One-Var Calculus

CH2

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Functions

Definition (Polynomial)

Polynomial is sum of monomials

Definition (Monomial)

f is monomial if:

$$f(x) = \bar{a}x^{\bar{k}}, \quad \bar{a} \in \mathbb{R}, \bar{k} \in \mathbb{N} \cup \{0\}$$

Definition (Rational function)

$$P(x)/Q(x), \quad P, Q \in \textit{set of polynomials} \wedge Q \neq 0$$

Definition (Exponential function)

f is exponential function if: $f(x) = \bar{a}\bar{b}^x, \quad \bar{a}, \bar{b} \in \mathbb{R}$

Increasing, Decreasing

Definition (Increasing Function)

f is <u>increasing</u> if:

$$\forall x_1, x_2, \quad x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$$

Definition (Decreasing Function)

f is <u>decreasing</u> if:

$$\forall x_1, x_2, \quad x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$$

Minimum, Maximum

Definition (Local (Relative) minimum)

$(x_0, f(x_0))$ *is local minimum of f if f changes from decreasing to increasing*

Definition (Local (Relative) maximum)

$(x_0, f(x_0))$ *is local maximum of f if f changes from increasing to decreasing*

Note: Boundary max/min

Definition (Global (Absolute) minimum)

$(x_0, f(x_0))$ *is Global minimum of $f : D \to \mathbb{R}$ if*

$$f(x_0) \le f(x) \quad \forall x \in D$$

Interval

Definition (Open Interval)

$$(\bar{a}, \bar{b}) := \{x \in \mathbb{R} | \bar{a} < x < \bar{b}\}$$

Definition (Closed Interval)

$$[\bar{a}, \bar{b}] := \{x \in \mathbb{R} | \bar{a} \leq x \leq \bar{b}\}$$

- half-open (or half-closed) intervals

$$(\bar{a}, \bar{b}], \quad [\bar{a}, \bar{b}), \cdots$$

- infinite intervals

$$(-\infty, \bar{a}], \quad (\bar{a}, \infty), \cdots$$

Linear Function

Definition (Linear Function)

Polynomial of degree 0 (Constant function) or 1

- Properties
  - Graph: Straight line
  - Constant slope

Definition (Slope of Linear function f)

$$Slope := \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Theorem (2.1)

The line (Graph of f) with slope $\bar{m}$ and vertical intercept $\bar{b}$ $\Rightarrow$

$f(x) = \bar{m}x + \bar{b}$

Nonlinear function

Definition (Nonlinear function)

f is nonlinear function if f is not linear function

Definition (Derivative at $(\bar{x}_0, f(\bar{x}_0))$)

$f'(\bar{x}_0)$:= Derivative of f at $(\bar{x}_0, f(\bar{x}_0))$ is the slope of the tangent line to the graph of f at $(\bar{x}_0, f(\bar{x}_0))$. i.e.,

$$f'(\bar{x}_0) := \lim_{h \to 0} \frac{f(\bar{x}_0 + h) - f(\bar{x}_0)}{h}$$

- Alternative Notations

$$f'(x_0) \equiv \frac{df}{dx}(x_0) \equiv \left.\frac{df}{dx}\right|_{x=x_0}$$

Computing Derivatives

Theorem (2.2)

$$f(x) = x^2 \Rightarrow f'(x) = 2x$$

Theorem (2.3)

$$f(x) = x^{\bar{k}}, \bar{k} \in \mathbb{N} \Rightarrow f'(x) = \bar{k}x^{\bar{k}-1}$$

Operation of Functions

Definition (Operation of Functions)

$$(f \pm g)(x) := f(x) \pm g(x)$$

$$(f \cdot g)(x) := f(x)g(x)$$

$$(f/g)(x) := f(x)/g(x)$$

Rules for Computing Derivatives

Theorem (2.4)

$$(f \pm g)' = f' \pm g'$$

$$(\bar{k}f)' = \bar{k}(f')$$

$$(f \cdot g)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(f^{\bar{n}})' = \bar{n}f^{\bar{n}-1}f'$$

Differentiable

Definition (Differentiable)

$f : D \to \mathbb{R}$ *is differentiable if*

$$\forall x_0 \in D \textit{ and } \forall\{h_n\} \to 0, \exists \textit{unique } f'(x_0) = \lim_{h_n \to 0} \frac{f(x_0 + h_n) - f(x_0)}{h_n}$$

- Geometrical meaning: graph of f is smooth
- $f \in \mathbf{C}^1$

Continuous

Definition (Continuous)

$f : D \to \mathbb{R}$ *is continuous if:*

$$\forall x_0 \in D, \quad x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$$

- Geometrical meaning: graph of f is not disconnected

Second Derivative of f

Definition (Second Derivative of $f \in \mathbf{C}^2$)

$$f'' := (f')' \equiv \frac{d}{dx}\left(\frac{df}{dx}\right) \equiv \frac{d^2 f}{dx^2}$$

- $\mathbf{C}^3$

$$f''', \quad \frac{d^3 f}{dx^3}$$

- $\mathbf{C}^k$

$$f^{(k)}, \quad \frac{d^k f}{dx^k}$$

- Polynomial is $\mathbf{C}^\infty$

Approximation

- Δx: Change in x (general representation)
- dx: "Small" change in x (or Δx which is sufficiently close to 0)
- Suppose x_0 is changed to $x_0 + h$, and h is sufficiently close to 0. then,

$$\frac{f(x_0+h)-f(x_0)}{h} \equiv \frac{\Delta f}{\Delta x} \approx f'(x_0)$$

$$\Delta f \approx f'(x_0)\Delta x$$

or,

$$df = f'(x_0)dx$$