One-Var Calculus

CH2

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1 Functions on $\mathbb{R}$

Functions

Definition 1 (Polynomial). *Polynomial is sum of monomials*

Definition 2 (Monomial). *f is monomial if:*

$$f(x) = \bar{a}x^{\bar{k}}, \quad \bar{a} \in \mathbb{R}, \bar{k} \in \mathbb{N} + \{0\}$$

Definition 3 (Rational function).

$$P(x)/Q(x), \quad P, Q \in \textit{set of polynomials} \wedge Q \neq 0$$

Definition 4 (Exponential function). *f is exponential function if: $f(x) = \bar{a}\bar{b}^x, \quad \bar{a}, \bar{b} \in \mathbb{R}$*

Increasing, Decreasing

Definition 5 (Increasing Function). *f is increasing if:*

$$\forall x_1, x_2, \quad x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$$

Definition 6 (Decreasing Function). *f is decreasing if:*

$$\forall x_1, x_2, \quad x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$$

Minimum, Maximum

Definition 7 (Local (Relative) minimum). *$(x_0, f(x_0))$ is local minimum of f if f changes from decreasing to increasing*

Definition 8 (Local (Relative) maximum). *$(x_0, f(x_0))$ is <u>local maximum</u> of f if f changes from increasing to decreasing*

Note: Boundary max/min

Definition 9 (Global (Absolute) minimum). *$(x_0, f(x_0))$ is <u>Global minimum</u> of $f : D \to \mathbb{R}$ if*

$$f(x_0) \leq f(x) \quad \forall x \in D$$

Interval

Definition 10 (Open Interval).

$$(\bar{a}, \bar{b}) := \{x \in \mathbb{R} | \bar{a} < x < \bar{b}\}$$

Definition 11 (Closed Interval).

$$[\bar{a}, \bar{b}] := \{x \in \mathbb{R} | \bar{a} \leq x \leq \bar{b}\}$$

- half-open (or half-closed) interval

$$(\bar{a}, \bar{b}], \quad [\bar{a}, \bar{b}), \cdots$$

- infinite intervals

$$(-\infty, \bar{a}], \quad (\bar{a}, \infty), \cdots$$

2 Linear Functions

Linear Function

Definition 12 (Linear Function). *Polynomial of degree 0 (Constant function) or 1*

- Properties
  - Graph: Straight line
  - Constant slope

    Definition 13 (Slope of Linear function f).

$$Slope := \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Theorem 1 (2.1). *The line (Graph of f) with slope $\bar{m}$ and vertical intercept $\bar{b} \Rightarrow f(x) = \bar{m}x + \bar{b}$*

3 The Slope of Nonlinear Functions

Nonlinear function

Definition 14 (Nonlinear function)**.** *f is nonlinear function if f is not linear function*

Definition 15 (Derivative at $(\bar{x}_0, f(\bar{x}_0))$)**.** *$f'(\bar{x}_0)$:= Derivative of f at $(\bar{x}_0, f(\bar{x}_0))$ is the slope of the tangent line to the graph of f at $(\bar{x}_0, f(\bar{x}_0))$. i.e.,*

$$f'(\bar{x}_0) := \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

- Alternative Notations

$$f'(x_0) \equiv \frac{df}{dx}(x_0) \equiv \left.\frac{df}{dx}\right|_{x=x_0}$$

4 Computing Derivatives

Computing Derivatives

Theorem 2 (2.2)**.**

$$f(x) = x^2 \Rightarrow f'(x) = 2x$$

Theorem 3 (2.3)**.**

$$f(x) = x^{\bar{k}}, \bar{k} \in \mathbb{N} \Rightarrow f'(x) = \bar{k}x^{\bar{k}-1}$$

Operation of Functions

Definition 16 (Operation of Functions)**.**

$$(f \pm g)(x) := f(x) \pm g(x)$$

$$(f \cdot g)(x) := f(x)g(x)$$

$$(f/g)(x) := f(x)/g(x)$$

Rules for Computing Derivatives

Theorem 4 (2.4)**.**

$$(f \pm g)\prime = f' \pm g'$$

$$(\bar{k}f)' = \bar{k}(f')$$

$$(f \cdot g)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(f^{\bar{n}})' = \bar{n}f^{\bar{n}-1}f'$$

5 Differentiablilty and Continuity

Differentiable

Definition 17 (Differentiable). $f : D \to \mathbb{R}$ *is differentiable if*

$$\forall x_0 \in D \text{ and } \forall \{h_n\} \to 0, \exists unique\ f'(x_0) = \lim_{h_n \to 0} \frac{f(x_0 + h_n) - f(x_0)}{h_n}$$

- Geographical meaning: graph of f is smooth
- $f \in \mathbf{C}^1$

Continuous

Definition 18 (Continuous). $f : D \to \mathbb{R}$ *is continuous if:*

$$\forall x_0 \in D, \quad x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$$

- Geographical meaning: graph of f is not disconnected

6 Higher-order Derivatives

Second Derivative of f

Definition 19 (Second Derivative of $f \in \mathbf{C}^2$).

$$f'' := (f')' \equiv \frac{d}{dx}\left(\frac{df}{dx}\right) \equiv \frac{d^2 f}{dx^2}$$

- $\mathbf{C}^3$
$$f''', \quad \frac{d^3 f}{dx^3}$$
- $\mathbf{C}^k$
$$f^{(k)}, \quad \frac{d^k f}{dx^k}$$
- Polynomial is $\mathbf{C}^\infty$

7 Approximation by Differentials

Approximation

- Δx: Change in x (general representation)
- dx: "Small" change in x (or Δx which is sufficiently close to 0)

- Suppose x_0 is changed to $x_0 + h$, and h is sufficiently close to 0. then,

$$\frac{f(x_0+h) - f(x_0)}{h} \equiv \frac{\Delta f}{\Delta x} \approx f'(x_0)$$

$$\Delta f \approx f'(x_0)\Delta x$$

or,

$$df = f'(x_0)dx$$