

# One-Var Calculus

## CH2

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# Functions

## Definition (Polynomial)

Polynomial is sum of monomials

## Definition (Monomial)

$f$  is monomial if:

$$f(x) = \bar{a}x^{\bar{k}}, \quad \bar{a} \in \mathbb{R}, \bar{k} \in \mathbb{N} + \{0\}$$

## Definition (Rational function)

$$P(x)/Q(x), \quad P, Q \in \text{set of polynomials} \wedge Q \neq 0$$

## Definition (Exponential function)

$f$  is exponential function if:  $f(x) = \bar{a}\bar{b}^x$ ,  $\bar{a}, \bar{b} \in \mathbb{R}$

# Increasing, Decreasing

## Definition (Increasing Function)

$f$  is increasing if:

$$\forall x_1, x_2, \quad x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$$

## Definition (Decreasing Function)

$f$  is decreasing if:

$$\forall x_1, x_2, \quad x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$$

# Minimum, Maximum

## Definition (Local (Relative) minimum)

$(x_0, f(x_0))$  is local minimum of  $f$  if  $f$  changes from decreasing to increasing

## Definition (Local (Relative) maximum)

$(x_0, f(x_0))$  is local maximum of  $f$  if  $f$  changes from increasing to decreasing

Note: Boundary max/min

## Definition (Global (Absolute) minimum)

$(x_0, f(x_0))$  is Global minimum of  $f : D \rightarrow \mathbb{R}$  if

$$f(x_0) \leq f(x) \quad \forall x \in D$$

# Interval

## Definition (Open Interval)

$$(\bar{a}, \bar{b}) := \{x \in \mathbb{R} \mid \bar{a} < x < \bar{b}\}$$

## Definition (Closed Interval)

$$[\bar{a}, \bar{b}] := \{x \in \mathbb{R} \mid \bar{a} \leq x \leq \bar{b}\}$$

- half-open (or half-closed) intervals

$$(\bar{a}, \bar{b}], \quad [\bar{a}, \bar{b}), \dots$$

- infinite intervals

$$(-\infty, \bar{a}], \quad (\bar{a}, \infty), \dots$$

# Linear Function

## Definition (Linear Function)

*Polynomial of degree 0 (Constant function) or 1*

- Properties
  - Graph: Straight line
  - Constant slope

## Definition (Slope of Linear function $f$ )

$$\text{Slope} := \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

## Theorem (2.1)

*The line (Graph of  $f$ ) with slope  $\bar{m}$  and vertical intercept  $\bar{b} \Rightarrow$*

$$f(x) = \bar{m}x + \bar{b}$$

# Nonlinear function

## Definition (Nonlinear function)

*$f$  is nonlinear function if  $f$  is not linear function*

## Definition (Derivative at $(\bar{x}_0, f(\bar{x}_0))$ )

*$f'(\bar{x}_0) :=$  Derivative of  $f$  at  $(\bar{x}_0, f(\bar{x}_0))$  is the slope of the tangent line to the graph of  $f$  at  $(\bar{x}_0, f(\bar{x}_0))$ . i.e.,*

$$f'(\bar{x}_0) := \lim_{h \rightarrow 0} \frac{f(\bar{x}_0 + h) - f(\bar{x}_0)}{h}$$

- Alternative Notations

$$f'(x_0) \equiv \frac{df}{dx}(x_0) \equiv \left. \frac{df}{dx} \right|_{x=x_0}$$



# Computing Derivatives

## Theorem (2.2)

$$f(x) = x^2 \Rightarrow f'(x) = 2x$$

## Theorem (2.3)

$$f(x) = x^{\bar{k}}, \bar{k} \in \mathbb{N} \Rightarrow f'(x) = \bar{k}x^{\bar{k}-1}$$

# Operation of Functions

## Definition (Operation of Functions)

$$(f \pm g)(x) := f(x) \pm g(x)$$

$$(f \cdot g)(x) := f(x)g(x)$$

$$(f/g)(x) := f(x)/g(x)$$

# Rules for Computing Derivatives

## Theorem (2.4)

$$(f \pm g)' = f' \pm g'$$

$$(\bar{k}f)' = \bar{k}(f')$$

$$(f \cdot g)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(f^{\bar{n}})' = \bar{n}f^{\bar{n}-1}f'$$

# Differentiable

## Definition (Differentiable)

$f : D \rightarrow \mathbb{R}$  is differentiable if

$$\forall x_0 \in D \text{ and } \forall \{h_n\} \rightarrow 0, \exists \text{unique } f'(x_0) = \lim_{h_n \rightarrow 0} \frac{f(x_0 + h_n) - f(x_0)}{h_n}$$

- Geometrical meaning: graph of  $f$  is smooth
- $f \in C^1$

# Continuous

## Definition (Continuous)

$f : D \rightarrow \mathbb{R}$  is continuous if:

$$\forall x_0 \in D, \quad x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow f(x_0)$$

- Geometrical meaning: graph of  $f$  is not disconnected

# Second Derivative of $f$

Definition (Second Derivative of  $f \in C^2$ )

$$f'' := (f')' \equiv \frac{d}{dx} \left( \frac{df}{dx} \right) \equiv \frac{d^2 f}{dx^2}$$

- $C^3$

$$f''', \quad \frac{d^3 f}{dx^3}$$

- $C^k$

$$f^{(k)}, \quad \frac{d^k f}{dx^k}$$

- Polynomial is  $C^\infty$

# Approximation

- $\Delta x$ : Change in  $x$  (general representation)
- $dx$ : “Small” change in  $x$  (or  $\Delta x$  which is sufficiently close to 0)
- Suppose  $x_0$  is changed to  $x_0 + h$ , and  $h$  is sufficiently close to 0. then,

$$\frac{f(x_0 + h) - f(x_0)}{h} \equiv \frac{\Delta f}{\Delta x} \approx f'(x_0)$$

$$\Delta f \approx f'(x_0)\Delta x$$

or,

$$df = f'(x_0)dx$$