1. Derive the Laplace transform for the following time functions: [Section: 2.2]

a. $u(t)$

b. $tu(t)$

c. $\sin \omega t\, u(t)$

d. $\cos \omega t\, u(t)$

$$\mathcal{L}(u(t)) = \frac{1}{s}$$

$$\mathcal{L}(t\,u(t)) = \frac{1}{s^2}$$

$$\mathcal{L}(\sin\omega\, u(t)) = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}(\cos\omega t\, u(t)) = \frac{s}{s^2+\omega^2}$$

2. Using the Laplace transform pairs of Table 2.1 and the Laplace transform theorems of Table 2.2, derive the Laplace transforms for the following time functions: [Section: 2.2]

a. $e^{-at}\sin \omega t\, u(t)$

b. $e^{-at}\cos \omega t\, u(t)$

c. $t^3 u(t)$

$$\mathcal{L}(e^{-at}\sin\omega t\, u(t)) = \frac{\omega}{(s+a)^2+\omega^2}$$

$$\mathcal{L}(t^3 u(t)) = \frac{6}{s^4}$$

$$\mathcal{L}(e^{-at}\cos\omega t\, u(t)) = \frac{(s+a)}{(s+a)^2+\omega^2}$$

3. Repeat Problem 19 in Chapter 1, using Laplace transforms. Assume zero initial conditions. [Sections: 2.2; 2.3]

19. Repeat Problem 18 using the network shown in Figure P1.7. Assume $R=1\Omega, L=0.5\text{H}$, and $1/LC=16$. [Review]

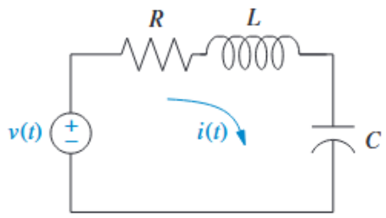

FIGURE P1.7 *RLC* network

$$v(t) = R\cdot i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau$$

$$v(t) = RI(s) + L(sI(s) - \cancel{i(0)}^{0}) + \frac{1}{C}\frac{I(s)}{s}$$

$$\frac{1}{s} = I(s)\cdot\left(R + Ls + \frac{1}{Cs}\right) \qquad I(s) = \frac{1}{Rs + Ls^2 + 1/C}$$

$$\frac{s^2}{2} + s + 8 = \frac{1}{2}\left(s^2 + 2s + 16\right) \Rightarrow \frac{2}{s^2 + 2s + 16} = \frac{As + B}{s^2 + 2s + 16}$$

$$K_1 = 0, \quad 2as = 2s \Rightarrow a = 1 \qquad a^2 + \omega^2 = 16$$

$$\cancel{K_1}^{0} a + K_2\omega = 2 \qquad \omega = \sqrt{16-1} = \sqrt{15}$$

$$K_2 = \frac{2}{\sqrt{15}} \qquad i(t) = \frac{2e^{-t}}{\sqrt{15}} \operatorname{sen} \sqrt{15}t$$

4. Repeat Problem 20 in Chapter 1, using Laplace transforms. Assume that the forcing functions are zero prior to $t = 0-$. [Section: 2.2]

a. $\frac{dx}{dt} + 7x = 5\cos 2t$

$$sX(s) - \cancel{x(0)}^{\,0} + 7X(s) = \frac{5s}{s^2+4}$$

$$X(s)\cdot(s+7) = \frac{5s}{s^2+4}$$

$$X(s) = \frac{5s}{(s^2+4)(s+7)} = \frac{A}{(s+7)} + \frac{Bs+C}{(s^2+4)}$$

$$\cancel{(s^2+4)(s+7)}\frac{5s}{\cancel{(s^2+4)(s+7)}} = (s^2+4)\cancel{(s+7)}\frac{A}{\cancel{(s+7)}} + \cancel{(s^2+4)}(s+7)\frac{Bs+C}{\cancel{(s^2+4)}}$$

$$5s = (s^2+4)A + (s+7)(Bs+C)$$

$$As^2 + 4A + Bs^2 + 7Bs + Cs + 7C - 5s = 0$$

$$s^2(A+B) + s(7B+C-5) + 4A + 7C = 0$$

$$\begin{cases} A+B=0 \\ 7B+C=5 \\ 4A+7C=0 \end{cases} \qquad A = -\frac{35}{53},\ B = \frac{35}{53},\ C = \frac{20}{53}$$

$$\frac{5s}{(s^2+4(s+7)} = -\frac{35}{53(s+7)} + \frac{35s+20}{53(s^2+4)}$$

$$\mathcal{L}^{-1}\left\{\frac{5s}{((s^2+4(s+7))}\right\} = \mathcal{L}^{-1}\left\{-\frac{35}{53(s+7)}\right\} + \mathcal{L}^{-1}\left\{\frac{35s+20}{53(s^2+4)}\right\}$$

$$\mathcal{L}^{-1}\left\{-\frac{35}{53(s+7)}\right\} = -\frac{35}{53}e^{-7t}$$

$$\mathcal{L}^{-1}\left\{\frac{35s+20}{53(s^2+4)}\right\} = \frac{35}{53}\mathcal{L}^{-1}\left\{\frac{s+20}{(s^2+4)}\right\} \Rightarrow K_1 = \frac{35}{53} \qquad \cancel{a}^{\,0} + w^2 = 4$$

$$w = \sqrt{4} = 2$$

$$K_1\cancel{a}^{\,0} + K_2\cdot w = \frac{20}{53} \qquad K_2 = \frac{20}{53\cdot 2} = \frac{20}{106} = \frac{10}{53}$$

$$\cancel{e^{-at}}^{\,1} \cdot -\frac{35}{53}\cos 2t + \frac{10}{53}\operatorname{sen} 2t$$

$$\mathcal{L}^{-1}\left\{\frac{5s}{(s^2+4(s+7))}\right\} = -\frac{35}{53}\left(e^{-7t} - \cos 2t\right) + \frac{10}{53}\operatorname{sen} 2t$$

b. $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 5\sin 3t$ $= s^2X(s) - s\cancel{x(0)}^{\,0} - \cancel{x'(0)}^{\,0} + 6(sX(s) - \cancel{x(0)}^{\,0}) + 8X(s) = \frac{15}{s^2+9}$

$$s^2X(s) + 6sX(s) + 8X(s) = \frac{15}{s^2+9} \qquad X(s)(s^2+6s+8) = \frac{15}{s^2+9}$$

$$X(s) = \frac{15}{(s^2+9)(s^2+6s+8)} = \frac{15}{(s^2+9)(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4} + \frac{Cs+D}{s^2+9}$$

$$\frac{\cancel{(s^2+9)}\cancel{(s+2)}\cancel{(s+4)}15}{\cancel{(s^2+9)}\cancel{(s+2)}\cancel{(s+4)}} = (s^2+9)\cancel{(s+2)}(s+4)\frac{A}{\cancel{s+2}} + (s^2+9)(s+2)\cancel{(s+4)}\frac{B}{\cancel{s+4}} + \cancel{(s^2+9)}(s+2)(s+4)\frac{Cs+D}{\cancel{s^2+9}}$$

$$15 = (s^2+9)(s+4)A + (s^2+9)(s+2)B + (s+2)(s+4)(Cs+D)$$

$$15 = (s^2+9)(As+4A) + (s^2+9)(Bs+2B) + (s+2)(Cs^2+4Cs+Ds+4D)$$

$$15 = As^3 + 9As + 4As^2 + 36A + Bs^3 + 9Bs + 2Bs^2 + 18B + Cs^3 + 2Cs^2 + 4Cs^2 + 8Cs + Ds^2 + 2Ds$$
$$+ 4Ds + 8D$$

$$15 = As^3 + 9As + 4As^2 + 36A + Bs^3 + 9Bs + 2Bs^2 + 18B + Cs^3 + 2Cs^2 + 4Cs^2 + 8Cs + Ds^2 + 2Ds$$
$$+ 4Ds + 8D$$

$$15 = s^3(A+B+C) + s^2(4A+2B+6C+D) + s(9A+9B+8C+6D) + 36A+18B+8D$$

$$\begin{cases} A+B+C=0 \\ 4A+2B+6C+D=0 \\ 9A+9B+8C+6D=0 \\ 36A+18B+8D=15 \end{cases} \qquad A=\frac{15}{26},\ B=-\frac{3}{10},\ C=-\frac{18}{65},\ D=-\frac{3}{65}$$

$$X(s) = \frac{15}{26(s+2)} + \frac{-3}{10(s+4)} - \frac{18s+3}{65(s^2+9)}$$

$$\mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{15}{26(s+2)} + \frac{-3}{10(s+4)} - \frac{18s+3}{65(s^2+9)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{15}{26(s+2)}\right\} = \frac{15}{26}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = \frac{15}{26}e^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{-3}{10(s+4)}\right\} = \frac{-3}{10}\mathcal{L}^{-1}\left\{\frac{1}{(s+4)}\right\} = -\frac{3}{10}e^{-4t}$$

$$\mathcal{L}^{-1}\left\{-\left(\frac{18s+3}{65(s^2+9)}\right)\right\} = -\mathcal{L}^{-1}\left\{\frac{18s+3}{65(s^2+9)}\right\} \qquad K_1 = \frac{18}{65} \qquad 2as = 0 \Rightarrow a = 0$$

$$\cancel{a}^{0} + \omega^2 = 9 \qquad \omega = \sqrt{9} = 3$$

$$\cancel{K_1 a}^{0} + K_2\omega = \frac{3}{65} \qquad K_2 = \frac{\cancel{3}}{65\cdot\cancel{3}} = \frac{1}{65}$$

$$-\left(\cancel{e^{0}}^{1} \cdot \frac{18}{65} \cdot \cos 3t + \frac{\operatorname{sen} 3t}{65}\right)$$

$$\mathcal{L}^{-1}\{X(s)\} = \frac{15}{26}e^{-2t} - 3\cdot\left(\frac{e^{-4t}}{10} + \frac{6\cos 3t}{65} + \frac{\operatorname{sen} 3t}{195}\right)$$

c. $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x = 10u(t)$ $= s^2X(s) - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{0} + 8(sX(s) - \cancel{x(0)}^{0}) + 25X(s) = \frac{10}{s}$

$X(s)(s^2+8s+25) = \frac{10}{s}$ $\qquad X(s) = \frac{10}{s(s^2+8s+25)} = \frac{A}{s} + \frac{Bs+C}{(s^2+8s+25)}$

$\cancel{\frac{s(s^2+8s+25)10}{s(s^2+8s+25)}} = \cancel{s}(s^2+8s+25)\frac{A}{\cancel{s}} + \frac{s\cancel{(s^2+8s+25)}Bs+C}{\cancel{(s^2+8s+25)}}$

$10 = (s^2+8s+25)A + s(Bs+C)$

$As^2 + 8As + 25A + Bs^2 + Cs = 10$
$s^2(A+B) + s(8A+C) + 25A = 10$

$\begin{cases} A+B=0 \\ 8A+C=0 \\ 25A=10 \end{cases}$ $\qquad A = \frac{2}{5},\ B = -\frac{2}{5},\ C = -\frac{16}{5}$

$X(s) = \frac{2}{5s} - \frac{2s+16}{5(s^2+8s+25)} \Rightarrow \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{5s} - \frac{2s+16}{5(s^2+8s+25)}\right\}$

$\mathcal{L}^{-1}\left\{\frac{2}{5s}\right\} = \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = \frac{2}{5}u(t)$

$\mathcal{L}^{-1}\left\{-\frac{2s+16}{5(s^2+8s+25)}\right\} = -\mathcal{L}^{-1}\left\{\frac{2s+16}{5(s^2+8s+25)}\right\} = $ $K_1 = \frac{2}{5}$, $\quad 2as = 8s \Rightarrow a = 4$

$a^2 + w^2 = 25$

$K_1 \cdot a + K_2 w = \frac{16}{5}$ $\qquad K_2 = \frac{16}{15} - \frac{8}{5} = -\frac{8}{15}$ $\qquad w = \sqrt{25-16} = 3$

$e^{-4t}\left(\frac{2}{5}\cos 3t - \frac{8}{15}\operatorname{sen} 3t\right)$

$$\mathcal{L}^{-1}\{X(s)\} = \frac{2}{5}\left(-e^{-4t}\left(\cos 3t - \frac{4}{3}\operatorname{sen} 3t\right) + u(t)\right)$$

5. Repeat Problem 21 in Chapter 1, using Laplace transforms. Use the following initial conditions for each part as follows: (a) $x(0) = 4$, $x'(0) = -4$; (b) $x(0)=4$, $x'(0)=1$; (c) $x(0)=2, x'(0)=3$, where $x'(0)=\frac{dx}{dt}(0)$.

Assume that the forcing functions are zero prior to $t=0-$. [Section: 2.2]

a. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = \sin 2t$

$$s^2X(s) - s\cancel{x(0)}^{4} - s\cancel{x'(0)}^{-4} + 2(sX(s) - \cancel{x(0)}^{4}) + 2X(s) = \frac{2}{s^2+4}$$

$$X(s)(s^2+2s+2) - 8 = \frac{2}{s^2+4}$$

$$X(s) = \frac{2}{(s^2+4)(s^2+2s+2)} + \frac{8}{(s^2+2s+2)}$$

$$\frac{2}{(s^2+4)(s^2+2s+2)} = \frac{As+B}{(s^2+4)} + \frac{Cs+D}{(s^2+2s+2)}$$

$$\frac{\cancel{(s^2+4)}\cancel{(s^2+2s+2)}2}{\cancel{(s^2+4)}\cancel{(s^2+2s+2)}} = \frac{\cancel{(s^2+4)}(s^2+2s+2)As+B}{\cancel{(s^2+4)}} + \frac{(s^2+4)\cancel{(s^2+2s+2)}(Cs+D)}{\cancel{(s^2+2s+2)}}$$

$$2 = (s^2+2s+2)(As+B) + (s^2+4)(Cs+D)$$

$$2 = As^3 + 2As^2 + 2As + Bs^2 + 2Bs + 2B + Cs^3 + 4Cs + Ds^2 + 4D$$

$$s^3(A+C) + s^2(2A+B+D) + s(2A+2B+4C) + 2B + 4D$$

$$\begin{cases} A+C=0 \\ 2A+B+D=0 \\ 2A+2B+4C=0 \\ 2B+4D=2 \end{cases}$$

$$A = -\frac{1}{5},\quad B = -\frac{1}{5},\quad C = \frac{1}{5},\quad D = \frac{3}{5}$$

$$\frac{-s-1}{5(s^2+4)} + \frac{s+3}{5(s^2+2s+2)}$$

$$\mathcal{L}^{-1}\left\{\frac{-s-1}{5(s^2+4)}\right\}+\mathcal{L}^{-1}\left\{\frac{s+3}{5(s^2+2s+2)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{-s-1}{5(s^2+4)}\right\} \Rightarrow K_1=\frac{-1}{5}\,, \quad 2as=0 \Rightarrow a=0$$

$$\cancel{a^2}^{0}+w^2=4 \qquad w=\sqrt{4}=2$$

$$\cancel{K_1}^{0}\cdot a+K_2 w=-\frac{1}{5} \qquad K_2=\frac{-1}{10}$$

$$\cancel{e^{-at}}^{1}\cdot -\frac{(2\cos 2t+\operatorname{sen} 2t)}{10}$$

$$\mathcal{L}^{-1}\left\{\frac{s+3}{5(s^2+2s+2)}\right\} \Rightarrow K_1=\frac{1}{5}\,, \quad 2as=2s \Rightarrow a=1$$

$$a^2+w^2=2 \qquad w=\sqrt{2-1}=1$$

$$K_1\cdot a+K_2 w=\frac{3}{5} \qquad K_2=\frac{3}{5}-\frac{1}{5}=\frac{2}{5}$$

$$e^{-t}\left(\frac{\cos t+2\operatorname{sen} t}{5}\right)$$

$$\frac{8}{(s^2+2s+2)} = K_1=0\,, \quad 2as=2s \Rightarrow a=1 \qquad a^2+w^2=2$$

$$w=\sqrt{2-1}=1$$

$$\cancel{K_1}^{0}\cdot a+K_2 w=8$$

$$K_2=8$$

$$e^{-t}\,8\operatorname{sen} t$$

$$\mathcal{L}^{-1}\{X(s)\}= e^{-t}\left(\frac{\cos t+42\operatorname{sen} t}{5}\right)-\frac{(2\cos 2t+\operatorname{sen} 2t)}{10}$$

b. $\dfrac{d^2x}{dt^2}+2\dfrac{dx}{dt}+x=5e^{-2t}+t$

$$s^2X(s)-s\cancel{x(0)}^{4}-\cancel{x'(0)}^{1}+2\left(sX(s)-\cancel{x(0)}^{4}\right)+X(s)=\frac{5}{s+2}+\frac{1}{s^2}$$

$$X(s)\left(s^2+2s+1\right)=\frac{5}{s+2}+\frac{1}{s^2}+4s+9$$

$$\frac{5}{(s^2+2s+1)(s+2)}+\frac{1}{(s^2+2s+1)s^2}+\frac{4s+9}{(s^2+2s+1)}$$

$$\mathcal{L}^{-1}\{X(s)\}=\mathcal{L}^{-1}\left\{\frac{5}{(s^2+2s+1)(s+2)}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{(s^2+2s+1)s^2}\right\}+\mathcal{L}^{-1}\left\{\frac{4s+9}{(s^2+2s+1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{5}{(s+2)(s+1)^2}\right\}=\frac{A}{(s+2)}+\frac{B}{(s+1)}+\frac{C}{(s+1)^2}$$

$$\frac{5}{(s+2)(s+1)^2}=\frac{A}{(s+2)}+\frac{B}{(s+1)}+\frac{C}{(s+1)^2}$$

$$5=(s+1)(s+1)A+(s+2)(s+1)B+(s+2)C$$
$$5=(s^2+2s+1)A+(s^2+3s+2)B+(s+2)C$$
$$5=As^2+2As+A+Bs^2+3Bs+2B+Cs+2C$$
$$s^2(A+B)+s(2A+3B+C)+A+2B+2C=5$$

$$\begin{cases}A+B=0\\2A+3B+C=0\\A+2B+2C=5\end{cases}\qquad \begin{aligned}A&=5\\B&=-5\\C&=5\end{aligned}$$

$$\frac{5}{(s+2)}-\frac{5}{(s+1)}+\frac{5}{(s+1)^2}\Rightarrow\mathcal{L}^{-1}\left\{\frac{5}{(s+2)}-\frac{5}{(s+1)}+\frac{5}{(s+1)^2}\right\}$$

$$=5\mathcal{L}^{-1}\left\{\frac{1}{(s+2)}\right\}-5\mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\}+5\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}\qquad=5e^{-2t}-5e^{-t}+5te^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+2s+1)s^2}\right\} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{s} + \frac{D}{s^2}$$

$$1 = (s+1)s^2A + Bs^2 + (s+1)(s+1)Cs + (s+1)(s+1)D$$
$$1 = (s+1)s^2A + Bs^2 + (s^2+2s+1)Cs + (s^2+2s+1)D$$
$$1 = As^3 + As^2 + Bs^2 + Cs^3 + 2Cs^2 + Cs + Ds^2 + 2Ds + D$$
$$1 = s^3(A+C) + s^2(A+B+2C+D) + s(C+2D) + D$$

$$\begin{cases} A+C=0 \\ A+B+2C+D=0 \\ C+2D=0 \\ D=1 \end{cases} \qquad \begin{matrix} A=2 \\ B=1 \\ C=-2 \\ D=1 \end{matrix} \qquad \frac{2}{(s+1)} + \frac{1}{(s+1)^2} + \frac{-2}{s} + \frac{1}{s^2}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{(s+1)} + \frac{-1}{(s+1)^2} + \frac{-2}{s} + \frac{1}{s^2}\right\}$$

$$= 2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 2e^{-t} + te^{-t} - 2 + t$$
$$= e^{-t}(2+t) - 2 + t$$

$$\frac{4s+9}{(s^2+2s+1)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} \qquad 4s+9 = (s+1)A + B$$
$$4s+9 = As + A + B = 4s+9 = sA + A + B$$

$$A = 4$$
$$B+A=9 \Rightarrow B=5 \qquad \mathcal{L}^{-1}\left\{\frac{4}{(s+1)} + \frac{5}{(s+1)^2}\right\} = 4\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 5\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} =$$

$$4e^{-t} + 5te^{-t}$$

$$\mathcal{L}^{-1}\{X(s)\} = e^{-t}(1+11t) + 5e^{-2t} - 2 + t$$

8. A system is described by the following differential equation:

$$\frac{d^3y}{dt^3}+3\frac{d^2y}{dt^2}+5\frac{dy}{dt}+y=\frac{d^3x}{dt^3}+4\frac{d^2x}{dt^2}+6\frac{dx}{dt}+8x$$

Find the expression for the transfer function of the system, $Y(s)/X(s)$. [Section: 2.3]

$$\mathcal{L}\left\{\frac{d^3y}{dt^3}\right\}=s^3Y(s)-\left(s^2y(0)+sy'(0)+y''(0)\right) \qquad \mathcal{L}\left\{\frac{5dy}{dt}\right\}=5\left(sY(s)-y(0)\right)$$

$$\mathcal{L}\left\{\frac{3d^2y}{dt^2}\right\}=3\left(s^2Y(s)-sy(0)-y'(0)\right) \qquad \mathcal{L}\{y\}=Y(s)$$

$$Y(s)(s^3+3s^2+5s+1)$$

$$\mathcal{L}\left\{\frac{d^3x}{dt^3}\right\}=s^3X(s)-\left(s^2x(0)+sx'(0)+x''(0)\right) \qquad \mathcal{L}\left\{\frac{6dx}{dt}\right\}=6\left(sX(s)-X(0)\right)$$

$$\mathcal{L}\left\{\frac{4d^2x}{dt^2}\right\}=3\left(s^2X(s)-sx(0)-x'(0)\right) \qquad \mathcal{L}\{8x\}=8X(s)$$

$$X(s)(s^3+3s^2+6s+8)$$

$$Y(s)(s^3+3s^2+5s+1)=X(s)(s^3+3s^2+6s+8)$$

$$\frac{Y(s)(s^3+3s^2+5s+1)}{X(s)}=(s^3+3s^2+6s+8)$$

$$\frac{Y(s)}{X(s)}=\frac{(s^3+3s^2+6s+8)}{(s^3+3s^2+5s+1)}$$

9. For each of the following transfer functions, write the corresponding differential equation. [Section: 2.3]

a. $\dfrac{X(s)}{F(s)} = \dfrac{7}{s^2 + 5s + 10}$

$X(s)(s^2+5s+10) = 7F(s)$

$s^2X(s) + 5sX(s) + 10X(s) = 7F(s)$

$s^2X(s) - sx(0) - sx'(0) + 5(sX(s) - x(0)) + 10X(s) = 7F(s)$

$\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 10x = 7f$

b. $\dfrac{X(s)}{F(s)} = \dfrac{15}{(s + 10)(s + 11)}$

$X(s)(s+10)(s+11) = 15F(s)$

$X(s)(s^2+21s+110) = 15F(s)$

$s^2X(s) + 21sX(s) + 110X(s) = 15F(s)$

$\dfrac{d^2x}{dt^2} + 21\dfrac{dx}{dt} + 110x = 15f$

c. $\dfrac{X(s)}{F(s)} = \dfrac{s + 3}{s^3 + 11s^2 + 12s + 18}$

$X(s)(s^3+11s^2+12s+18) = (s+3)F(s)$

$s^3X(s) + 11s^2X(s) + 12sX(s) + 18X(s) = sF(s) + 3F(s)$

$\dfrac{d^3x}{dt^3} + 11\dfrac{d^2x}{dt^2} + 12\dfrac{dx}{dt} + 18x = \dfrac{df}{dt} + 3f$

10. Write the differential equation for the system shown in Figure P2.1. [Section: 2.3]

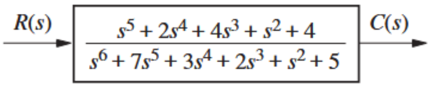

FIGURE P2.1

$$\frac{C(s)}{R(s)} = \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+3s^3+s^2+5}$$

$$(s^6+7s^5+3s^4+3s^3+s^2+5)C(s) = (s^5+2s^4+4s^3+s^2+4)R(s)$$

$$s^6C(s)+7s^5C(s)+3s^4C(s)+3s^3C(s)+s^2C(s)+5C(s) = s^5R(s)+2s^4R(s)+4s^3R(s)+s^2R(s)+4R(s)$$

$$\frac{d^6c}{dt^6}+\frac{7d^5c}{dt}+\frac{3d^4c}{dt}+\frac{3d^3c}{dt}+\frac{d^2c}{dt}+5c = \frac{d^5R}{dt}+\frac{2d^4R}{dt}+\frac{4d^3R}{dt}+\frac{d^2R}{dt}+4R$$

11. Write the differential equation that is mathematically equivalent to the block diagram shown in Figure P2.2. Assume that $r(t) = 3t^3$. [Section: 2.3]

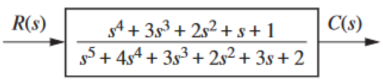

FIGURE P2.2

$$\frac{C(s)}{R(s)} = \frac{s^4+3s^3+2s^2+s+1}{s^5+4s^4+3s^3+2s^2+3s+2}$$

$$(s^4+3s^3+2s^2+s+1)C(s) = (s^5+4s^4+3s^3+2s^2+3s+2)R(s) \qquad 3t^3$$

$$s^4C(s)+3s^3C(s)+2s^2C(s)+sC(s)+C(s) = s^5R(s)+4s^4R(s)+3s^3R(s)+2s^2R(s)+3sR(s)+2R(s)$$

$$\frac{d^4c}{dt^4}+3\frac{d^3c}{dt^3}+2\frac{d^2c}{dt^2}+\frac{dc}{dt}+c = \frac{d^5R}{dt^5}+\frac{4d^4R}{dt^4}+\frac{3d^3R}{dt^3}+\frac{2d^2R}{dt^2}+\frac{3dR}{dt}+2R$$

$$\frac{d^4c}{dt^4}+3\frac{d^3c}{dt}+2\frac{d^2c}{dt^2}+\frac{dc}{dt}+c = 6t^3+9t^2+36t+54$$

12. A system is described by the following differential equation: [Section 2.3]

$$\frac{d^2x}{dt^2}+4\frac{dx}{dt}+5x=1$$

with the initial conditions $x(0)=1,\ \dot{x}(0)=-1$. Show a block diagram of the system, giving its transfer function and all pertinent inputs and outputs. (Hint: the initial conditions will show up as added inputs to an effective system with zero initial conditions.)

$$s^2X(s)-s\overset{1}{\cancel{x(0)}}-\overset{-1}{\cancel{x'(0)}}+4\left(sX(s)-\overset{1}{\cancel{x(0)}}\right)+5X(s)= \qquad X(s)\rightarrow\boxed{\frac{(s+1)}{s^2+4s+5}}\rightarrow Y(s)$$

$$X(s)(s^2+4s+5)=(s+1)$$