2. Faça a leitura do Capítulo 2 do Nise, sobre a modelagem de sistemas eletromecânicos, e explique o que significado físico da curva Torque-velocidade e como ela deve ser usada para o projeto de sistemas que usem motor CC.

A curva torque-velocidade é uma relação linear entre essas duas grandezas relativas à um motor CC para uma dada tensão de armadura (e_a) constante. A partir dessa curva podemos determinar os parâmetros de R_a, K_b e K_t, os quais serão usados posteriormente na montagem da função de transferência do sistema.

3. Da lista de problemas do livro, disponibilizado no arquivo 'Nise - cap2 - Lista de Exercícios', resolva as seguintes sequências:

a) Faça os exercícios de 36 até o 41, modelando os sistemas com engrenagens.

36. For the rotational system shown in Figure P2.21, find the transfer function, $G(s) = \theta_2(s)/T(s)$. [Section: 2.7]

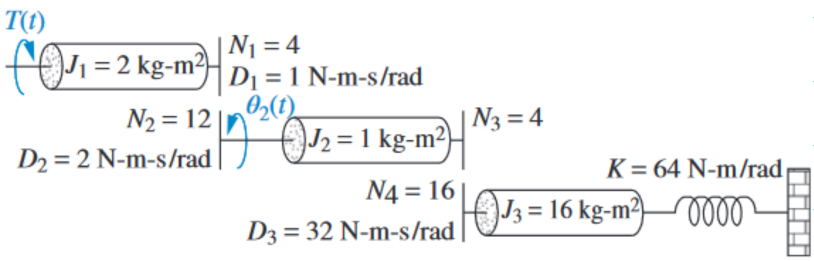


FIGURE P2.21

$$R_{1,2} = \left(\frac{N_1}{N_2}\right)^2 = \left(\frac{4}{12}\right)^2 = \left(\frac{1}{9}\right) \qquad R_{1,3} = \left(\frac{N_3}{N_4}\cdot\frac{N_1}{N_2}\right)^2 = \left(\frac{4}{16}\cdot\frac{4}{12}\right)^2 = \frac{1}{144}$$

$$J_{eq} = J_1 + J_2 R_{1,2} + J_3 R_{1,3} = \frac{20}{9} \qquad K_{eq} = K R_{1,3} = \frac{4}{9}$$

$$D_{eq} = D_1 + D_2 R_{1,2} + D_3 R_{1,3} = \frac{13}{9} \qquad -D_{eq}\dot{\theta}_1(t) - K\theta_1(t) + T(t) = J_{eq}\ddot{\theta}_1(t)$$

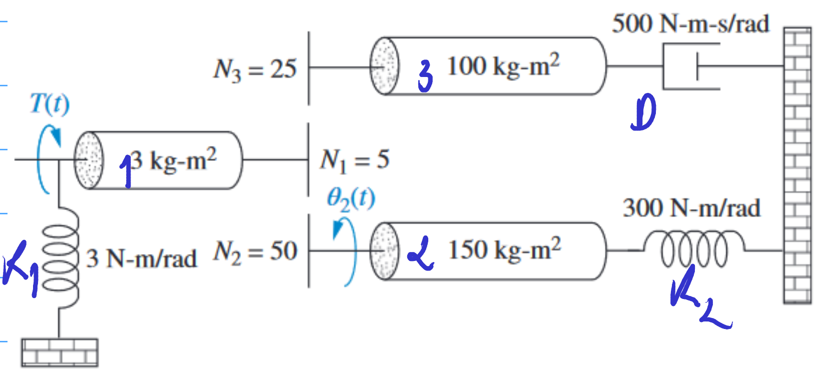
$$J_{eq}\ddot{\theta}_1(t) + D_{eq}\dot{\theta}_1(t) + K\theta_1(t) = T(t)$$

$$J_{eq}\ddot{\theta}_1(t) + D_{eq}\dot{\theta}_1(t) + K\theta_1(t) = T(t) \qquad \theta_1(s)\left(s^2 J_{eq} + sD_{eq} + K\right) = T(s)$$

$$\frac{\theta_1(s)}{T(s)} = \frac{1}{\left(\frac{20}{9}s^2 + \frac{13}{9}s + \frac{4}{9}\right)} = \frac{9}{\left(20s^2 + 13s + 4\right)}$$

$$\theta_2(s) = \frac{N_1}{N_2}\theta_1 \Rightarrow \frac{\theta_2(s)}{T(s)} = \frac{9}{3\left(20s^2 + 13s + 4\right)} = \frac{3}{\left(20s^2 + 13s + 4\right)}$$

37. Find the transfer function, $G(s) = \theta_2(s)/T(s)$, for the rotational mechanical system shown in Figure P2.22. [Section: 2.7]



$$R_{1,2} = \left(\frac{5}{50}\right)^2$$

$$R_{1,3} = \left(\frac{5}{25}\right)^2$$

$$K_{eq} = K_1 + K_2 \cdot R_{1,2} = 6 \qquad -K_{eq}\theta_1(t) - D_{eq}\dot{\theta}_1(t) + T(t) = J_{eq}\ddot{\theta}_1(t)$$

$$D_{eq} = D \cdot R_{1,3} = 20 \qquad J_{eq}\ddot{\theta}_1(t) + K_{eq}\theta_1(t) + D_{eq}\dot{\theta}_1(t) = T(t)$$

$$J_{eq} = M_1 + M_2 R_{1,2} + M_3 R_{1,3} = 17/2 \qquad \theta_1(s)\left(\frac{17}{2}s^2 + 20s + 6\right) = T(s)$$

$$\theta_1(s) = 10\,\theta_2(s) \Rightarrow 10\,\theta_2(s)\left(\frac{17}{2}s^2 + 20s + 6\right) = T(s)$$

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{10\left(\frac{17}{2}s^2 + 20s + 6\right)}$$

38. Find the transfer function, $G(s) = \theta_4(s)/T(s)$, for the rotational system shown in Figure P2.23. [Section: 2.7]

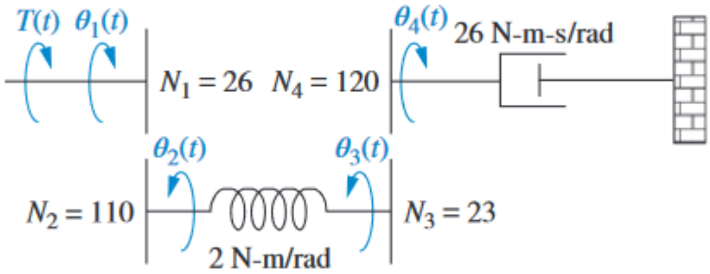


FIGURE P2.23

$$R_{1,2} = \left(\frac{N_1}{N_2}\right) = \frac{26}{110} \qquad R_{3,4} = \left(\frac{N_4}{N_3}\right) = \frac{120}{23}$$

$$T(s) + K\left(\theta_4(t)R_{3,4} - R_{1,2}\theta_1(t)\right) = 0$$

$$-K\left(\theta_4(t)R_{3,4} - R_{1,2}\theta_1(t)\right) = T(s)$$

$$R_{1,2}\theta_1(s)K \quad - K\theta_4(s)R_{3,4} = T(s)$$

$$-D\theta_4(t) - K\left(\theta_4(t)R_{3,4} - R_{1,2}\theta_1(t)\right) = 0$$

$$D\dot{\theta}_4(t) + K\left(\theta_4(t)R_{3,4} - R_{1,2}\theta_1(t)\right) = 0$$

$$-R_{1,2}\theta_1(s)K \quad + \theta_4(s)(26s + 2R_{3,4})$$

$$\begin{cases} R_{1,2}\theta_1(s)K & -K\theta_4(s)R_{3,4} \\ -R_{1,2}\theta_1(s)K & +\theta_4(s)(26s+2R_{3,4}) \end{cases} = \begin{bmatrix} 2R_{1,2} & -2R_{3,4} \\ -2R_{1,2} & (26s+2R_{3,4}) \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2R_{1,2} & -2R_{3,4} \\ -2R_{1,2} & (26s+2R_{3,4}) \end{vmatrix} = 2R_{1,2}(26s+2R_{3,4}) - (2R_{3,4}2R_{1,2})$$

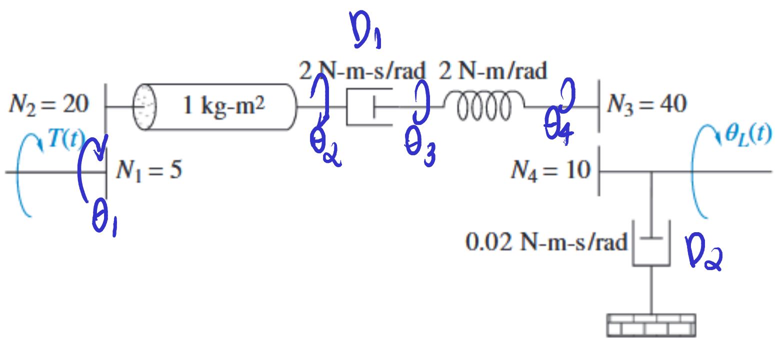
$$= 676s/5$$

$$\Theta_4(s) = \frac{\begin{vmatrix} 2R_{1,2} & T(s) \\ -2R_{1,2} & 0 \end{vmatrix}}{\Delta}$$

$$\Theta_4(s) = \frac{-2R_{1,2}T(s)5}{676s}$$

$$\frac{\Theta_4(s)}{T(s)} = -\frac{1}{26s}$$

39. For the rotational system shown in Figure P2.24, find the transfer function, $G(s) = \theta_L(s)/T(s)$. [Section: 2.7]



$$R_{1,2} = \left(\frac{5}{20}\right) = \frac{1}{4} \qquad R_{L,4} = \left(\frac{10}{40}\right) = \frac{1}{4}$$

$$D_1(\dot{\theta}_3(t) - \dot{\theta}_1(t)R_{1,2}) + T(s) = J_1\ddot{\theta}_1(t)R_{1,2}$$

$$J_1\ddot{\theta}_1(t)R_{1,2} - D_1(\dot{\theta}_3(t) - \dot{\theta}_1(t)R_{1,2}) = T(s)$$

$$\Theta_1(s)R_{1,2}(s^2+2s) \quad -\Theta_3(s)(2s)$$

$$-D_1(\dot{\theta}_3(t) - \dot{\theta}_1(t)R_{1,2}) + K(\dot{\theta}_L(t)R_{L,4} - \dot{\theta}_3(t)) = 0$$

$$D_1(\dot\theta_3(t) - \dot\theta_1(t) R_{1,2}) - K(\dot\theta_L(t) R_{L,4} - \dot\theta_3(t)) = 0$$

$$-R_{1,2}\,\theta_1(s)(2s) + \theta_3(s)(2s+2) - 2R_{L,4}\,\theta_L(s) = 0$$

$$-D_2\dot\theta_L(t) - K(\dot\theta_L(t) R_{L,4} - \dot\theta_3(t)) = 0$$

$$D_2\dot\theta_L(t) + K(\dot\theta_L(t) R_{L,4} - \dot\theta_3(t)) = 0$$

$$\theta_L(s)\left(\frac{2}{100}s + 2R_{L,4}\right) - 2\theta_3(s) = 0$$

$$\begin{cases} \theta_1(s) R_{1,2}(s^2+2s) \quad -\theta_3(s)(2s) \\ -R_{1,2}\,\theta_1(s)(2s) + \theta_3(s)(2s+2) - 2R_{L,4}\,\theta_L(s) = 0 \\ \theta_L(s)\left(\frac{2}{100}s + 2R_{L,4}\right) - 2\theta_3(s) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} R_{1,2}(s^2+2s) & 0 & -2s \\ -R_{1,2}(2s) & (2s+2) & -2R_{L,4} \\ 0 & -2 & \left(\frac{2}{100}s + 2R_{L,4}\right) \end{vmatrix} = \frac{s^4}{400} + \frac{37s^3}{1600} + \frac{371s^2}{800}$$

$$\theta_L(s) = \frac{\begin{vmatrix} R_{1,2}(s^2+2s) & 0 & T(s) \\ -R_{1,2}(2s) & (2s+2) & 0 \\ 0 & -2 & 0 \end{vmatrix}}{\Delta} \Rightarrow \frac{\theta_L(s)}{T(s)} = \frac{s}{\left(\frac{s^4}{400} + \frac{37s^3}{1600} + \frac{371s^2}{800}\right)4}$$

40. For the rotational system shown in Figure P2.25, write the equations of motion from which the transfer function, $G(s) = \theta_1(s)/T(s)$, can be found. [Section: 2.7]

$$R_{12} = \left(\frac{N_1}{N_2}\right)^2$$

$$R_{43} = \left(\frac{N_4}{N_3}\right)^2$$

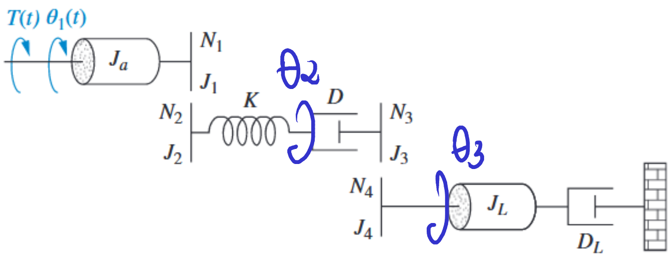


FIGURE P2.25

$$J_{eq_1} = J_1 + J_2 R_{12} + J_a \qquad T(t) + K(\theta_2(t) - R_{12}\theta_1(t)) = J_{eq_1}\ddot{\theta}_1(t)$$

$$J_{eq_1}\ddot{\theta}_1(t) - K(\theta_2(t) - R_{12}\theta_1(t)) = T(t)$$

$$\theta_1(s)(s^2 J_{eq_1} + R_{12}K) - K\theta_2(s) = T(s)$$

$$-K(\theta_2(t) - R_{12}\theta_1(t)) + D(\dot{\theta}_3(t)R_{43} - \dot{\theta}_2(t)) = 0$$

$$K(\theta_2(t) - R_{12}\theta_1(t)) - D(\dot{\theta}_3(t)R_{43} - \dot{\theta}_2(t)) = 0$$

$$-KR_{12}\theta_1(s) + \theta_2(s)(Ds + K) - R_{43}D\theta_3(s) = 0$$

$$J_{eq_2} = J_L + R_{43}J_3 + J_4 \qquad -D_L\dot{\theta}_3(t) - D(\theta_3(t)R_{43} - \theta_2(t)) = J_{eq_2}\ddot{\theta}_3(t)$$

$$J_{eq_2}\ddot{\theta}_3(t) + D_L\dot{\theta}_3(t) + D(\theta_3(t)R_{43} - \theta_2(t)) = 0$$

$$-\theta_2(s)(Ds) + \theta_3(s)(s^2 J_{eq_2} + s(D_L + DR_{43})$$

$$\begin{cases} \Theta_1(s)(s^2 J_{eq1} + R_{12}K) - K\Theta_2(s) = T(s) \\ -KR_{12}\Theta_1(s) + \Theta_2(s)(Ds+K) - R_{43}D\Theta_3(s) = 0 \\ -\Theta_2(s)(Ds) + \Theta_3(s)(s^2 J_{eq2} + s(D_L + DR_{43})) = 0 \end{cases}$$

$$R_{12} = \left(\frac{N_1}{N_2}\right)$$

$$R_{13} = R_{12}\left(\frac{N_3}{N_4}\right)$$

41. Given the rotational system shown in Figure P2.26, find the transfer function, $G(s) = \theta_6(s)/\theta_1(s)$. [Section: 2.7]

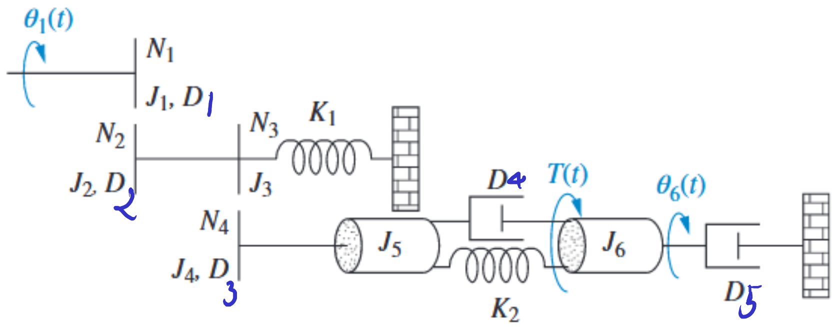


FIGURE P2.26

$$J_{eq} = J_1 + R_{12}(J_2 + J_3) + R_{13}(J_3 + J_4 + J_5) \qquad K_{eq} = K_1 R_{12}$$

$$D_{eq} = D_1 + D_2 R_{12} + D_3 R_{13}$$

$$-D_{eq} + D_4(\dot\theta_6(t) - R_{13}\dot\theta_1(t)) + K_2(\theta_6(t) - R_{13}\theta_1(t)) - K_{eq} = J_{eq}\ddot\theta_1(t)$$

$$J_{eq}\ddot\theta_1(t) + D_{eq} - D_4(\dot\theta_6(t) - R_{13}\theta_1(t)) - K_2(\theta_6(t) - R_{13}\theta_1(t)) - K_{eq} = 0$$

$$\Theta_1(s)(s^2 J_{eq} + s(D_{eq} + R_{13}D_4) + K_{eq} + K_2 R_{13} - \Theta_6(s)(sD_4 + K_2) = 0$$

$$-D_4(\dot\theta_6(t) - R_{13}\theta_1(t)) - D_5\dot\theta_6(t) - K_2(\theta_6(t) - R_{13}\theta_1(t)) + T(t) = 0$$

$$D_4(\dot\theta_6(t) - R_{13}\theta_1(t)) + D_5\dot\theta_6(t) + K_2(\theta_6(t) - R_{13}\theta_1(t)) = T(t)$$

$$-\Theta_1(s)R_{13}(sD_4 + K_2) + \Theta_6(s)(s(D_5 + D_6) + K_2) = T(s)$$

$$\begin{cases} \Theta_1(s)(s^2 J_{eq} + s(D_{eq} + R_{13} D_4) + K_{eq} + K_2 R_{13} - \Theta_6(s)(sD_4 + K_2) = 0 \\ -\Theta_1(s) R_{13}(sD_4 + K_2) + \Theta_6(s)(s(D_5 + D_6) + K_2) = T(s) \end{cases}$$

$$\begin{bmatrix} (s^2 J_{eq} + s(D_{eq} + R_{13} D_4) + K_{eq} + K_2 R_{13}) & (sD_4 + K_2) \\ -R_{13}(sD_4 + K_2) & (s(D_5 + D_6) + K_2) \end{bmatrix}$$

$$\Delta = \begin{vmatrix} (s^2 J_{eq} + s(D_{eq} + R_{13} D_4) + K_{eq} + K_2 R_{13}) & (sD_4 + K_2) \\ -R_{13}(sD_4 + K_2) & (s(D_5 + D_6) + K_2) \end{vmatrix}$$

$$\Theta_1(s) = \frac{\begin{bmatrix} 0 & (sD_4 + K_2) \\ T(s) & (s(D_5 + D_6) + K_2) \end{bmatrix}}{\Delta}$$

$$\Theta_6(s) = \frac{\begin{bmatrix} (s^2 J_{eq} + s(D_{eq} + R_{13} D_4) + K_{eq} + K_2 R_{13}) & 0 \\ -R_{13}(sD_4 + K_2) & T(s) \end{bmatrix}}{\Delta}$$

$$\frac{\Theta_6(s)}{\Theta_1(s)} = \frac{-K_2 + K_{eq} + sD_{eq} + J_{eq} s^2 + D_4 R_{13} s}{K_2 + D_4 s}$$

42. In the system shown in Figure P2.27, the inertia, J, of radius, r, is constrained to move only about the stationary axis A. A viscous damping force of translational value f_v exists between the bodies J and M. If an external force, $f(t)$, is applied to the mass, find the transfer function, $G(s) = \theta(s)/F(s)$. [Sections: 2.5; 2.6]

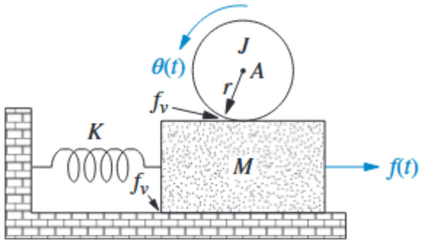


FIGURE P2.27

$$-Kx_1(t) - f_{v_1}(\dot{x}_1(t) - \dot{\theta}(t)) - f_{v_2}\dot{x}_1(t) + f(t) = M\ddot{x}_1(t)$$

$$M\ddot{x}_1(t) + Kx_1(t) + f_{v_1}(\dot{x}_1(t) - \dot{\theta}(t)) + f_{v_2}\dot{x}_1(t) = f(t)$$

$$X_1(s)(Ms^2 + s(f_{v_1} + f_{v_2}) + K) \quad - \Theta(s)(sf_{v_1}) = F(s)$$

$$f_{v_1}(\dot{x}_1(t) - \dot{\theta}(t)) = J\ddot{\theta}(t)$$

$$J\ddot{\theta}(t) - f_{v_1}(\dot{x}_1(t) - \dot{\theta}(t)) = 0$$

$$-X_1(s)(sf_{v_1}) \quad + \Theta(s)(Js^2 + sf_{v_1}) = 0$$

$$\begin{bmatrix} (Ms^2 + s(f_{v_1} + f_{v_2}) + K) & -(sf_{v_1}) \\ -(sf_{v_1}) & (Js^2 + sf_{v_1}) \end{bmatrix} \Rightarrow \Delta = \begin{vmatrix} (Ms^2 + s(f_{v_1} + f_{v_2}) + K) & -(sf_{v_1}) \\ -(sf_{v_1}) & (Js^2 + sf_{v_1}) \end{vmatrix}$$

$$\Theta(s) = \frac{\begin{vmatrix} (Ms^2 + s(f_{v_1} + f_{v_2}) + K) & F(s) \\ -(sf_{v_1}) & 0 \end{vmatrix}}{\Delta}$$

$$\frac{\theta(s)}{F(s)} = \frac{s f_{v1}}{J(Ms^3 + s^2(f_{v1}+f_{v2}) + Ks) + M f_{v1} s^2 + K f_{v1}}$$

43. For the combined translational and rotational system shown in Figure P2.28, find the transfer function, $G(s) = X(s)/T(s)$. [Sections: 2.5; 2.6; 2.7]

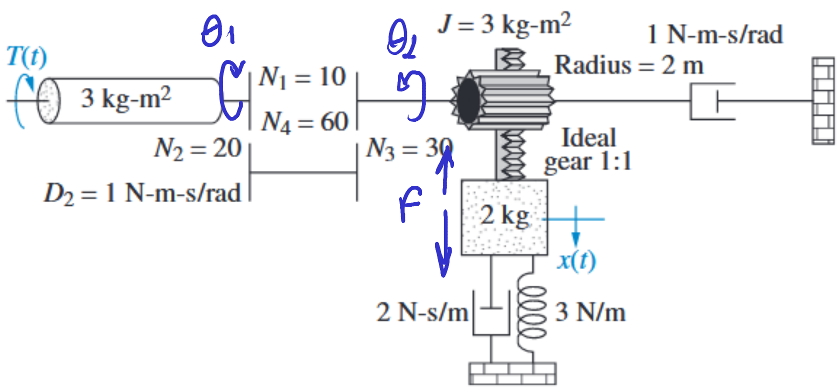


FIGURE P2.28

$$R_{13} = \left(\frac{N_3 N_1}{N_4 N_2}\right)^2 = \frac{1}{16} \qquad J_{eq} = J_1 + J_2 R_{13} \qquad D_{eq} = D_2 R_{12} + D_3 R_{13}$$

$$R_{12} = \left(\frac{N_1}{N_2}\right)^2 = \frac{1}{4}$$

$$x(t) = R\theta_2(t) = 2\theta_2(t) = 2R_{13}\theta_1(t) \Rightarrow \theta_1(t) = \frac{x(t)}{2\sqrt{R_{13}}}$$

$$J_{eq}\ddot{\theta}_1(t) + D_{eq}\dot{\theta}_1(t) = 0$$

$$-D_{eq}\dot{\theta}_1(t) - F(s) + T(s) = J_{eq}\ddot{\theta}_1(t) \qquad \boxed{M\ddot{x}(t) + D_4\dot{x}(t) + Kx(t) = F(s)}$$

$$J_{eq}\ddot{\theta}_1(t) + D_{eq}\dot{\theta}_1(t) + F(s) = T(t)$$

$$J_{eq}\ddot{\theta}_1(t) + D_{eq}\dot{\theta}_1(t) + M\ddot{x}(t) + D_4\dot{x}(t) + Kx(t) = T(t)$$

$$\frac{J_{eq}\ddot{x}(t)}{2\sqrt{R_{13}}} + \frac{D_{eq}\dot{x}(t)}{2\sqrt{R_{13}}} + M\ddot{x}(t) + D_4\dot{x}(t) + Kx(t) = T(t)$$

$$X(s)\left(\frac{1}{2\sqrt{R_{13}}}(s^2 J_{eq} + sD_{eq}) + Ms^2 + sD_4 + K\right) = T(s)$$

$$X(s)\left(\frac{4867s^2}{2048} + \frac{261s}{128} + 3\right) = T(s)$$

$$\frac{X(s)}{T(s)} = \frac{2048}{4867s^2} + \frac{128}{261s} + \frac{1}{3}$$

44. Given the combined translational and rotational system shown in Figure P2.29, find the transfer function, $G(s) = X(s)/T(s)$. [Sections: 2.5; 2.6]

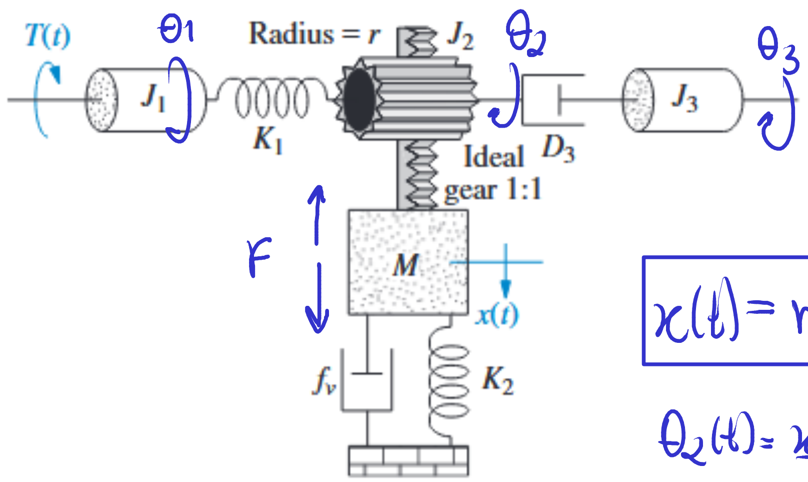


FIGURE P2.29

$$\boxed{x(t) = r\theta_2(t)}$$

$$\theta_2(t) = \frac{x(t)}{r}$$

$$K_1(\theta_2(t) - \theta_1(t)) + T(t) = J_1\ddot{\theta}_1(t)$$

$$-f_v\dot{x}_1(t) - K_2x_1(t) + F(t) = M\ddot{x}(t)$$

$$J_1\ddot{\theta}_1(t) - K_1(\theta_2(t) - \theta_1(t)) = T(t)$$

$$M\ddot{x}(t) + f_v\dot{x}_1(t) + K_2x_1(t) = F(t)$$

$$\theta_1(s)(s^2J_1 + K_1) - \theta_2(s)K_1 = T(s)$$

$$X(s)(Ms^2 + sf_v + K_2) = F(s)$$

$$-K_1(\theta_2(t)-\theta_1(t))+D_3(\dot{\theta}_3(t)-\dot{\theta}_2(t))-F(t)=J_2\ddot{\theta}_2(t)$$

$$J_2\ddot{\theta}_2(t)+K_1(\theta_2(t)-\theta_1(t))-D_3(\dot{\theta}_3(t)-\dot{\theta}_2(t))-F(t)=0$$

$$J_2\ddot{\theta}_2(t)+K_1(\theta_2(t)-\theta_1(t))-D_3(\dot{\theta}_3(t)-\dot{\theta}_2(t))-M\ddot{x}(t)+f_v\dot{x}_1(t)+K_2x_1(t)$$

$$J_2\ddot{\theta}_2(t)+K_1(\theta_2(t)-\theta_1(t))-D_3(\dot{\theta}_3(t)-\dot{\theta}_2(t))-\frac{(M\ddot{\theta}_2(t)+f_v\dot{\theta}_2(t)+K_2\theta_2(t))}{r}$$

$$-K\theta_1(s)+\theta_2(s)\left(\left(J_2-\frac{M}{r}\right)s^2+\left(D_3+\frac{f_v}{r}\right)s+K_1-\frac{K_2}{r}\right)-D_3\theta_3(s)$$

$$-D_3(\dot{\theta}_3(t)-\dot{\theta}_2(t))=J_3\dot{\theta}_3(t)$$

$$-\theta_2(s)(sD_3)+\theta_3(s)(J_3s^2+sD_3)$$

$$\begin{cases}\theta_1(s)(s^2J_1+K_1)-\theta_2(s)K_1=T(s)\\ -K\theta_1(s)+\theta_2(s)\left(\left(J_2-\frac{M}{r}\right)s^2+\left(D_3+\frac{f_v}{r}\right)s+K_1-\frac{K_2}{r}\right)-D_3\theta_3(s)\\ -\theta_2(s)(sD_3)+\theta_3(s)(J_3s^2+sD_3)\end{cases}$$

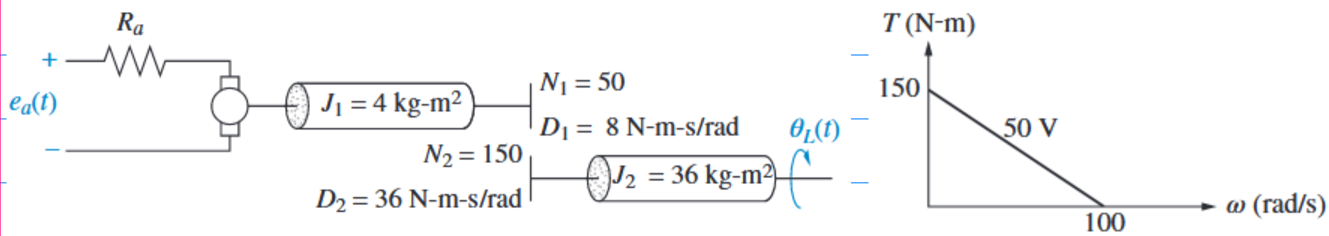
$$\begin{bmatrix}(s^2J_1+K_1) & -K_1 & 0\\ -K_1 & \left(\left(J_2-\frac{M}{r}\right)s^2+\left(D_3+\frac{f_v}{r}\right)s+K_1-\frac{K_2}{r}\right) & -D_3s\\ 0 & -sD_3 & (J_3s^2+sD_3)\end{bmatrix}$$

$$\Delta = \begin{vmatrix} (s^2 J_1 + K_1) & -K_1 & 0 \\ -K_1 & \left(\left(J_2 - \frac{M}{r}\right)s^2 + \left(D_3 + \frac{f_v}{r}\right)s + K_1 - \frac{K_2}{r}\right) & -D_3 s \\ 0 & -sD_3 & (J_3 s^2 + sD_3) \end{vmatrix}$$

$$X(s) = r\,\theta_2(s) = \frac{r \begin{vmatrix} (s^2 J_1 + K_1) & T(s) & 0 \\ -K_1 & 0 & -D_3 s \\ 0 & 0 & (J_3 s^2 + sD_3) \end{vmatrix}}{\Delta}$$

$$\frac{X(s)}{T(s)} = r\left(\frac{K_1(J_3 s^2 + D_3 s)}{\Delta}\right)$$

45. For the motor, load, and torque-speed curve shown in Figure P2.30, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$. [Section: 2.8]



$$T_m = -\frac{K_b - K_t}{R_a}\omega_m + \frac{K_t e_a}{R_a} \qquad K_b = \frac{e_a}{\omega_n} = \frac{50}{100} = \frac{1}{2}$$

$$\frac{K_t}{R_a} = \frac{T_s}{e_a} = \frac{150}{50} = 3 \qquad \omega_{nL} = 100$$

$$T_s = 150$$

$$R_{21} = \left(\frac{N_2}{N_1}\right)^2 = \left(\frac{15\cancel{0}}{5\cancel{0}}\right)^2 = 9 \qquad J_{eq} = J_1 R_{21} + J_2 \qquad \theta_m = \frac{N_1}{N_2}\theta_L$$

$$D_{eq} = D_2 + D_1 R_{21}$$

$$-D_{eq}\dot{\theta}_L(t) + T(t) = J_{eq}\ddot{\theta}_L(t) \qquad T(t) = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a$$

$$J_{eq}\ddot{\theta}_L(t) + D_{eq}\dot{\theta}_L(t) = T(s)$$

$$T(s) = -\frac{\cancel{3}}{2}\frac{1}{\cancel{3}}\theta_L(s) + 3E_a(s) = -\frac{\theta_L(s)}{2} + 3E_a(s)$$

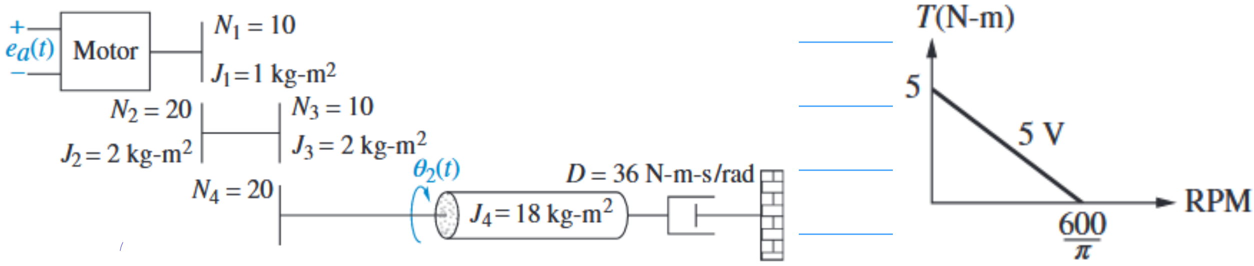
$$\theta_L(s)\left(s^2 J_{eq} + sD_{eq}\right) = T(s)$$

$$\theta_L(s)\left(s^2 J_{eq} + sD_{eq}\right) = -\frac{\theta_L(s)}{2} + 3E_a(s)$$

$$\theta_L(s)\left(s^2 J_{eq} + sD_{eq} + \frac{1}{2}\right) = 3E_a(s)$$

$$\frac{\theta_L(s)}{E_a(s)} = \frac{3}{s^2 J_{eq} + sD_{eq} + \frac{1}{2}} = \frac{3}{72s^2 + 332s + \frac{1}{2}}$$

46. The motor whose torque-speed characteristics are shown in Figure P2.31 drives the load shown in the diagram. Some of the gears have inertia. Find the transfer function, $G(s) = \theta_2(s)/E_a(s)$. [Section: 2.8]



$$\frac{1R}{min} = \frac{2\pi\,rad}{60s} \Rightarrow \frac{600}{\pi}\frac{2\pi\,rad}{60s} = 20\,rad/s = \omega_{nL} \qquad \frac{K_t}{R_a} = \frac{T_s}{e_a} = \frac{5}{5} = 1$$

$$K_b = \frac{e_a}{w_n} = \frac{5}{20} = \frac{1}{4} \qquad T_m = -\frac{K_b K_t}{R_a} w_m + \frac{K_t e_a}{R_a}$$

$$R_{41} = \left(\frac{N_1}{N_2}\frac{N_3}{N_4}\right)^2 = \left(\frac{10}{20}\cdot\frac{10}{20}\right)^2 = \frac{1}{16} \qquad J_{eq} = J_1 R_{41} + (J_2 + J_3) R_{43} + J_4 = \frac{305}{16}$$

$$R_{43} = \left(\frac{N_3}{N_4}\right)^2 = \frac{1}{4} \qquad -D\dot{\theta}_2(t) + T(t) = J_{eq}\ddot{\theta}_2(t)$$

$$J_{eq}\ddot{\theta}_2(t) + D\dot{\theta}_2(t) = T(t)$$

$$T(t) = -\frac{K_b K_t}{R_a} w_m + \frac{K_t e_a}{R_a} \qquad w_n\left(\frac{N_1}{N_2}\frac{N_3}{N_4}\right) = \theta_2 = 2\theta_2$$

$$T(t) = -\frac{w_n}{4} + e_a = -\frac{\theta_2}{2} + e_a$$

$$J_{eq}\ddot{\theta}_2(t) + D\dot{\theta}_2(t) = -\frac{\theta_2}{2} + e_a$$

$$\theta_2(s)\left(s^2 J_{eq} + sD + \frac{1}{2}\right) = E_a(s) \qquad \frac{\theta_2(s)}{E_a(s)} = \frac{1}{\frac{305}{16}s^2 + 36s + \frac{1}{2}}$$

$$\frac{305s^2 + 576s + 8}{16} \qquad \frac{\theta_2(s)}{E_a(s)} = \frac{16}{305s^2 + 576s + 8}$$

47. A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m^2 and 3 N-m-s/rad, respectively, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$, of this motor if it drives an inertia load of 105 kg-m^2 through a gear train, as shown in Figure P2.32. [Section: 2.8]

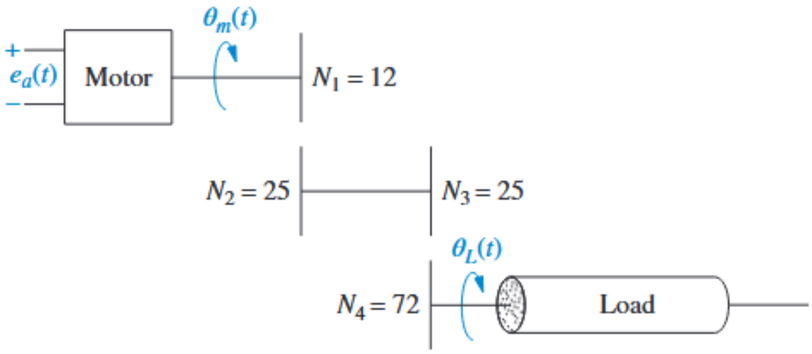


FIGURE P2.32

$$\frac{600R}{min} \cdot \frac{2\pi}{60} = 20\pi \text{ rad/s} \qquad \frac{K_t}{R_a} = \frac{T_s}{e_a} = \frac{100}{12} = \frac{25}{3}$$

$$J_m = 7 \quad D_m = 3 \qquad K_b = \frac{e_a}{\omega_n} = \frac{12}{20\pi} = \frac{3}{5\pi}$$

$$J_L = 105$$

$$R_{41} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 = 1,911 \qquad J_{eq} = J_L + J_m \cdot R_{41} = 118,37$$

$$D_{eq} = D_m \cdot R_{41} = 5,733$$

$$-D_{eq}\dot{\theta}_L(t) + T(t) = J_{eq}\ddot{\theta}_L(t) \qquad \omega_m = \left(\frac{N_1 N_3}{N_2 N_4}\right)\theta_L = \frac{\theta_L}{6}$$

$$J_{eq}\ddot{\theta}_L(t) + D_{eq}\dot{\theta}_L(t) = T(t)$$

$$T(t) = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t e_a}{R_a} = -\frac{5}{\pi}\omega_m + \frac{25}{3}e_a$$

$$J_{eq}\ddot{\theta}_L(t) + D_{eq}\dot{\theta}_L(t) = -\frac{5}{\pi}\omega_m + \frac{25}{3}e_a$$

$$\theta_L(s)\left(s^2 J_{eq} + sD_{eq} + \frac{5}{6\pi}\right) = \frac{25}{3}E_a(s)$$

$$\frac{\theta_L(s)}{E_a(s)} = \frac{25}{3\left(s^2 J_{eq} + sD_{eq} + \frac{5}{6\pi}\right)} = \frac{25}{3\left(s^2 118{,}37 + 5{,}733s + \frac{5}{6\pi}\right)}$$

48. In this chapter, we derived the transfer function of a dc motor relating the angular displacement output to the armature voltage input. Often we want to control the output torque rather than the displacement. Derive the transfer function of the motor that relates output torque to input armature voltage. [Section: 2.8]

$$T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a \qquad \frac{K_t}{R_a} = C$$

$$T_m = -K_b C\omega_m + C e_a \qquad T_m = C(-K_b\omega_m + e_a)$$

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s) \qquad T_m(s) = C(-K_b\theta_m(s) + E_a(s))$$

$$\theta_m(s) = \frac{T_m(s)}{(J_m s^2 + D_m s)} \qquad T_m(s) = C\left(-K_b\frac{T_m(s)}{(J_m s^2 + D_m s)} + E_a(s)\right)$$

$$T_m(s) = \qquad + E_a(s))$$

$$C\left(\frac{K_b T_m(s)}{(J_m s^2 + D_m s)}\right) + T_m(s) = CE_a(s)$$

$$T_m(s)\left(1 + \frac{CK_b}{(J_m s^2 + D_m s)}\right) = CE_a(s)$$

$$\frac{I_m(s)}{E_a(s)} = \frac{C}{\left(1+\frac{CK_b}{(J_m s^2+D_m s)}\right)} = \frac{C\,(J_m s^2+D_m s)}{(J_m s^2+D_m s)+CK_b}$$

$$\left(1+\frac{CK_b}{(J_m s^2+D_m s)}\right) = \frac{(J_m s^2+D_m s)+CK_b}{(J_m s^2+D_m s)}$$

49. Find the transfer function, $G(s) = X(s)/E_a(s)$, for the system shown in Figure P2.33. [Sections: 2.5–2.8]

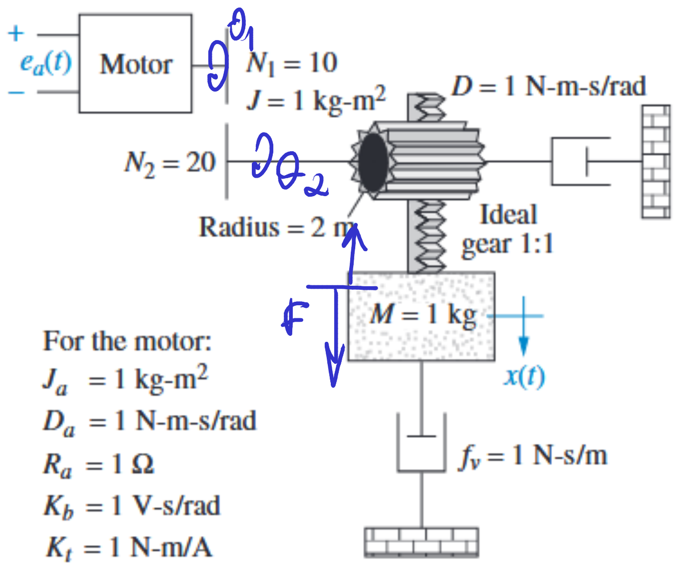


FIGURE P2.33

$$R_{21} = \left(\frac{N_2}{N_1}\right)^2 = 4$$

$$x(t) = \theta_2(t)\,r = 2\theta_2(t)$$

$$-f_v\dot{x}(t) + F(t) = M\ddot{x}(t)$$

$$M\ddot{x}(t) + f_v\dot{x}(t) = F(t)$$

$$J_{eq} = (J_a + J_1)R_{21} = 8$$

$$T(t) = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t e_a}{R_a} = -\omega_n + e_a$$

$$T(t) - D\dot{\theta}_2(t) - F(t) = J_{eq}\ddot{\theta}_2(t)$$

$$J_{eq}\ddot{\theta}_2(t) + D\dot{\theta}_2(t) = T(t) - F(t) \rightarrow$$

$$J_{eq}\ddot{\theta}_2(t) + D\dot{\theta}_2(t) = -\left(M\ddot{x}(t) + f_v\dot{x}(t)\right) + T(s)$$

$$J_{eq}\ddot{\theta}_2(t) + D\dot{\theta}_2(t) = -\left(M2\ddot{\theta}_2(t) + f_v2\dot{\theta}_2(t)\right) + T(s)$$

$$J_{eq}\ddot{\theta}_2(t) + D\dot{\theta}_2(t) + M2\ddot{\theta}_2(t) + f_v2\dot{\theta}_2(t) = T(s)$$

$$\theta_2(s)\left(10s^2 + 3s\right) = T(s) \qquad -w_n + e_a$$

$$X(s)\left(\frac{10s^2+3s}{2}\right) = -\frac{X(s)}{4} + E_a(s) \qquad w_n = \frac{N_1}{N_2}\theta_2 = \frac{N_1}{N_2}\frac{x(t)}{r} = \frac{x(t)}{4}$$

$$X(s)\left(\frac{20s^2+6s+1}{4}\right) = E_a(s) \Rightarrow \frac{X(s)}{E_a(s)} = \frac{4}{20s^2+6s+1}$$