (i_2, j_2), \ldots, (i_m, j_m)\}$ be a set of pairs of clones where

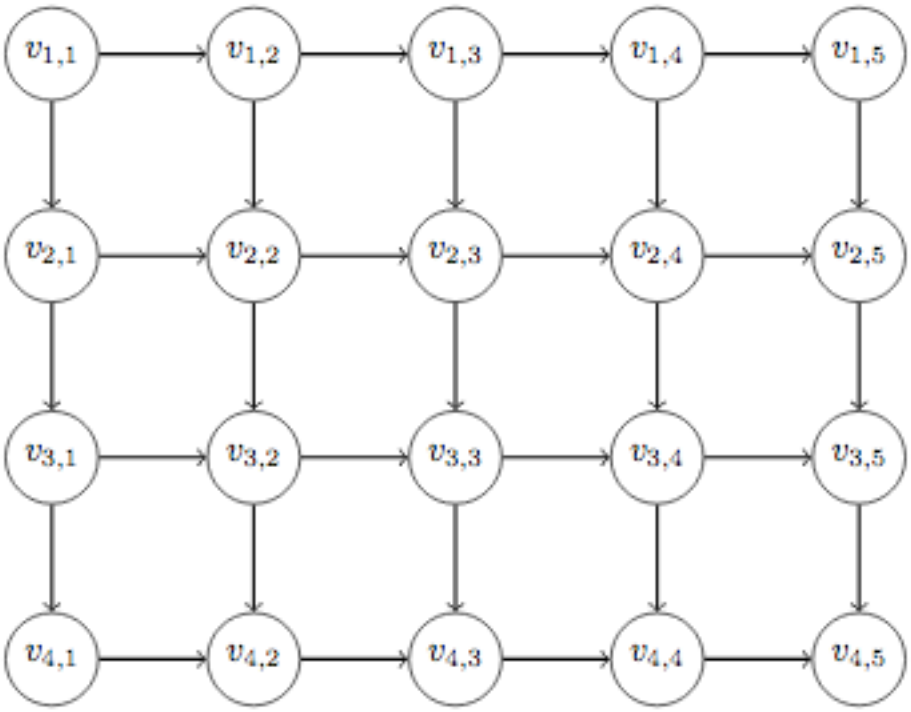

Figure 1: Example: Directed Grid Network of Size 4×5.

each pair (i, j) indicates that clone i and clone j should not be separated; that is, Poe must either take both i and j with him on his spaceship or take neither of them. Can you help Poe find out whether it is possible to choose exactly k clones and take them with him, or Poe should just stay home while regretting his latest invention?

Design an algorithm that takes (n, k, E) as an input, and determines in $O(m + nk)$ runtime which k clones Poe should take with him (or output "S.T.A.Y." if Poe should not travel). Prove correctness of your algorithm and analyze its running time. (You may assume that $k \leq n$ because Poe only has n clones and thus cannot travel with exactly k clones if $k > n$.)

2) Huffman Code Problem (50 points)

(a) Generate a Huffman code for the following sample of items shown and design an automaton that recognizes it.

(b). Encode using the generated code in (a) the following message:

LLACCUUUUMNDDCMMDLAOONEEEERRIHIHIOOEEEAAMUCLDSEAONAEERIIHHEHAOLC

(c). Build a finite automaton that decodes the message.

E 0.15	S 0.090
A 0.15	D 0.090
O 0.11	L 0.007
N 0.10	C 0.007
R 0.10	U 0.003
I 0.095	M 0..003
H 0.095	