A Literature Survey on Fitts' Law and its Applications

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Abstract

The way of pointing a target object and the movement time of the pointer lead to a challanging problem in HCI. A large number of seminal paper have been proposed towards this. Moreover, the HCI community uses few formulas, and are related to *Fitts' law*. Few are derived from *Shannon's* information theory. This paper introduces a survey on Fitts' law and its derivatives. Finally, we study the various application of Fitts' law in HCI.

Index Terms

Fitts' law, index of performance, index of difficulty

I. Introduction

Pointing a virtual object is typically achieved by an input device that acts as an intermediary between human and the graphical object being pointed to. In this scenario, it is very important to consider the time required to reach at the target object. This idea was initially proposed in the seminal paper, and modeled using law called *Fitts' law*. This law gives the movement time required to reach a target of given size at a given distance.

The qualitative feature of Fitts' law shows the movement time needed to a pointer is lesser for a large size object than a smaller one. Along with this, the movement time achieved is smaller for a small distant object than a large distant one. However, the law that was formulated was mainly for one-dimensional targets, and along the axis of the width of the target. This can be extended to two dimensional target where height and width needs to be considered.

In case the pointer follows a particular path, then the distance traversed in the curvature is considered for finding the movement time and is closely related to Fitts' law. This is called *Steering law* and was proposed by.

In this survey work, we analyze the Fitts' law and its variations (See Sect. II). We then study the applications of Fitts' law (See Sect. III)

II. Fitts' Law

Here we discuss Fitts' law and its relation to Shannon's Information theory. The HCI research community uses different formulas for Fitts' law. However, each of them is formulated from Shannon's theory.

Shannon's information theory is used to calculate the information capacity of a communication channel, as a function of bandwidth and *Signal-to-noise* ration, and is given below,

$$C = B \times log_2(1 + S/N)$$

where C=capacity (data-rate, bits per sec.), S=signal power, N=noise power, and B=bandwidth

Information capacity of the human motor system is called index of performance (IP), and is measured in units processed per second. Similarly, difficulty of motor task is called index of difficulty (ID). We can write,

$$\begin{aligned} IP = ID/MT \\ MT = ID/IP \end{aligned} \tag{1}$$

where MT=motor task.

Fitts' law, that was initially proposed, is given as $MT = a + blog_2(\frac{2A}{W})$, where:

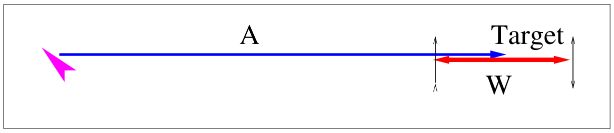

Fig. 1: Fitts' law in 1D

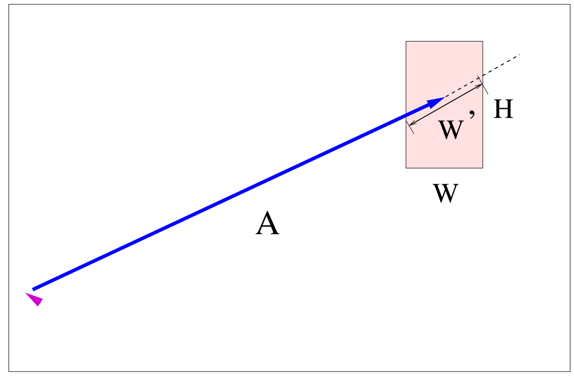

Fig. 2: Fitts' law in 2D

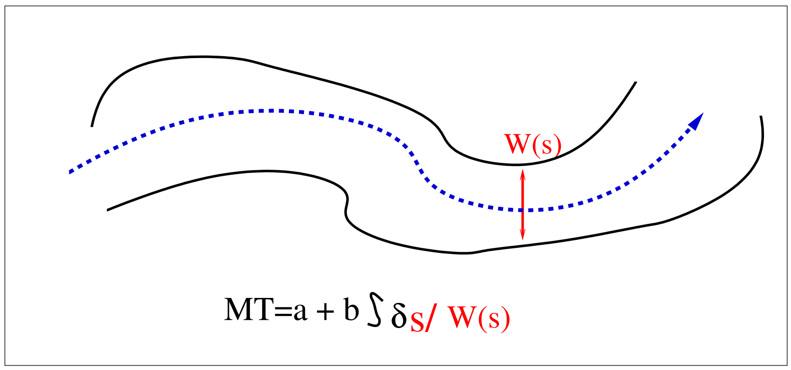

Fig. 3: Steering Law

- :A, the distance of the pointer to the center of the target,
- :W, the width of the target,
- :a, think time,
- :b, shows the efficicency of the pointing device, and finally,
- :MT, movement time of the device.

Fitts' law varies based on the curvature of the movement, moving towards the target. So Fitts' law can be like in 1D, 2D, or, any curvature of movement path.

In case the movement happens in a curvature shown in Fig.3, then law defined for movement time is called *steering law* and is closely related to Fitts' law.

The movement time inside a straight tunnel of constant width is given by,

$$MT = a + b(\frac{A}{W})$$

A. Logarithmic Model

A simple way of deriving Fitts' law is described in [1], and is based on discrete-step moves of the pointer to reach towards the center of the target. In each step, the pointer moves towards the center of the target and takes same amount of time. The subsequent moves end inside the center (See the Fig. 4). The derivation of the Fitts' law is as follows,

Let, the distance between pointer and center of target be A, and in each move, it shifts λ part of the remaining distance. The width of the target be W, and hence,

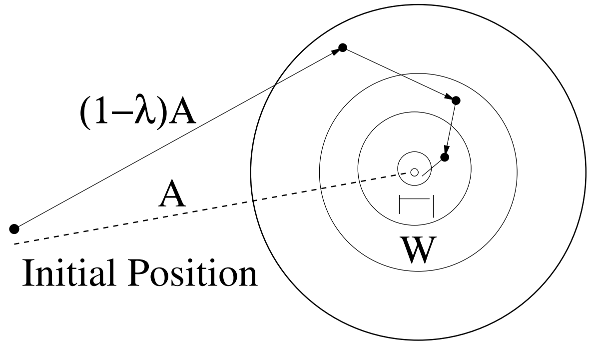

Fig. 4: Logarithmic Model

$$A_{i+1} = \lambda A_i$$
$$=> A_i = \lambda^i A \tag{2}$$

Let n be the number of steps required to touch the target. Then,

$$A_n = \lambda^n A = W/2$$
$$=> n = log_2(\frac{W}{2A})/log_2(\lambda) \tag{3}$$

Let each step takes τ time and a be the initial time to be started. Then,

$$T = a + \tau.n$$
$$T = a + blog_2(\frac{2A}{W}) \tag{4}$$

where $b = -\frac{\tau}{log_2(\lambda)}$. The variations of the formula can derived as follows,

By factoring out 2, we write, $T = a^{'} + blog_2(\frac{A}{W})$, where $a^{'} = a + b$. Similarly, we write, $A_c = A_e + W/2$, then, $T = a^{'} + blog_2(\frac{A_e}{W} + 0.5)$. Another formula was proposed for Fitts' law based on accuracy of the statistical results, and fitted in a similar way as *Shannon's theory* of Information. This can be written as, $T = a + blog_2(\frac{A}{W} + 1)$.

B. Square-Root model

The *Square-Root* model is based on control theory where the optimal time required to reach a target assumes a maximal acceleration in the first half of the distance, followed by maximal deceleration for the next half of the pointer. So based on this, the switching between acceleration to deceleration happens at a distance of, $x_{mid} = A/2$. Another assumption is about the acceleration which is proportional to the width of the target, i.e, acceleration, say, $\ddot{x} \propto W$, where $\ddot{x}$ be the double derivative of distance x, and W be the width of the target.

1) For the fist-half: As the pointer is at stand still initially, the initial velocity is zero. using equations of static motion of equations (final_velocity=Initial_velocity+acceleration×time), we can write, $\dot{x}(t) = \ddot{x}t$. The distance traversed by the pointer in time t,

$$\begin{aligned} distance = velocity \times time + (1/2) \times acceleration \times time^2 \\ \frac{A}{2} = (1/2)\ddot{x}t^2 \\ t = \sqrt{\frac{A}{\ddot{x}}} \end{aligned} \tag{5}$$

2) For the second-half: As the pointer remains stopped once it reached to the target, the final velocity is zero. Let $t^{'}$ be the time required for the next-half of the distance. We can write as,

$$\begin{aligned} final\_velocity = Initial\_velocity + acceleration \times time \\ 0 = \ddot{x} \times t - \ddot{x} \times t^{'} \\ t^{'} = t \end{aligned} \tag{6}$$

The above shows switching happens at a time of $T/2$, where T be the total time required to reach at the target. So we can write,

$$T = 2\sqrt{\frac{A}{\ddot{x}}} \tag{7}$$

From the assumption of W and $\ddot{x}$, we can write, $\ddot{x} = kW$, where k is a constant scalar. Finally, we can deduce the total time $T^{'}$ as initial time a, plus movement time T,

$$T^{'} = a + T = a + 2\sqrt{\frac{A}{kW}} = a + b\sqrt{\frac{A}{W}} \tag{8}$$

where $b = \frac{2}{\sqrt{k}}$.

III. Applications of Fitts' law

A. Evaluation of Eye Gaze Interaction

In this paper [2], they have study about the eye gaze versus mouse selection. So, they have conducted two experiments, eye gaze selection technique and select with mouse and compared the selection time. Finally, they found that eye gaze selection is faster than selecting with mouse.

They also analyzed the data from his circle experiment and found that eye movement-based technique follows the Fitts law.

B. Application of Fitts law to eye gaze interaction

In this experiment [3], they have compared the performance of an eye tracker and a mouse in simple pointing task. They have also found that Fitts law model gives as good fit to the data for eye movement. But they have found the result which is opposing the result of Evaluation of Eye Gaze Interaction in terms of the speed of performance. The possible reason for this slowdown in performance may be a greater degree of response damping introduced by the adaptive averager.

TABLE I: Keymap for chording on a mobile phone keypad

Character Key	$None$	$*$	0	$\#$	1
$None$				-	1
2	a	b	c		2
3	d	e	F		3
4	g	h	i		4
5	j	k	l		5
6	m	n	o		6
7	p	q	r	s	7
8	t	u	v		8
9	w	x	y	z	9

C. Children with congenital spastic hemiplegia obey Fitts Law in a visually guided tapping task

Generally Fitts law is found to apply to motor tasks involving precise aiming movements. Children with cerebral palsy (CP) in such task like movement speed and accuracy of goal directed arm movements on their affected side. So, they have performed some experiment to find that for Children with cerebral palsy (CP) obey Fitts law or not. And they have found three major results.

First is that movements made with the hemiplegic hand have longer movement time which led to a higher Y-intercept a in the Fitts equation. These described to a loss in force generation.

Second is that the loglinear relationship between the amplitude of the movement and target width also held for both hands of children with congenital spastic hemiplegia and shows that children with congenital spastic hemiplegia obey Fitts Law in easy tapping tasks.

And third is that spasticity does not seem to be an important explanatory factor for movement speed or accuracy, since no signicant correlations emerged between the spasticity measures and any of the kinematic variables. The only signicant correlation between clinical and kinematic variables indicated that if the active wrist excursion is more constrained, children with congenital spastic hemiplegia need more time to make a goal directed movement.

D. A Model of Two-Thumb Chording on a Phone Keypad

In this paper [4], they have presented a method and model of text entry for a standard 12-key mobile phone keypad. This model is composed of a keypad model, a Fitts Law-based timing model, and a behavioral model of an expert user. They have also found that this model is stable under large variations to the input parameters.

TABLE II: X, Y positions for a digitized Motorola RAZR keypad

	X Pos.		
Y Pos.	5 0	17 3	29 2
255	1	2	3
315	4	5	6
375	7	8	9
435	*	0	#

E. Touch a Screen or Turn a Knob: Choosing the Best Device for the Job

Users can interact with system through input devices. In this paper they have conducted two experiments to find whether and how task demands and user age inuenced task performance for a direct input device (touch screen) and an indirect input device (rotary encoder).

IV. Conclusion

This paper has introduced a survey on different formula for finding the movement time, targeting an object. We extensively studied on different ways of formulating movement time. One famous formula called Fitts' law was also well studied, and along with its variations. We also studied different applications of Fitts' law in HCI.

References

[1] H. Drewes, "Only one fitts' law formula please!" in *CHI '10 Extended Abstracts on Human Factors in Computing Systems*, ser. CHI EA '10. New York, NY, USA: ACM, 2010, pp. 2813–2822. [Online]. Available: http://doi.acm.org/10.1145/1753846.1753867

[2] L. E. Sibert and R. J. K. Jacob, "Evaluation of eye gaze interaction," in *Proceedings of the SIGCHI conference on Human Factors in Computing Systems*, ser. CHI '00. New York, NY, USA: ACM, 2000, pp. 281–288. [Online]. Available: http://doi.acm.org/10.1145/332040.332445

[3] D. Miniotas, "Application of fitts' law to eye gaze interaction," in *CHI '00 Extended Abstracts on Human Factors in Computing Systems*, ser. CHI EA '00. New York, NY, USA: ACM, 2000, pp. 339–340. [Online]. Available: http://doi.acm.org/10.1145/633292.633496

[4] N. Patel, J. Clawson, and T. Starner, "A model of two-thumb chording on a phone keypad," in *Proceedings of the 11th International Conference on Human-Computer Interaction with Mobile Devices and Services*, ser. MobileHCI '09. New York, NY, USA: ACM, 2009, pp. 8:1–8:4. [Online]. Available: http://doi.acm.org/10.1145/1613858.1613869