One-Variable Calculus: Chain Rule

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1 Composite Functions and the Chain Rule

Derivative of Composite Functions

Definition 1 (Composition of Functions)**.**

$$(h \circ g)(x) := h(g(x))$$

h: outside function, g: inside function

In general, $h \circ g \neq g \circ h$

Chain Rule

Suppose $\hat{f}_i = (f_i \circ f_{i+1} \circ \cdots \circ f_n)(x) = f_i(f_{i+1}(\cdots(f_n(x))))$. then,

$$\frac{d(f_1 \circ f_2 \circ \cdots \circ f_n)(x)}{dx} = \frac{df_1(f_2(\cdots(f_n(x))))}{dx} = \frac{d\hat{f}_1}{d\hat{f}_2}\frac{d\hat{f}_2}{d\hat{f}_3}\cdots\frac{d\hat{f}_n}{dx}$$

Exercise: $[\cos((\sin(x^3+4x))^{500})]' = ?$

2 Inverse Functions and Their Derivatives

Inverse of a Function

Definition 2 (Inverse Function)**.** *Suppose $f : E_1 \to E_2$. Then $g : E_2 \to E_1$ is an <u>inverse</u> of f if*

$$g(f(x)) = x \quad \forall x \in E_1$$

$$f(g(x)) = x \quad \forall x \in E_2$$

- Notation: $g(x) = f^{-1}(x)$

- $(f^{-1} \circ f)(x) = (f \circ f^{-1})(x) = x$
- Geometrical meaning: Graph of f^{-1} is reflection of the graph of f across 45 degree line
- $\exists f^{-1}$ (*i.e.*, f is invertible) iff f is strictly and monotonically increasing [decreasing]

Derivative of the Inverse Function

Theorem 1 (4.3: Inverse Function Theorem)**.** *Suppose $f : I \to \mathbb{R}$ is $\mathbf{C}^1$ Function, $f' \neq 0, x \in I$. Then,*

1. $\exists f^{-1}$ *on* I
2. $f^{-1} \in \mathbf{C}^1$ *on interval* $f(I)$
3. $(f^{-1})' = \frac{1}{f'(f^{-1})}$*. more intuitively,*

$$\frac{df^{-1}}{dx} = \frac{1}{\frac{df(f^{-1})}{df^{-1}}} = \frac{1}{\frac{dx}{df^{-1}}}$$

$$f(f^{-1})' = x' = 1 \quad \Rightarrow \quad \frac{df(f^{-1})}{dx} = 1 \quad \Rightarrow \quad \frac{df(f^{-1})}{df^{-1}} \frac{df^{-1}}{dx} = 1$$

The Derivative of $x^{m/n}$

Theorem 2 (4.4)**.**

$$\forall n \in \mathbb{N}, \quad (x^{1/n})' = \frac{1}{n} x^{(1/n)-1}$$

Theorem 3 (4.5)**.**

$$\forall m, n \in \mathbb{N}, \quad (x^{m/n})' = \frac{m}{n} x^{(m/n)-1}$$

In general,

$$(x^r)' = r x^{(r-1)} \quad \forall r \in \mathbb{R}$$