17. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each network shown in Figure P2.3. [Section: 2.4]

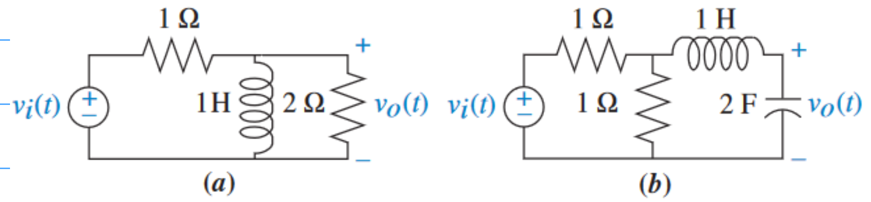


FIGURE P2.3

a) $V(s) = RI(s) + \overbrace{Z_{R_2/\!/L}\cdot I(s)}^{v_0}$ $\quad V(s) = I(s)\cdot\left(R_1 + \frac{R_2\cdot Ls}{R_2+Ls}\right) =$

$Z_{R/\!/L} = \frac{R_2\cdot Ls}{R_2+Ls}$ $\quad \left(\frac{R_1R_2+R_1Ls+R_2Ls}{R_2+Ls}\right)\cdot I(s) = V(s)$

$v_o(t) \Leftrightarrow V_L(s) = \frac{R_2\cdot Ls}{R_2+Ls}\cdot I(s)$ $\quad I(s) = \frac{R_2+Ls}{R_1R_2+R_1Ls+R_2Ls}\, V(s)$

$V_L(s) = \frac{R_2\cdot Ls}{R_2+Ls}\cdot\frac{R_2+Ls}{R_1R_2+R_1Ls+R_2Ls}\, V(s)$

$$\frac{V_L(s)}{V(s)} = \frac{R_2\cdot Ls}{R_2+Ls}\cdot\frac{R_2+Ls}{R_1R_2+R_1Ls+R_2Ls} = \frac{2s}{2+s}\cdot\frac{2+s}{2+s+2s} = \frac{2s}{2+3s}$$

b) $V(s) = R_1(I) + (Z_{L+C}/\!/Z_{R_2})\, I(s)$ $\quad \boxed{Z_{L+C} = Ls - \frac{1}{Cs}}$ $\quad \boxed{Z_{R_2} = R_2}$

$$Z_{L+C}/\!/Z_{R_2} = \frac{R_2\cdot\left(Ls-\frac{1}{Cs}\right)}{R_2+\left(Ls-\frac{1}{Cs}\right)} = \frac{\left(RLs-\frac{R}{Cs}\right)}{R_2+Ls-\frac{1}{Cs}}$$

$\frac{RLCs^2-R}{Cs}$ $\quad \frac{R_2Cs+LCs^2-1}{Cs}$

$$\frac{\frac{RLCs^2-R}{Cs}}{\frac{R_2Cs+LCs^2-1}{Cs}} = \frac{RLCs^2-R}{\cancel{Cs}} \; \frac{\cancel{Cs}}{R_2Cs+LCs^2-1} = \frac{R_2LCs^2-R_2}{R_2Cs+LCs^2-1}$$

$$V(s) = \left(R_1 + \frac{R_2LCs^2-R_2}{R_2Cs+LCs^2-1}\right) I(s) = \frac{R_1R_2Cs+R_1LCs^2+R_1+R_2LCs^2-R_2}{R_2Cs+LCs^2-1} \cdot I(s)$$

$$I(s) = \frac{R_2Cs+LCs^2-1}{R_1R_2Cs+R_1LCs^2+R \;+R_2LCs^2-R_2} V(s)$$

$$I_C = \left(\frac{R_2}{R_2+\left(Ls-\frac{1}{Cs}\right)}\right) \cdot I(s) \qquad \frac{R_2}{\frac{R_2Cs+LCs^2-1}{Cs}} = \frac{R_2Cs}{R_2Cs+LCs^2-1} \cdot I$$

$$V_C = \left(\frac{R_2Cs}{R_2Cs+LCs^2-1} \cdot I\right) \cdot -\frac{1}{Cs}$$

$$V_C = \left(\frac{R_2Cs}{R_2Cs+LCs^2-1}\right) \frac{R_2Cs+LCs^2+1}{R_1R_2Cs+R_1LCs^2+R \;+R_2LCs^2-R_2} V(s) \cdot -\frac{1}{Cs}$$

$$\frac{V_C(s)}{V(s)} = \left(\frac{R_2\cancel{Cs}}{\cancel{R_2Cs+LCs^2-1}}\right) \frac{\cancel{R_2Cs+LCs^2-1}}{R_1R_2Cs+R_1LCs^2+R \;+R_2LCs^2-R_2} \; \frac{-1}{\cancel{Cs}}$$

$$\frac{V_C(s)}{V(s)} = \frac{-R_2}{R_1R_2Cs+R_1LCs^2+R \;+R_2LCs^2-R_2} = -\frac{1}{2s+2s^2+\cancel{1}+2s^2-\cancel{1}}$$

$$\frac{V_C(s)}{V(s)} = -\frac{1}{2s+4s^2}$$

18. Find the transfer function, $G(s) = V_L(s)/V(s)$, for each network shown in Figure P2.4. [Section: 2.4]

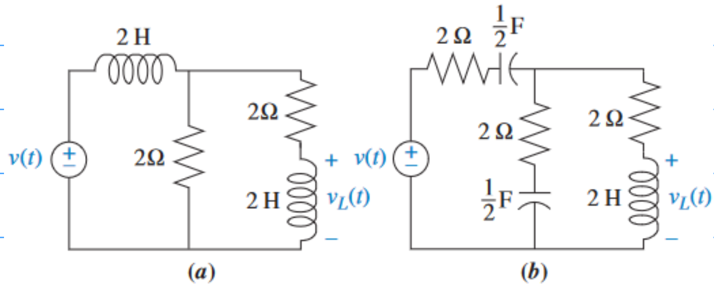


FIGURE P2.4

a) $V(s) = Ls \cdot I(s) + Z_{(R1//(R2+L_2))} \cdot I(s)$

$$Z_{R2+L_2} = R_2 + L_2 s \qquad Z_{(R1//(R2+L_2))} = \frac{R_1 \cdot (R_2 + L_2 s)}{R_1 + R_2 + L_2 s}$$

$$V(s) = I(s)\ \frac{R_1 \cdot (R_2 + L_2 s)}{R_1 + R_2 + L_2 s} + Ls$$

$$Z_{eq} = \frac{R_1 \cdot (R_2 + L_2 s)}{R_1 + R_2 + L_2 s} + Ls = \frac{R_1 R_2 + R_1 L_2 s}{R_1 + R_2 + L_2 s} + Ls$$

$$\frac{R_1 R_2 + R_1 L_2 s}{R_1 + R_2 + L_2 s} + Ls = \frac{R_1 \cdot (R_2 + L_2 s) + Ls(R_1 + R_2 + L_2 s)}{R_1 + R_2 + L_2 s} = Z_{eq}$$

$$V(s) = \left(\frac{R_1 \cdot (R_2 + L_2 s) + Ls(R_1 + R_2 + L_2 s)}{R_1 + R_2 + L_2 s} \right) \cdot I(s)$$

$$I(s) = \left(\frac{R_1 + R_2 + L_2 s}{R_1 \cdot (R_2 + L_2 s) + Ls(R_1 + R_2 + L_2 s)} \right) V(s)$$

$$I_L = \frac{R_1}{R_1 + R_2 + L_{2}s} \cdot I(s) = \frac{R_1}{R_1 + R_2 + L_{2}s} \cdot \left(\frac{R_1 + R_2 + L_2 s}{R_1 \cdot (R_2 + L_2 s) + Ls(R_1 + R_2 + L_2 s)} \right) V(s)$$

$$V_L = I_L \cdot Z_L = \frac{R_1}{R_1+R_2+L_2s} \cdot \left(\frac{R_1+R_2+L_2s}{R_1\cdot(R_2+L_2s)+Ls(R_1+R_2+L_2s)}\right) V(s) \cdot L_2s$$

$$\frac{V_L(s)}{V(s)} = \frac{R_1}{R_1+\cancel{R_2}+L_2s} \cdot \left(\frac{R_1+\cancel{R_2}+L_2s}{R_1\cdot(R_2+L_2s)+Ls(R_1+R_2+L_2s)}\right) L_2s$$

$$\frac{V_L(s)}{V(s)} = \frac{4s}{4+4s+4s+4s+4s^2} = \frac{4s}{4+12s+4s^2} = \frac{s}{s^2+3s+1}$$

b) $V(s) = I(s)\left(Z_{R_1}+Z_C+Z_{(R_2+C_2)//+R_3L}\right)$

$$Z_{R_2+C_2} = R+\frac{1}{Cs} = \frac{RCs+1}{Cs} \qquad Z_{R_3+L} = R_3+Ls$$

$$Z_{(R_2+C_2)//+R_3L} = \frac{\frac{RCs+1}{Cs}\cdot(R_3+Ls)}{\frac{RCs+1}{Cs}+R_3+Ls} =$$

$$\frac{\cancel{Cs}}{RCs+1+R_3Cs+LCs^2} \cdot \frac{RCs+1}{\cancel{Cs}} \cdot (R_3+Ls) = \frac{RCs+1\cdot(R_3+Ls)}{RCs+1+R_3Cs+LCs^2}$$

$$V(s) = I(s)\left(\frac{(R_2C_2s+1)\cdot(R_3+Ls)}{R_2C_2s+1+R_3Cs+LC_2s^2} + R_1 + \frac{1}{C_1s}\right)$$

$$I(s) = \left(\frac{R_2C_2s+1+R_3Cs+LC_2s^2}{(R_2C_2s+1)(R_3+Ls)} + \frac{1}{R_1} + C_1s\right) \cdot V(s)$$

$$V_L(s) = Z_L \cdot I_L$$

$$I_L(s) = \frac{R_2 + \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s} + R_3 + Ls} \cdot I(s) = \frac{R_2 C_2 s + 1}{\cancel{C_2 s}} \cdot \frac{\cancel{C_2 s}}{C_2 s \cdot (R_2 + R_3 + Ls) + 1} \cdot I(s)$$

$$V_L(s) = Ls \cdot \left(\frac{R_2 C_2 s + 1}{C_2 s \cdot (R_2 + R_3 + Ls) + 1} \right) \cdot \left(\frac{R_2 C_2 s + 1 + R_3 C s + L C_2 s^2}{(R_2 C_2 s + 1)(R_3 + Ls)} + \frac{1}{R_1} + C_1 s \right) \cdot V(s)$$

$$\frac{V_L(s)}{V(s)} = Ls \cdot \left(\frac{R_2 C_2 \cancel{s} + 1}{C_2 s \cdot (R \cancel{+R} + Ls) + 1} \right) \cdot \left(\frac{R_2 C_2 \cancel{s+1} + R_3 C s + L C_2 s^2}{(R_2 \cancel{C_2} s + 1)(R_3 + Ls)} + \frac{1}{R_1} + C_1 s \right)$$

$$Ls \cdot \left(\frac{1}{R_3 + Ls} + \left(\frac{R_2 C_2 s + 1}{C_2 s \cdot (R + R_3 + Ls) + 1} \right) \right) \cdot \left(\frac{1}{R_1} + C_1 s \right)$$

$$\frac{V_L(s)}{V(s)} = 2s \left(\frac{1}{2 + 2s} + \frac{s+1}{6s+2} + \frac{s^2 + s}{2} \right) = \frac{4 + s + 1 + 3s^3 + 3s^2 + s^2 + s}{6s + 2}$$

$$\frac{V_L(s)}{V(s)} = 2s \cdot \left(\frac{3s^3 + 4s^2 + 2s + 5}{6s + 2} \right) = \frac{6s^4 + 8s^2 + 4s^2 + 10s}{6s + 2}$$

19. Find the transfer function, $G(s) = V_o(s)/\,V_i(s)$, for each network shown in Figure P2.5. Solve the problem using mesh analysis. [Section: 2.4]

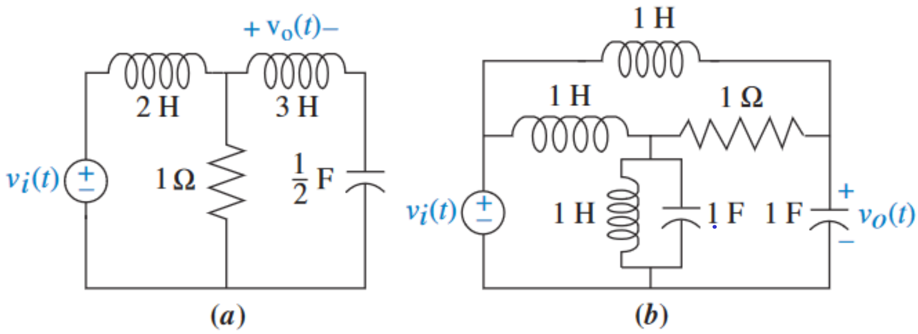


FIGURE P2.5

$$\begin{bmatrix} \sum V_1(s) \\ \sum V_2(s) \end{bmatrix} = \begin{bmatrix} \sum Z_1(s) & -\sum Z_{21}(s) \\ -\sum Z_{12}(s) & \sum Z_2(s) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

a) $-V(s)+L_1s+(I_1-I_2)R=0$

$V(s)=L_1s\cdot I_1+(I_1-I_2)R$

$V_{L_2}=L_2s\cdot I_2$

$I_2=\dfrac{V_{L2}}{L_2s}$

$-(I_1-I_2)R+L_2s\cdot I_2+\dfrac{1}{Cs}\cdot I_2=0$

$-(I_1-I_2)R+V_{L2}+\dfrac{1}{Cs}\cdot I_2=0$

$V_{L2}=R\cdot(I_1-I_2)-\dfrac{I_2}{Cs}$

$$V_{L2} = R\cdot(I_1 - I_2) - \frac{I_2}{Cs}$$

$$V_{L2} = R.\left(I_1 - \frac{V_{L2}}{L_2 s}\right) - \frac{V_{L2}}{L_2 s C s}$$

$$R\cdot I_1 = V_{L2} + \frac{V_{L2}}{L_2 s} - \frac{V_{L2}}{L_2 s C s}$$

$$I_1 = \frac{V_{L2}}{R} + \frac{V_{L2}}{R L_2 s} - \frac{V_{L2}}{R L_2 s C s}$$

$$V(s) = L_1 s \cdot I_1 + (I_1 - I_2) R$$

$$V(s) = (L_1 s + R)\left(\frac{V_{L2}}{R} + \frac{V_{L2}}{R L_2 s} - \frac{V_{L2}}{R L_2 s C s}\right) - R.\frac{V_{L2}}{L_2 s}$$

$$V(s) = (L_1 s + R)\left(\frac{V_{L2}}{R} + \frac{V_{L2}}{R L_2 s} - \frac{V_{L2}}{R L_2 s C s}\right) - R.\frac{V_{L2}}{L_2 s}$$

$$V(s) = V_{L2}\left((L_1 s + R)\left(\frac{1}{R} + \frac{1}{R L_2 s} - \frac{1}{R L_2 s C s}\right) - \frac{R}{L_2 s}\right)$$

$$V(s) = V_{L2}\left((2s+2)\left(1 + \frac{1}{3s} - \frac{2}{3s^2}\right) - \frac{1}{3s}\right)$$

$$V(s) = V_{L2}\left((2s+2) + \frac{(2s+2)}{3s} - \frac{4s+4}{3s^2} - \frac{1}{3s}\right)$$

$$V_{(s)} = V_{L_2}\left((2s+2) + \frac{(2s+2)}{3s} - \frac{4s+4}{3s^2} - \frac{1}{3s}\right)$$

$$\frac{6s^3+2s^2+2s-4s-4-s}{3s^2} = \frac{6s^3+2s^2-s-4}{3s^2}$$

$$V_{(s)} = V_{L_1}\left(\frac{6s^3+2s^2-s-4}{3s^2}\right)$$

$$\frac{V_L}{V_s} = \frac{1}{\left(\frac{6s^3+2s^2-s-4}{3s^2}\right)} = \frac{3s^2}{6s^3+2s^2-s-4}$$

b) $-V(s) + L_1(I_1-I_2) + \left(\frac{L_2 \cdot C_1}{L_2+C_1}\right)(I_1-I_2) = 0$

$$\left(\frac{L_2 \cdot C_1}{L_2+C_1}\right) = Z_{LC}$$

$$V(s) = L_1(I_1-I_2) + \left(\frac{L_2 \cdot C_1}{L_2+C_1}\right)(I_1-I_2)$$

$$Ls \cdot \frac{1}{Cs} = \frac{L}{C}$$

$$(L_1+Z_{LC})\, I_1 - (Z_{LC}+L_2)\, I_2 = V(s)$$

$$Ls+\frac{1}{Cs}$$

$$-L_1(I_1-I_2) + L_3 \cdot I_2 + R(I_2-I_3) = 0$$

$$-L_1 I_1 + (L_1+L_3+R)I_2 - RI_3 = 0$$

$$\frac{LCs^2+1}{Cs} \cdot \frac{L}{C}$$

$$\frac{L^2Cs^2+L}{C^2s}$$

$$-Z_{LC}(I_1 - I_2) - R(I_2 - I_3) + C_3 I_3 = 0$$

$$-Z_{LC} I_1 + (Z_{LC} - R) I_2 + (R + C_3) I_3 = 0$$

$$\begin{cases} (L_1 + Z_{LC}) I_1 - (Z_{LC} + L_2) I_2 = V(s) \\ -L_1 I_1 + (L_1 + L_3 + R) I_2 - R I_3 = 0 \\ -Z_{LC} I_1 + (Z_{LC} - R) I_2 + (R + C_3) I_3 = 0 \end{cases}$$

$$\begin{bmatrix} (L_1 + Z_{LC}) & -(Z_{LC} + L_2) & 0 \\ -L_1 I_1 & (L_1 + L_3 + R) & -R \\ -Z_{LC} & (Z_{LC} - R) & (R + C_3) \end{bmatrix}$$

$$\Delta = (L_1 + Z_{LC})(L_1 + L_3 + R)(R + C_3) - R Z_{LC}(Z_{LC} + L_2)$$
$$-(-R(Z_{LC} - R)(L_1 + Z_{LC}) + (R + C_3)(L_1 I_1)(Z_{LC} + L_2))$$

$$(L_1^2 + L_1 L_3 + L_1 R + L_1 Z_{LC} + L_3 Z_{LC} + R Z_{LC}) \cdot (R + C_3)$$
$$R L_1^2 + R L_1 L_3 + L_1 R^2 + R L_1 Z_{LC} + R L_3 Z_{LC} + R^2 Z_{LC}$$

$$C_3 L_1^2 + C_3 L_1 L_3 + C_3 L_1 R + C_3 L_1 Z_{LC} + C_3 L_3 Z_{LC} + C_3 R Z_{LC}$$

$$(-R Z_{LC} + R^2)(L_1 + Z_{LC}) + (R L_1 I_1 + C_3 L_1 I_1)(Z_{LC} + L_2))$$

$$-(-RL_1Z_{LC}+RZ_{LC}+L_1R^2+R^2Z_{LC})+RL_1Z_{LC}I_1+RL_1L_2I_1+C_3L_1I_1Z_{LC}$$
$$+L_2C_3L_1I_1)$$

$$RL_1Z_{LC}-RZ_{LC}-L_1R^2+R^2Z_{LC}-RL_1Z_{LC}-RL_1L_2-C_3L_1Z_{LC}-L_2C_3L_1$$

$$RL_1^2+RL_1L_3+L_1R^2+RL_1Z_{LC}+RL_3Z_{LC}+R^2Z_{LC}-RZ_{LC}^2-RL_2Z_{LC}+\dots$$

$$\dots+C_3L_1^2+C_3L_1L_3+C_3L_1R+C_3L_1Z_{LC}+C_3L_3Z_{LC}+C_3RZ_{LC}+\dots$$

$$\dots+RL_1Z_{LC}-RZ_{LC}-L_1R^2+R^2Z_{LC}-RL_1Z_{LC}-RL_1L_2-C_3L_1Z_{LC}-L_2C_3L_1$$

$$s^2+s^2+s+\left(s+\frac{1}{s}\right)\left(s+s+1-\left(s+\frac{1}{s}\right)-s+1+1+\frac{1}{s}\right)+s+s+1$$

$$2s^2+3s+1+\left(s+\frac{1}{s}\right)3=2s^2+3s+1+3s+\frac{3}{s}=2s^2+6s+\frac{3}{s}+1$$

$$\Delta=\frac{2s^3+6s^2+s+3}{s}$$

$$I_3(s)=\frac{\begin{vmatrix}(L_1+Z_{LC}) & -(Z_{LC}+L_2) & V(s)\\ -L_1I_1 & (L_1+L_3+R) & 0\\ -Z_{LC} & (Z_{LC}-R) & 0\end{vmatrix}}{\Delta}=\begin{array}{l}V(s)(L_1+L_3+R)(Z_{LC})\\ V(s)(2s+1)\left(-s-\frac{1}{s}\right)\end{array}$$

$$I_3(s)=V(s)(2s+1)\left(-s-\frac{1}{s}\right)\cdot\frac{s}{2s^3+6s^2+s+3}$$

$$v_0(t)\Leftrightarrow V_C(s)=Z_{C2}\cdot I_3(s)$$

$$I_3(s) = \frac{V_C(s)}{Z_{C2}}$$

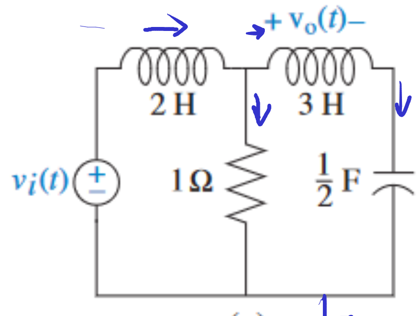
$$\underline{V_C(s)} = V(s)(2s+1)\left(-s-\frac{1}{s}\right)\cdot\frac{s}{2s^3+6s^2+s+3}\cdot\frac{1}{s}$$

$$\frac{V_C(s)}{V(s)} = (2s+1)\left(-s-\frac{1}{s}\right)\cdot\frac{\cancel{s}}{2s^3+6s^2+s+3}\cdot\frac{1}{\cancel{s}}$$

$$\left(-2s^2-2-s-\frac{1}{s}\right)\left(\frac{1}{2s^3+6s^2+s+3}\right) =$$

$$\left(\frac{-2s^3-2s-s^2-1}{s}\right)\left(\frac{1}{2s^3+6s^2+s+3}\right) = \frac{-2s^3-s^2-2s-1}{2s^4+6s^3+s^2+3s}$$

20. Repeat Problem 19 using nodal equations. [Section: 2.4]



(*a*)

$$I_1 - I_2 - I_3 = 0$$

$$\frac{V(s)-V_R(s)}{L_1 s} \overbrace{-\frac{(V_R(s)-V_C(s))}{L_2 s}}^{V_{L_2}(s)} - \frac{V_R(s)}{R} = 0$$

$$\frac{RL_2s(V(s)-V_R(s)) - RL_1s(V_{L2}(s)) - L_1L_2V_R(s)}{RL_1sL_2s} = 0$$

$$\underline{RL_2s(V(s)-V_R(s)) - RL_1s(V_{L2}(s)) - L_1L_2V_R(s)} = 0$$

$$-(L_1sL_2s+RL_2s)V_R(s) - (RL_1s)V_{L_2}(s) = -RL_2sV(s)$$

$$RL_2s(V(s)-V_R(s)) - RL_1s(V_R(s)-V_C(s)) - L_1L_2V_R(s)$$

$$-(L_1sL_2s+RL_2s)V_R(s) - (RL_1s)V_{L_2}(s) = -RL_2sV(s)$$

$$\frac{V_R-V_C}{L_2s} = CsV_C \qquad Cs(V_R-V_{L_2}) - \frac{V_{L_2}}{L_2s} = 0 \qquad CsV_C - \left(\frac{\overset{\nearrow V_{L2}}{V_R-V_C}}{L_2s}\right) = 0$$

$$-\left(Cs+\frac{1}{L_2s}\right)V_{L2} + (Cs)V_R = 0 \qquad V_{R2}-V_C = V_L$$

$$V_C = V_{R2}-V_{L2}$$

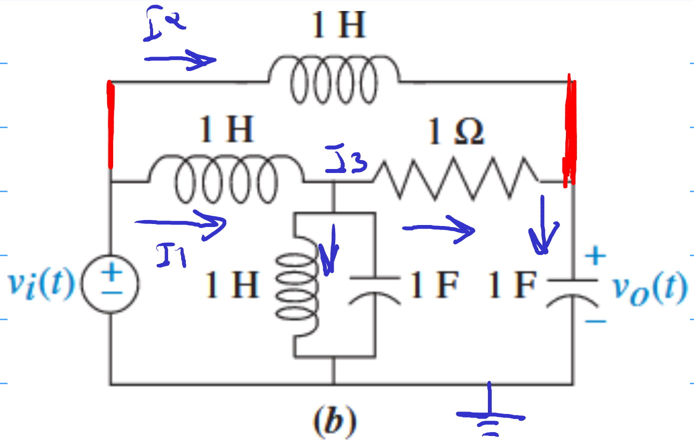
$$-(RL_1s)V_{L_2}(s) - (L_1sL_2s+RL_2s)V_R(s) = -RL_2sV(s)$$

$$\begin{vmatrix} -RL_1s & -(L_1sL_2s+RL_2s) \\ -\left(Cs+\frac{1}{L_2s}\right) & Cs \end{vmatrix} = \begin{vmatrix} -2s & -(6s^2+3s) \\ -\left(\frac{s}{2}+\frac{1}{3s}\right) & \frac{s}{2} \end{vmatrix} = \Delta$$

$$= -s^2 - \left(3s^3+2s+\frac{3s^2}{2}+1\right) = -3s^3 - \frac{5s^2}{2} - 2s - 1$$

$$V_{L2} = \frac{\begin{vmatrix} -3sV(s) & -(6s^2+3s) \\ 0 & \frac{s}{2} \end{vmatrix}}{\Delta}$$

$$V_{L2} = -\frac{3s^2}{\cancel{2}}V(s) \cdot \frac{\cancel{2}}{-6s^3-5s^2-4s-2} \qquad \frac{V_{L2}(s)}{V(s)} = -\frac{3s^2}{6s^3-5s^2-4s-2}$$



$$-I_1-I_2=0$$
$$I_1-I_3-I_4=0$$
$$I_4+I_2-I_5=0$$

$$-\left(\frac{v_i(t)-V_Z}{L_1 s}\right)-\left(\frac{v_i(t)-V_C}{L_2 s}\right)=0$$

$$-\left(\frac{v_i(t)-V_Z}{L_1 s}\right)=\frac{V_Z}{Z}+\frac{V_Z-V_C}{R}$$

$$-\left(\frac{v_i(t)-V_Z}{L_1 s}\right)-\left(\frac{v_i(t)-V_C}{L_2 s}\right)=0 \qquad \frac{-s(V(s)-V_Z(s))-s(V(s)-V_C)}{s^2}=0$$

$$\frac{-s(V(s)-V_Z(s))-s(V(s)-V_C)}{s^2}=0$$

$$-\cancel{s}\left((V(s)-V_Z(s))+(V(s)-V_C(s)\right)=0$$

$$2V(s)-V_Z(s)-V_C(s)=0$$

$$-\left(\frac{v_i(t)-V_Z}{L_1 s}\right)=\frac{V_Z}{Z}+\frac{V_Z-V_C}{R} \qquad V_Z(s)+V_C(s)=2V(s)$$

$$Z=s+\frac{1}{s}$$

$$-\left(\frac{v_i(t)-V_z}{L_1 s}\right)=\frac{V_z}{Z}+\frac{V_z-V_c}{R}$$

$$-\left(\frac{v_i(t)-V_z}{s}\right)=\frac{V_z\cdot s}{s^2+1}+\frac{V_z-V_c}{1}$$

$$-(s^2+1)\left(V(s)-V_z\right)$$

$$\frac{V_z}{Z}+\frac{V_z-V_c}{R}+\left(\frac{v_i(t)-V_z}{L_1 s}\right)=0$$

$$\frac{(s^2+1)\left(V(s)-V_z(s)\right)+s^2V_z(s)+s(s^2+1)\left(V_z(s)-V_c(s)\right)}{s(s^2+1)}=0$$

$$(s^2+1)\left(V(s)-V_z(s)\right)+s^2V_z(s)+s(s^2+1)\left(V_z(s)-V_c(s)\right)$$

$$(s^2+1)V(s)-(s^2+1)V_z(s)+s^2V_z(s)+s(s^2+1)V_z(s)-s(s^2+1)V_c(s)$$

$$(\cancel{s^2}+1)V_z(s)-\cancel{s^2}V_z(s)-s(s^2+1)V_z(s)+s(s^2+1)V_c(s)=(s^2+1)V(s)$$

$$V_z(s)-s^3V_z(s)-sV_z(s)+s^3+sV_c(s)$$

$$-(s^3+s-1)V_z(s)+(s^3+s)V_c(s)=(s^2+1)V(s)$$

$$\begin{cases} V_z(s)+V_c(s)=2V(s) \\ -(s^3+s-1)V_z(s)+(s^3+s)V_c(s)=(s^2+1)V(s) \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ -(s^3+s-1) & (s^3+s) \end{bmatrix}=\begin{bmatrix} 2 \\ (s^2+1) \end{bmatrix}$$

$$(s^3+s)+(s^3+s-1) = 2s^3+2s-1 = \Delta$$

$$V_c = \frac{\begin{vmatrix} 1 & V(s)\,2 \\ -(s^3+s-1) & V(s)(s^2+1) \end{vmatrix}}{\Delta} = \frac{V(s)(s^2+1)+2V(s)(s^3+s+1)}{2s^3+2s-1}$$

$$\frac{V(s)\left((s^2+1)+2(s^3+s+1)\right)}{2s^3+2s-1}$$

$$\frac{V_c(s)}{V(s)} = \frac{2s^3+s^2+2s+4}{2s^3+2s-1}$$

26. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at $x_2(t)$.) [Section: 2.5]

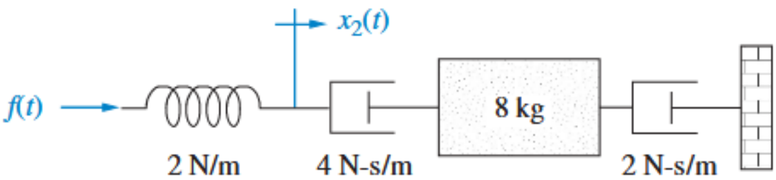


FIGURE P2.11

$$f(t) - Kx_2 - f_{v_1}\dot{x}_2 - f_{v_2}\dot{x}_2 = \overset{0}{\cancel{m}}\cdot\ddot{x}_2$$

$$f(t) - Kx_2 - f_{v_1}\dot{x}_2 - f_{v_2}\dot{x}_2 = 0$$

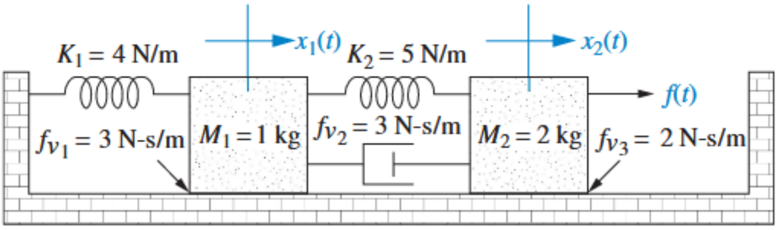
$$F(s) - KX_2(s) - f_{v_1}sX_2(s) - f_{v_2}sX_2(s)$$

$$KX_2(s) + f_{v_1}sX_2(s) + f_{v_2}sX_2(s) = F(s)$$

$$(6s+2)X_2(s) = F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{1}{6s+2}$$

27. For the system of Figure P2.12 find the transfer function, $G(s) = X_1(s)/F(s)$. [Section: 2.5]



$$-K_1 x_1(t) - f_{v_1}\dot{x}_1(t) + f_{v_2}(\dot{x}_2(t) - \dot{x}_1(t)) + K_2(x_2(t) - x_1(t)) = M_1 \cdot \ddot{x}_1(t)$$

$$-K_1 X_1(s) - f_{v_1} s X_1(s) + f_{v_2} s (X_2(s) - X_1(s)) + K_2(X_2(s) - X_1(s)) = M_1 s^2$$

$$M_1 s^2 X_1(s) + K_1 X_1(s) + f_{v_1} s X_1(s) - f_{v_2} s (X_2(s) - X_1(s)) - K_2(X_2(s) - X_1(s)) = 0$$

$$X_1(s)(M_1 s^2 + (f_{v_1} + f_{v_2})s + K_1 + K_2) - X_2(s)(f_{v_2}s + K_2) = 0$$

$$X_1(s)(s^2 + 6s + 9) - X_2(s)(3s + 5) = 0$$

$$f(t) - f_{v_3}\dot{x}_2 - f_{v_2}(\dot{x}_2 - \dot{x}_1) - K_2(x_2 - x_1) = M_2 \ddot{x}_2$$

$$M_2 \ddot{x}_2 + f_{v_3}\dot{x}_2 + f_{v_2}(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1) = f(t)$$

$$M_2 s^2 X_2(s) + f_{v_3} s X_2(s) + f_{v_2} s X_2(s) - f_{v_2} s X_1(s) + K_2 X_2(s) - K_2 X_1(s) = F(s)$$

$$\begin{cases} -X_1(s)(3s+5) + X_2(s)(2s^2+5s+5) = F(s) \\ X_1(s)(s^2+6s+9) - X_2(s)(3s+5) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} -(3s+5) & (2s^2+5s+5) \\ (s^2+6s+9) & -(3s+5) \end{vmatrix} = (3s+5)(3s+5) - ((2s^2+5s+5)(s^2+6s+9))$$

$$(3s+5)(3s+5)-\left((2s^2+5s+5)(s^2+6s+9)\right)$$

$$9s^2+30s+25-(2s^4+12s^3+18s+5s^3+30s^2+45s+5s^2+30s+45)$$

$$9s^2+30s+25-2s^4-17s^3-35s^2-93s-45$$

$$-(2s^4+17s^3+26s^2+63s+20)$$

$$X_1(s)=\frac{\begin{vmatrix} F(s) & (2s^2+5s+5) \\ 0 & (3s+5) \end{vmatrix}}{\Delta}=\frac{F(s)(3s+5)}{-(2s^4+17s^3+26s^2+63s+20)}$$

$$\frac{X_1(s)}{F(s)}=\frac{-(3s+5)}{(2s^4+17s^3+26s^2+63s+20)}$$

28. Find the transfer function, $G(s)=X_3(s)/F(s)$, for the translational mechanical system shown in Figure P2.13. [Section: 2.5]

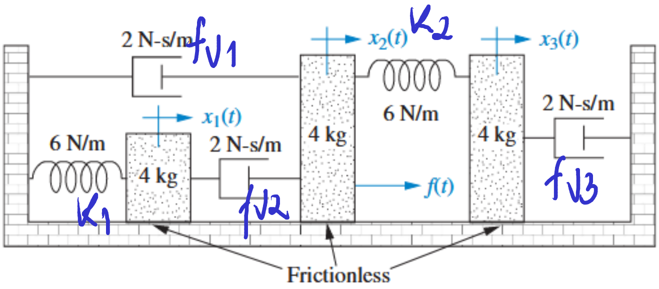


FIGURE P2.13

Masa 1: $-K_1x_1(t)+f_{V_2}(\dot{x}_2(t)-\dot{x}_1(t))=M_1\ddot{x}_1(t)$

$$M_1\ddot{x}_1(t)+K_1x_1(t)-f_{V_2}(\dot{x}_2(t)-\dot{x}_1(t))=0$$

$$M_1s^2X_1(t)+K_1X_1(s)+f_{V_2}sX_1(t)-f_{V_2}sX_2(s)=0$$

$$X_1(s)(M_1s^2+f_{V_2}s+K_1)-X_2(s)(f_{V_2}s)=0$$

$$\boxed{X_1(s)(4s^2+2s+6)-X_2(s)(2s)=0}$$

Massa 2: $-f_{v_1}\dot{x}_2(t) - f_{v_2}(\dot{x}_2(t) - \dot{x}_1(t)) - K_2(x_2(t) - x_3(t)) + f(t) = M_2\ddot{x}_2(t)$

$$M_2\ddot{x}_2(t) + f_{v_1}\dot{x}_2(t) + f_{v_2}(\dot{x}_2(t) - \dot{x}_1(t)) + K_2(x_2(t) - x_3(t)) = f(t)$$

$$M_2 s^2 X_2(s) + f_{v} s X_2(s) + f_{v_2} s (X_2(s) - X_1(s)) + K_2(X_2(s) - X_3(s)) = F(s)$$

$$-f_{v_2} s X_1(s) + X_2(s)(M_2 s^2 + f_v s + f_{v_2} s + K_2) - K_2 X_3(s) = F(s)$$

$$\boxed{-X_1(s)(2s) + X_2(s)(4s^2 + 4s + 6) - X_3(s)\,6}$$

Massa 3: $-f_{v_3}\dot{x}_3(t) + K_2(x_2(t) - x_3(t)) = M_3\ddot{x}_3(t)$

$$M_3\ddot{x}_3(t) + f_{v_3}\dot{x}_3(t) - K_2(x_2(t) - x_3(t)) = 0$$

$$M_3 s^2 X_3(s) + f_{v_3} s X_3(s) - K_2 X_2(s) + K_2 X_3(s) = 0$$

$$-K_2 X_2(s) \quad X_3(s)(M_3 s^2 + f_{v_3} s + K_2)$$

$$\boxed{-2X_2(s) + X_3(4s^2 + 2s + 6)}$$

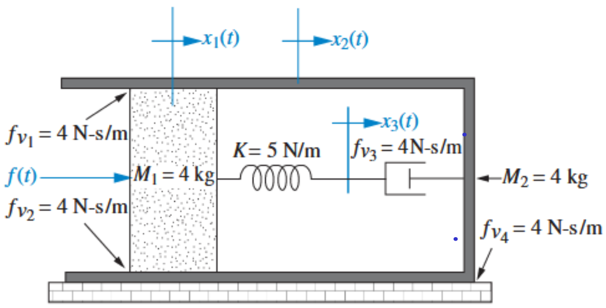
$$\begin{cases} X_1(s)(4s^2 + 2s + 6) - X_2(s)(2s) = 0 \\ -X_1(s)(2s) + X_2(s)(4s^2 + 4s + 6) - X_3(s)\,6 = F(s) \\ -2X_2(s) + X_3(4s^2 + 2s + 6) \end{cases}$$

$$\Delta = \begin{vmatrix} -(2s) & (4s^2 + 4s + 6) & -6 \\ (4s^2 + 2s + 6) & -2s & 0 \\ 0 & -2 & (4s^2 + 2s + 6) \end{vmatrix} = -64s^6 - 128s^5 - 352s^4 - 392s^3 - 480s^2 - 264s - 144$$

$$X_3(s) = \frac{\begin{vmatrix} -(2s) & (4s^2+4s+6) & F(s) \\ (4s^2+2s+6) & -2s & 0 \\ 0 & -2 & 0 \end{vmatrix}}{\Delta} = -\frac{4F(s)\cdot(2s^2+s+3)}{\Delta}$$

$$\frac{X_3(s)}{F(s)} = \frac{1}{8s^4+12s^3+26s^2+18s+12}$$

29. Find the transfer function, $X_3(s)/F(s)$, for each system shown in Figure P2.14. [Section: 2.5]



(*a*)

Massa 1:

$$f_{v_1}(\dot{x}_2(t) - \dot{x}_1(t)) + f_{v_2}(\dot{x}_2(t) - \dot{x}_1(t)) + f(t) + K(x_3(t) - x_1(t)) = M_1\ddot{x}_1(t)$$

$$M_1\ddot{x}_1(t) - f_{v_1}(x_2(t) - x_1(t)) - f_{v_2}(\dot{x}_2(t) - \dot{x}_1(t)) - K(x_3(t) - x_1(t)) = f(t)$$

$$X_1(s)(M_1s^2 + (f_{v_1}+f_{v_2})s + K) - X_2(s)((f_{v_1}+f_{v_2})s) - X_3(s)K = F(s)$$

$$X_1(s)(4s^2 + 8s + 5) - X_2(s)(8s) - 5X_3(s) = F(s)$$

Massa 2:

$$-f_{v_4}\dot{x}_2(t) - f_{v_2}(\dot{x}_2(t) - \dot{x}_1(t)) - f_{v_1}(\dot{x}_2(t) - \dot{x}_1(t)) + f_{v_3}(x_3(t) - x_2(t)) = M_2\ddot{x}_2(t)$$

$$M_2\ddot{x}_2(t)+f_{v4}\dot{x}_2(t)+f_{v2}(\dot{x}_2(t)-\dot{x}_1(t))+f_{v1}(\dot{x}_2(t)-\dot{x}_1(t))-f_{v3}(x_3(t)-x_2(t))=0$$

$$X_1(s)(-(f_{v_1}+f_{v_2})s)+X_2(s)(M_2s^2+s(f_{v4}+f_{v_2}+f_{v_1}+f_{v_3}))+f_{v_3}X_3(s)=0$$

$$X_1(s)(-8s)+X_2(s)(4s^2+12s)+X_3(s)(4s)=0$$

Massa 3: $-f_{v_3}(\dot{x}_3(t)-\dot{x}_2(t))-K(x_3(t)-x_1(t))=\cancel{M_3}\ddot{x}_3(t)$ 0

$$f_{v_3}(\dot{x}_3(t)-\dot{x}_2(t))+K(x_3(t)-x_1(t))=0$$

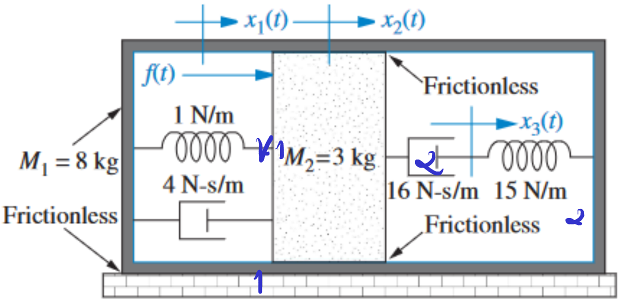
$$-K\,X_1(s)-X_2(s)(sf_{v_3})+X_3(s)(f_{v_3}s+K)$$

$$-5X_1(s)-X_2(s)(4s)+X_3(s)(4s+5)$$

$$\Delta=\begin{vmatrix}(4s^2+8s+5) & -8s & -5\\ -8s & (4s^2+12s) & 4s\\ -5 & -4s & (4s+5)\end{vmatrix}=16s^2\cdot(4s^3+29s^2+46s+30)$$

$$X_3(s)=\frac{\begin{vmatrix}(4s^2+8s+5) & -8s & F(s)\\ -8s & (4s^2+12s) & 0\\ -5 & -4s & 0\end{vmatrix}}{\Delta}=\frac{F(s)(13s+15)}{4s(4s^3+29s^2+46s+30)}$$

$$\frac{X_3(s)}{F(s)}=\frac{(13s+15)}{4s(4s^3+29s^2+46s+30)}$$



(b)

FIGURE P2.14

Masa 1: $K_1(x_2(t)-x_1(t))+f_{v_1}(\dot{x}_2(t)-\dot{x}_1(t))+f_{v_2}(\dot{x}_3(t)-\dot{x}_1(t))=M_1\ddot{x}_1(t)$

$$M_1\ddot{x}_1(t)-K_1(x_2(t)-x_1(t))-f_{v_1}(\dot{x}_2(t)-\dot{x}_1(t))-K_2(\dot{x}_3(t)-\dot{x}_1(t))=0$$

$$X_1(s)(M_1s^2+f_{v_1}s+K_1+K_2)-X_2(s)(f_{v_1}s+K_1)-X_3(s)(K_2)=0$$

$$X_1(s)(8s^2+4s+16)-X_2(s)(4s+1)-16X_3(s)=0$$

Masa 2:

$$f(t)-f_{v_1}(\dot{x}_2(t)-\dot{x}_1(t))-K_1(x_2(t)-x_1(t))+f_{v_2}(\dot{x}_3(t)-\dot{x}_2(t))=M_2\ddot{x}_2(t)$$

$$M_2\ddot{x}_2(t)+f_{v_1}(\dot{x}_2(t)-\dot{x}_1(t))+K_1(x_2(t)-x_1(t))-f_{v_2}(\dot{x}_3(t)-\dot{x}_2(t))=f(t)$$

$$-X_1(s)(f_{v_1}s+K_1)+X_2(s)(M_2s^2+s(f_{v_1}+f_{v_2})+K_1)-X_3(s)(f_{v_2}s)=F(s)$$

$$-X_1(s)(4s+1)+X_2(s)(3s^2+20s+1)-X_3(s)(16s)=F(s)$$

Masa 3: $-f_{v_2}(\dot{x}_3(t)-\dot{x}_2(t))-K_2(x_3(t)-x_1(t))=\cancel{M_3}^{\,0}\ddot{x}_3(t)$

$$f_{v_2}(\dot{x}_3(t)-\dot{x}_2(t))+K_2(x_3(t)-x_1(t))=0$$

$$-K_2X_1(s)-X_2(s)(f_{v_2}s)+X_3(s)(f_{v_2}s+K_2)=0$$

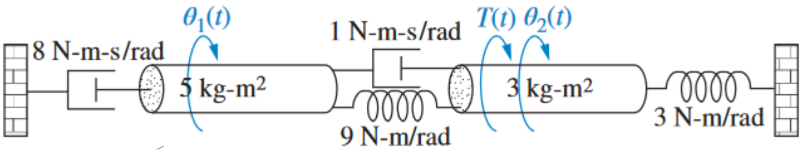
$$-X_1(s)-X_2(s)(16s)+X_3(s)(16s+15)=0$$

$$\Delta = \begin{vmatrix} 8s^2+4s+16 & -(4s+1) & -16 \\ -(4s+1) & (3s^2+20s+1) & -16s \\ -1 & -16s & (16s+15) \end{vmatrix} = 384s^5+1064s^4+3476s^3+1624s^2+4388s+209$$

$$X_3(s) = \frac{\begin{vmatrix} 8s^2+4s+16 & -(4s+1) & 0 \\ -(4s+1) & (3s^2+20s+1) & F(s) \\ -1 & -16s & 0 \end{vmatrix}}{\Delta} = \frac{F(s)(128s^3+64s^2+260s+1)}{384s^5+1064s^4+3476s^3+1064s^2+4388s+209}$$

$$\frac{X_3(s)}{F(s)} = \frac{(128s^3+64s^2+260s+1)}{384s^5+1064s^4+3476s^3+1064s^2+4388s+209}$$

32. For each of the rotational mechanical systems shown in Figure P2.17, write, but do not solve, the equations of motion. [Section: 2.6]



(a)

Masa 1: $-D_1\dot{\theta}_1(t) + D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) + K_1(\dot{\theta}_2(t)-\dot{\theta}_1(t)) = J\ddot{\theta}_1(t)$

$$J\ddot{\theta}_1(t) + D_1\dot{\theta}_1(t) - D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) - K_1(\dot{\theta}_2(t)-\dot{\theta}_1(t)) = 0$$

$$\Theta_1(s)(Js^2+s(D_1+D_2)+K_1) - \Theta_2(s)(D_2s+K_1) = 0$$

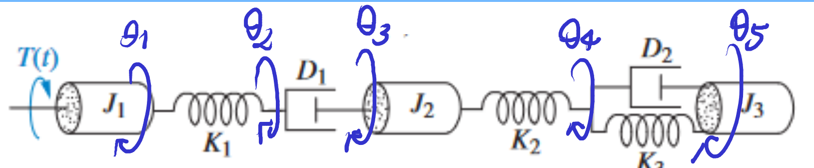
$$\Theta_1(s)(8s^2+9s+9) - \Theta_2(s)(s+9) = 0$$

Massa 2: $-K_1(\dot{\theta}_2(t)-\dot{\theta}_1(t)) - K_2\theta_2(t) - D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) + T(t) = J\ddot{\theta}_2(t)$

$$J\ddot{\theta}_2(t)+K_1(\dot{\theta}_2(t)-\dot{\theta}_1(t))+K_2\theta_2(t)+D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t))=T(t)$$

$$-\theta_1(s)(D_2s+K_1)+\theta_2(s)(sD_2+K_1+K_2)=T(s)$$

$$-\theta_1(s)(s+9)+\theta_2(s)(s+12)=T(s)$$



(b)

FIGURE P2.17

$$K_1(\theta_2(t)-\theta_1(t))+T(t)=M_1\ddot{\theta}_1(t)$$

$$M_1\ddot{\theta}_1(t)-K_1(\theta_2(t)-\theta_1(t))=T(t)$$

$$\boxed{\theta_1(s)(J_1s^2+K_1)-\theta_2(s)(K_1)=T(s)}$$

$$-K_1(\theta_2(t)-\theta_1(t))+D_1(\theta_3(t)-\theta_2(t))=0$$

$$\boxed{\theta_1(s)K_1-\theta_2(s)(K_1+D_1s)+\theta_3(s)D_1s=0}$$

$$-D_1(\theta_3(t)-\theta_2(t))+K_2(\theta_4(t)-\theta_3(t)=J_2\ddot{\theta}_3(t)$$

$$J_2\ddot{\theta}_3(t)+D_1(\theta_3(t)-\theta_2(t))-K_2(\theta_4(t)-\theta_3(t)=0$$

$$J_2\ddot{\theta}_3(t)+D_1(\theta_3(t)-\theta_2(t))-K_2(\theta_4(t)-\theta_3(t))=0$$

$$\boxed{-\theta_2(s)(D_1s)+\theta_3(s)(J_2s^2+D_1s+K_2)-K_2\theta_4(s)=0}$$

$$-K_2(\theta_4(t)-\theta_3(t)) + K_3(\theta_5(t)-\theta_4(t)) + D_2(\theta_5(t)-\theta_4(t)) = 0$$

$$K_2(\theta_4(t)-\theta_3(t)) - K_3(\theta_5(t)-\theta_4(t)) - D_2(\theta_5(t)-\theta_4(t)) = 0$$

$$\boxed{-K_2\theta_3(s) + \theta_4(s)(D_2 s + K_2 + K_3) - \theta(s)(D_2 s + K_3) = 0}$$

$$-K_3(\theta_5(t)-\theta_4(t)) - D_2(\dot{\theta}_5(t)-\dot{\theta}_4(t)) = J_3\ddot{\theta}_5(t)$$

$$J_3\ddot{\theta}_5(t) + K_3(\theta_5(t)-\theta_4(t)) + D_2(\dot{\theta}_5(t)-\dot{\theta}_4(t)) = 0$$

$$\boxed{-\theta_4(s)(D_2 s + K_3) + \theta_5(s)(J_3 s^2 + D_2 s + K_3) = 0}$$

33. For the rotational mechanical system shown in Figure P2.18, find the transfer function $G(s) = \theta_2(s)/T(s)$ [Section: 2.6]

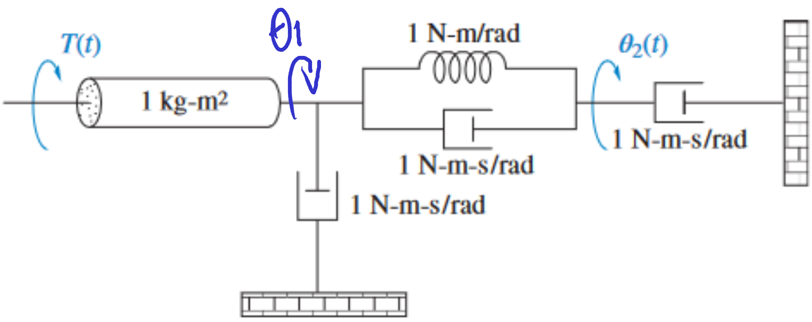


FIGURE P2.18

$$\text{Rot}_1: \quad T(t) - D_1\dot{\theta}_1(t) + D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) + K(\theta_2(t)-\theta_1(t)) = J_1\ddot{\theta}_1(t)$$

$$J_1\ddot{\theta}_1(t) + D_1\dot{\theta}_1(t) - D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) - K(\theta_2(t)-\theta_1(t)) = T(t)$$

$$\theta_1(s)(J_1 s^2 + s(D_1+D_2) + K \quad -\theta_2(s)(D_2 s + K) = T(s)$$

$$Rot_2: \quad -D_3\theta_2(t) - D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) - K(\dot{\theta}_2-\dot{\theta}_1)=0$$

$$D_3\theta_2(t) + D_2(\dot{\theta}_2(t)-\dot{\theta}_1(t)) + K(\dot{\theta}_2-\dot{\theta}_1)=0$$

$$-\theta_1(s)(sD_2+K) + \theta_2(s)(s(D_3+D_2)+K)=0$$

$$\Delta = \begin{vmatrix} (s^2+2s+1) & -(s+1) \\ -(s+1) & (2s+1) \end{vmatrix} = 2s(s+1)^2$$

$$\theta_2(S) = \frac{\begin{vmatrix} (s^2+2s+1) & T(s) \\ -(s+1) & 0 \end{vmatrix}}{\Delta} = \frac{T(s)}{2s(s+1)} \Rightarrow \frac{\theta_2(S)}{T(S)} = \frac{1}{2s(s+1)}$$

34. Find the transfer function, $\frac{\theta_1(s)}{T(s)}$, for the system shown in Figure P2.19.

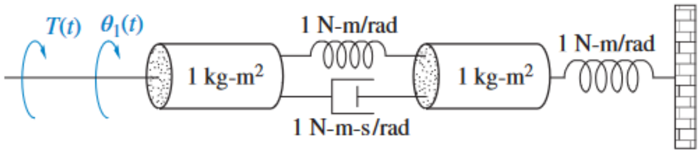


FIGURE P2.19

$$Rot_1: \; T(t) + K_1(\theta_2(t)-\theta_1(t)) + D(\dot{\theta}_2(t)-\dot{\theta}_1(t)) = J_1\ddot{\theta}_1(t)$$

$$J_1\ddot{\theta}_1(t) - K_1(\theta_2(t)-\theta_1(t)) - D(\dot{\theta}_2(t)-\dot{\theta}_1(t)) = T(t)$$

$$\theta_1(s)(J_1s^2+Ds+K_1) - \theta_2(s)(Ds+K_1) = T(s)$$

$$\theta_1(s)(s^2+s+1) - \theta_2(s)(s+1) = T(s)$$

$$\mathrm{Rot}_2:\ -D\left(\dot\theta_2(t)-\dot\theta_2(t)\right)-K_1\left(\theta_2(t)-\theta_1(t)-K_2\theta_2(t)=J_2\ddot\theta_2(t)$$

$$J_2\ddot\theta_2(t)+D\left(\dot\theta_2(t)-\dot\theta_1(t)\right)+K_1\left(\theta_2(t)-\theta_1(t)+K_2\theta_2(t)=0$$

$$-\theta_1(s)(Ds+K)\quad+\theta_2(s)(J_2s^2+s+K_1+K_2)=0$$

$$-\theta_1(s)(s+1)+\theta_2(s)(s^2+s+2)=0$$

$$\Delta=\begin{vmatrix}(s^2+s+1) & -(s+1)\\ -(s+1) & (s^2+s+2)\end{vmatrix}=(s+1)^4$$

$$\theta_2(s)=\begin{vmatrix}(s^2+s+1) & T(s)\\ -(s+1) & 0\end{vmatrix}=\frac{T(s)}{(s+1)^3}$$

$$\frac{\theta_2(s)}{T(s)}=\frac{1}{(s+1)^3}$$