Shadow Price Sensitivity Analysis

SUPPLY CHAIN ANALYTICS IN PYTHON

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Define Shadow Price

Modeling in issues:

- Input for model constraints are often estimates
- Will changes to input change our solution?

Shadow Prices:

- *The change in optimal value of the objective function per unit increase in the right-hand-side for a constraint, given everything else remain unchanged.*

Context - Glass Company - Resource Planning:

Resource	Prod. A	Prod. B	Prod. C
Production hours	6	5	8
WH Capacity sq. ft.	10.5	20	10
Profit $US	$500	$450	$600

Constraints:

- Production Capacity Hours ? 60
- Warehouse Capacity ? 150 sq. ft.
- Max Production of A ? 8

```python
# Initialize Class, Define Vars., and Objective
model = LpProblem("Max Glass Co. Profits", LpMaximize)
A = LpVariable('A', lowBound=0)
B = LpVariable('B', lowBound=0)
C = LpVariable('C', lowBound=0)
model += 500 * A + 450 * B + 600 * C

# Constraint 1
model += 6 * A + 5 * B + 8 * C <= 60

# Constraint 2
model += 10.5 * A + 20 * B + 10 * C <= 150

# Constraint 3
model += A  <= 8

# Solve Model
model.solve()
print("Model Status: {}".format(pulp.LpStatus[model.status]))
print("Objective = ", value(model.objective))
for v in model.variables():
    print(v.name, "=", v.varValue)
```

Example Solution

Solution:

Products	Prod. A	Prod. B	Prod. C
Production Cases	6.667	4	0

Objective value is $5133.33

Review Constraints

Decision Variable:

- A through C = Number of cases of respective A through C products

Constraints:

- 6A + 5B + 8C ? 60 (*limited production capacity*)
- 10A + 20B + 10C ? 150 (*limited warehouse capacity*)
- A ? 8 (*max production of A*)

Print Shadow Price

Python Code:

```
o = [{'name':name, 'shadow price':c.pi}
     for name, c in model.constraints.items()]
print(pd.DataFrame(o))
```

Shadow Prices Explained

Output:

```
name   shadow price
 _C1      78.148148
 _C2       2.962963
 _C3      -0.000000
```

Remember the Constraints:

1. *limited production capacity*
2. *limited warehouse capacity*
3. *max production of A*

Constraint Slack

Slack:

- The amount of a resource that is unused.

Python:

```
o = [{'name':name, 'shadow price':c.pi, 'slack': c.slack}
     for name, c in model.constraints.items()]
print(pd.DataFrame(o))
```

Constraint Slack Explained

Output:

```
name  shadow price     slack
 _C1     78.148148 -0.000000
 _C2      2.962963 -0.000000
 _C3     -0.000000  1.333333
```

More About Binding

- Slack = 0, then ***binding***
- Changing ***binding*** constraint, ***changes*** solution

Remember the Constraints:

1. *limited production capacity*
2. *limited warehouse capacity*
3. *max production of A*

Summary

- How to compute:
  - shadow prices
  - constraint slack
- Identify Binding Constraints
  - slack = 0, then ***binding***
  - slack > 0, then ***not-binding***

Try It Out!

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Capacitated Plant Location Case Study Part 3

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Capacitated Plant Location Model

Modeling

- Production at regional facilities
    - Two plant sizes (low / high)
- Exporting production to other regions
- Production facilities open / close

Expected Ranges

What should we expected for values of our decision variables?

Production Quantities:

- High production in regions with low variable production and shipping costs
- Maxed production in regions that also have relatively low fixed production costs

Production Plant Open Or Closed:

- High capacity production plant in regions with high demand
- High capacity production plant in regions with relatively low fixed costs

Sensitivity Analysis of Constraints

Total Production = Total Demand:

- Shadow Prices = Represent changes in total cost per increase in demand for a region
- Slack = Should be zero

Total Production ? Total Production Capacity:

- Shadow Prices = Represent changes in total costs per increase in production capacity
- Slack = Regions which have excess production capacity

```python
from pulp import *
import pandas as pd

# Initialize Class
model = LpProblem("Capacitated Plant Location Model", LpMinimize)

# Define Decision Variables
loc = ['A', 'B', 'C', 'D', 'E']
size = ['Low_Cap','High_Cap']
x = LpVariable.dicts("production_", [(i,j) for i in loc for j in loc],
                     lowBound=0, upBound=None, cat='Continuous')
y = LpVariable.dicts("plant_",
                         [(i,s) for s in size for i in loc], cat='Binary')

# Define Objective Function
model += (lpSum([fix_cost.loc[i,s]*y[(i,s)] for s in size for i in loc])
          + lpSum([var_cost.loc[i,j]*x[(i,j)] for i in loc for j in loc]))

# Define the Constraints
for j in loc:
   model += lpSum([x[(i, j)] for i in loc]) == demand.loc[j,'Dmd']
for i in loc:
   model += lpSum([x[(i, j)] for j in loc]) <= lpSum([cap.loc[i,s]*y[(i,s)]
                                                       for s in size])
```

```
# Solve
model.solve()

# Print Decision Variables and Objective Value
print(LpStatus[model.status])
o = [{'prod':"{} to {}".format(i,j), 'quant':x[(i,j)].varValue}
     for i in loc for j in loc]
print(pd.DataFrame(o))
o = [{'loc':i, 'lc':y[(i,size[0])].varValue, 'hc':y[(i,size[1])].varValue}
     for i in loc]
print(pd.DataFrame(o))
print("Objective = ", value(model.objective))

# Print Shadow Price and Slack
o = [{'name':name, 'shadow price':c.pi, 'slack': c.slack}
     for name, c in model.constraints.items()]
print(pd.DataFrame(o))
```

Business Questions

Likely Questions:

- What is the expected cost of this supply chain network model?
- If demand increases in a region how much profit is needed to cover the costs of production and shipping to that region?
- Which regions still have production capacity for future demand increase?

Summary

Reviewed:

- Expected ranges for decision variables
- Interpreted the output of sensitivity analysis (shadow prices and slack)
- Code to solve and output results
- Likely business related question

Great Work! Your Turn

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Simulation Testing Solution

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Caution

- Problems that take a long time to solve should not be used with LP or IP

Overall Concept

General Concept:

- Add random noise to key inputs you choose
- Solve the model repeatedly
- Observe the distribution

Why We Might Try

Why:

- Inputs are often estimates. There is a risk that they are inaccurate.
- Earlier Sensitivity Analysis only looked at changing one input at a time.

Context

Context - Glass Company - Resource Planning:

Resource	Prod. A	Prod. B	Prod. C
Profit $US	$500	$450	$600

Constraints:

- There are demand, production capacity, and warehouse Capacity constraints

Risks:

- Estimates of profits may be inaccurate

```python
# Initialize Class, & Define Variables
model = LpProblem("Max Glass Co. Profits", LpMaximize)
A = LpVariable('A', lowBound=0)
B = LpVariable('B', lowBound=0)
C = LpVariable('C', lowBound=0)

# Define Objective Function
model += 500 * A + 450 * B + 600 * C

# Define Constraints & Solve
model += 6 * A + 5 * B + 8 * C <= 60
model += 10.5 * A + 20 * B + 10 * C <= 150
model += A  <= 8
model.solve()
```

```
a, b, c = normalvariate(0,25), normalvariate(0,25), normalvariate(0,25)
```

```
# Define Objective Function
model += (500+a)*A + (450+b)*B + (600+c)*C
```

```
# Initialize Class, & Define Variables
model = LpProblem("Max Glass Co. Profits", LpMaximize)
A = LpVariable('A', lowBound=0)
B = LpVariable('B', lowBound=0)
C = LpVariable('C', lowBound=0)
a, b, c = normalvariate(0,25), normalvariate(0,25), normalvariate(0,25)

# Define Objective Function
model += (500+a)*A + (450+b)*B + (600+c)*C

# Define Constraints & Solve
model += 6 * A + 5 * B + 8 * C <= 60
model += 10.5 * A + 20 * B + 10 * C <= 150
model += A  <= 8
model.solve()
```

```python
def run_pulp_model():
    # Initialize Class
    model = LpProblem("Max Glass Co. Profits", LpMaximize)
    A = LpVariable('A', lowBound=0)
    B = LpVariable('B', lowBound=0)
    C = LpVariable('C', lowBound=0)
    a, b, c = normalvariate(0,25), normalvariate(0,25), normalvariate(0,25)

    # Define Objective Function
    model += (500+a)*A + (450+b)*B + (600+c)*C

    # Define Constraints & Solve
    model += 6 * A + 5 * B + 8 * C <= 60
    model += 10.5 * A + 20 * B + 10 * C <= 150
    model += A  <= 8
    model.solve()
    o = {'A':A.varValue, 'B':B.varValue,
         'C':C.varValue, 'Obj':value(model.objective)}
    return(o)
```

```python
def run_pulp_model():
    # Initialize Class
    model = LpProblem("Max Glass Co. Profits", LpMaximize)
    A = LpVariable('A', lowBound=0)
    B = LpVariable('B', lowBound=0)
    C = LpVariable('C', lowBound=0)
    a, b, c = normalvariate(0,25), normalvariate(0,25), normalvariate(0,25)

    # Define Objective Function
    model += (500+a)*A + (450+b)*B + (600+c)*C

    # Define Constraints & Solve
    model += 6 * A + 5 * B + 8 * C <= 60
    model += 10.5 * A + 20 * B + 10 * C <= 150
    model += A  <= 8
    model.solve()
    o = {'A':A.varValue, 'B':B.varValue,
         'C':C.varValue, 'Obj':value(model.objective)}
    return(o)

output = []
for i in range(100):
    output.append(run_pulp_model())
df = pd.DataFrame(output)
```

```
print(df['A'].value_counts())
print(df['B'].value_counts())
print(df['C'].value_counts())
```

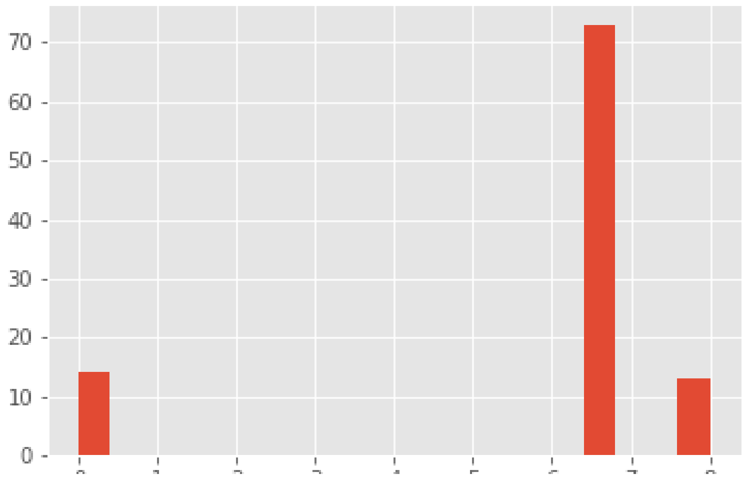
Output: (results may be different)

```
6.666667    73
0.000000    14
8.000000    13
Name: A, dtype: int64
```

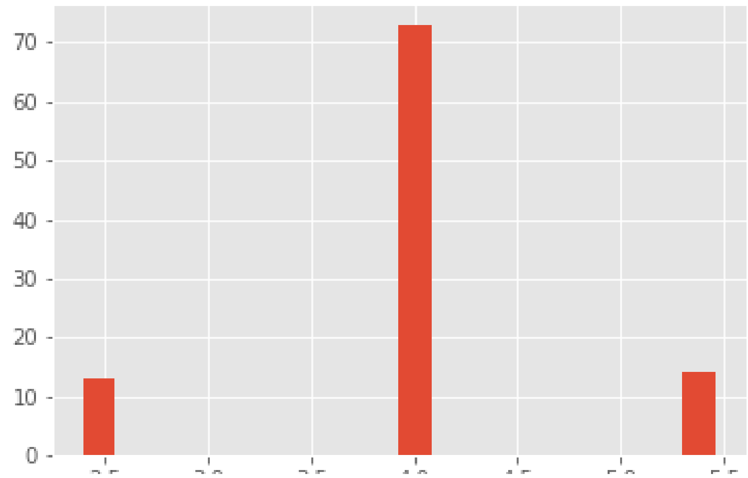
```
4.000000    73
5.454546    14
2.400000    13
Name: B, dtype: int64
0.000000    86
4.090909    14
Name: C, dtype: int64
```

Visualize As Histogram

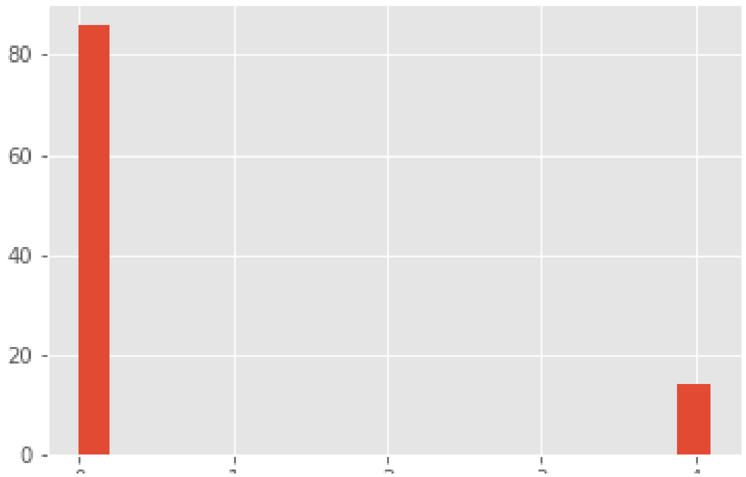
Product A:



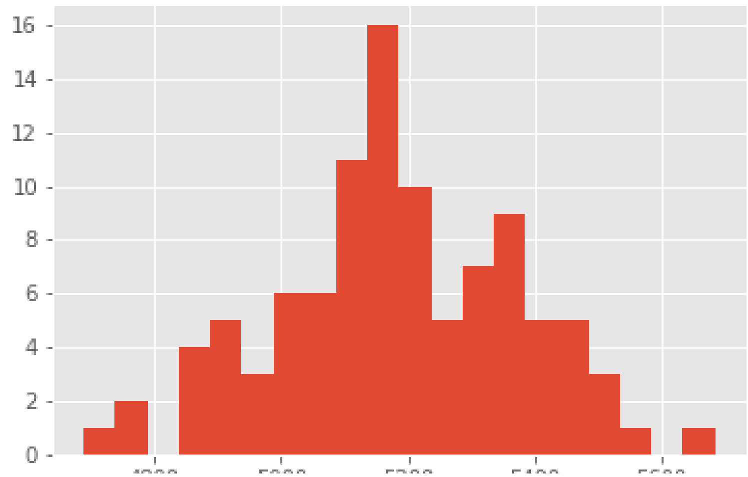
Product B:



Product C:



Objective Values:



Summary

- Should not be used on problems that take a long time to solve
- Benefits
  - View how optimal results change as model inputs change
- Steps
  1. Start with standard PuLP model code
  2. Add noise to key inputs using Python's `normalvariate`
  3. Wrap PuLP model code in a function that returns the model's output
  4. Create loop to call newly created function and store results in DataFrame
  5. Visualize results DataFrame

Try It Out!

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Capacitated Plant Location Case Study Part 4

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Simulation vs. Sensitivity Analysis

With Sensitivity Analysis:

- Observe how changes in demand and costs affect production:
    - Where should production be added?
    - Does production move to a different region.
    - Which regions have stable production quantities?
- Observe multiple changes at once vs. one at a time with sensitivity analysis

Simulation Modeling

We can apply simulation testing to our Capacitated Plant Location Model

Possible inputs for adding noise

- **Demand**
- **Variable costs**
- Fixed costs
- Capacity

```python
# Initialize Class
model = LpProblem("Capacitated Plant Location Model", LpMinimize)

# Define Decision Variables
loc = ['A', 'B', 'C', 'D', 'E']
size = ['Low_Cap','High_Cap']
x = LpVariable.dicts("production_", [(i,j) for i in loc for j in loc],
                     lowBound=0, upBound=None, cat='Continuous')
y = LpVariable.dicts("plant_",
                         [(i,s) for s in size for i in loc], cat='Binary')

# Define Objective Function
model += (lpSum([fix_cost.loc[i,s]*y[(i,s)] for s in size for i in loc])
          + lpSum([var_cost.loc[i,j]*x[(i,j)] for i in loc for j in loc]))

# Define the Constraints
for j in loc:
   model += lpSum([x[(i, j)] for i in loc]) == demand.loc[j,'Dmd']
for i in loc:
   model += lpSum([x[(i, j)] for j in loc]) <= lpSum([cap.loc[i,s]*y[(i,s)]
                                                      for s in size])
# Solve
model.solve()
print(LpStatus[model.status])
```

Objective:

```
model += (lpSum([fix_cost.loc[i,s]*y[(i,s)] for s in size for i in loc])
          + lpSum([(var_cost.loc[i,j] + normalvariate(0.5, 0.5))*x[(i,j)]
                   for i in loc for j in loc]))
```

Total Demand:

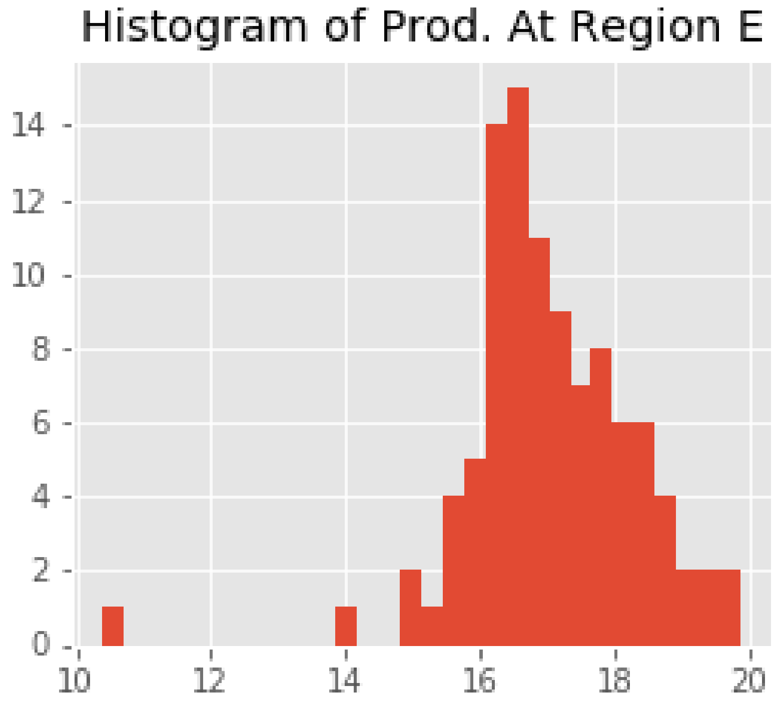
```
for j in loc:
    rd = normalvariate(0, demand.loc[j,'Dmd']*.05)
    model += lpSum([x[(i,j)] for i in loc]) == (demand.loc[j,'Dmd']+rd)
```

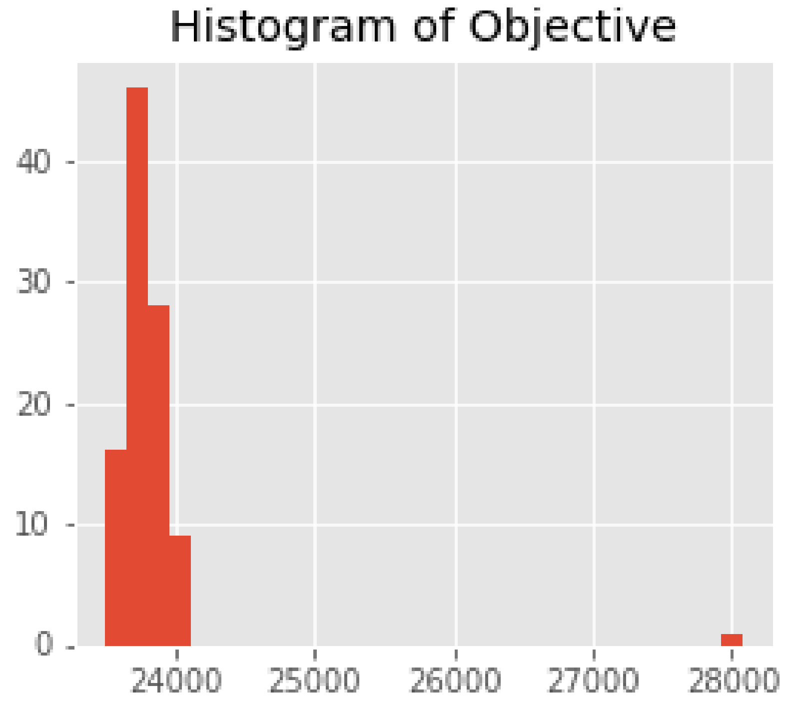
```python
def run_pulp_model(fix_cost, var_cost, demand, cap):
    # Initialize Class
    model = LpProblem("Capacitated Plant Location Model", LpMinimize)

    # Define Decision Variables
    loc = ['A', 'B', 'C', 'D', 'E']
    size = ['Low_Cap','High_Cap']
    x = LpVariable.dicts("production_", [(i,j) for i in loc for j in loc],
                         lowBound=0, upBound=None, cat='Continuous')
    y = LpVariable.dicts("plant_",
                          [(i,s) for s in size for i in loc], cat='Binary')

    # Define Objective Function
    model +=(lpSum([fix_cost.loc[i,s]*y[(i,s)] for s in size for i in loc])
             + lpSum([(var_cost.loc[i,j]+normalvariate(0.5, 0.5))*x[(i,j)]
                     for i in loc for j in loc]))

    # Define the Constraints
    for j in loc:
        rd = normalvariate(0, demand.loc[j,'Dmd']*.05)
        model += lpSum([x[(i,j)] for i in loc]) == (demand.loc[j,'Dmd']+rd)
    for i in loc:
        model += lpSum([x[(i,j)] for j in loc]) \
        <= lpSum([cap.loc[i,s]*y[(i,s)] for s in size])
```

```
    # Solve
    model.solve()
    o = {}
    for i in loc:
        o[i] = value(lpSum([x[(i, j)] for j in loc]))
    o['Obj'] = value(model.objective)
    return(o)
output = []
for i in range(100):
    output.append(run_pulp_model(fix_cost, var_cost, demand, cap))
df = pd.DataFrame(output)
```

Results

```
import matplotlib.pyplot as plt
plt.title('Histogram of Prod. At Region E')
plt.hist(df['E'])
plt.show()
```





Summary

Capacitated Plant Model

- Simulation vs. sensitivity analysis
- Stepped through code example

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Final Summary

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Summary

- Reviewed what is Linear Programing (LP)
- Reviewed PuLP and how it can be used with LP
- Solving large scale models
  - `LpSum()`
  - `LpVariable.dicts()`
- Logical constraints
- Common constraint mistakes
- Solving PuLP model
  - printing decision variables, and objective

Summary

- Sanity checking solution
- Sensitivity Analysis
  - Shadow Prices
  - Slack
- Simulation Testing
- Capacitated Plant Location model - Case Study

Congratulations!

Additional Resources

For more on PuLP check out these additional resources:

- **https://www.coin-or.org/PuLP/**
- **https://www.coin-or.org/**
- PuLP GitHub: **https://github.com/coin-or/pulp**
- Google group: **https://groups.google.com/forum/#!forum/pulp-or-discuss**

Additional Resources

For books related to the subject, check out these:

- Bradley, Stephen P., et al. *Applied Mathematical Programming*. Addison-Wesley, 1977.
- Chopra, Sunil, and Meindl, Peter. *Supply Chain Management: Strategy, Planning, and Operations.* Pearson Prentice-Hall, 2007.

Thank You!

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