One-Variable Calculus: Chain Rule

CH3

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Derivative of Composite Functions

Definition (Composition of Functions)

$$(h \circ g)(x) := h(g(x))$$

h: *outside function*, g: *inside function*

In general, $h \circ g \neq g \circ h$

Chain Rule

Suppose $\hat{f}_i = (f_i \circ f_{i+1} \circ \cdots \circ f_n)(x) = f_i(f_{i+1}(\cdots(f_n(x))))$. then,

$$\frac{d(f_1 \circ f_2 \circ \cdots \circ f_n)(x)}{dx} = \frac{df_1(f_2(\cdots(f_n(x))))}{dx} = \frac{d\hat{f}_1}{d\hat{f}_2}\frac{d\hat{f}_2}{d\hat{f}_3}\cdots\frac{d\hat{f}_n}{dx}$$

Exercise: $[\cos((\sin(x^3+4x))^{500})]' = ?$

Inverse of a Function

Definition (Inverse Function)

Suppose $f : E_1 \to E_2$. *Then* $g : E_2 \to E_1$ *is an inverse of* f *if*

$$g(f(x)) = x \quad \forall x \in E_1$$

$$f(g(x)) = x \quad \forall x \in E_2$$

- Notation: $g(x) = f^{-1}(x)$
- $(f^{-1} \circ f)(x) = (f \circ f^{-1})(x) = x$
- Geometrical meaning: Graph of f^{-1} is reflection of the graph of f across 45 degree line
- $\exists f^{-1}$ ($i.e.$, f is invertible) iff f is monotonically increasing [decreasing]

Derivative of the Inverse Function

Theorem (4.3: Inverse Function Theorem)

Suppose $f : I \to \mathbb{R}$ *is* $\mathbf{C}^1$ *Function,* $f' \neq 0, x \in I$. *Then,*

1. $\exists f^{-1}$ *on* I
2. $f^{-1} \in \mathbf{C}^1$ *on interval* $f(I)$
3. $(f^{-1})' = \frac{1}{f'(f^{-1})}$. *more intuitively,*

$$\frac{df^{-1}}{dx} = \frac{1}{\frac{df(f^{-1})}{df^{-1}}} = \frac{1}{\frac{dx}{df^{-1}}}$$

The Derivative of $x^{m/n}$

Theorem (4.4)

$$\forall n \in \mathbb{N}, \quad (x^{1/n})' = \frac{1}{n} x^{(1/n)-1}$$

Theorem (4.5)

$$\forall m, n \in \mathbb{N}, \quad (x^{m/n})' = \frac{m}{n} x^{(m/n)-1}$$

In general,

$$(x^r)' = r x^{(r-1)} \quad \forall r \in \mathbb{R}$$