



# Shipping Optimization Team

## Take-Home Test

### Instructions

- It is strongly advised that you write down any additional hypothesis that you feel is necessary for a good understating of your answers.
- You can answer this test in either English, Portuguese or Spanish.

### Challenge: Bicycles Relocation

#### Introduction

RentalBike is a bike sharing company specialized in short-term bicycle rentals along various areas of a city, offering different categories of bicycles. Clients may rent bicycles for any duration and may deliver the rented bicycles in a place different from where the rental started. However, the delivery place must be in one of the areas currently operated by RentalBike. Since clients are able to change the bicycles' original position, RentalBike must often redistribute the surplus of bicycles according to the expected demand of each area.

The decision of how many bicycles will be redistributed from one area to another is based on expected profits for each possible volume of bicycles to be moved. Therefore, before starting the relocation of bicycles, it is necessary to optimize the volume of bicycles to be moved from one area to the others.

The following sections describe the problem to be solved. Read the input descriptions and assumptions before doing the tasks.

#### Input description

Given as input

- a surplus of bicycles of different categories in the source area;
- constants for the space occupied by one bicycle of each category in the truck used for relocating different categories of bicycles;
- and the expected profits for relocating bicycles from the source area to any other area.

Your challenge is to determine how many bicycles of each category should be moved to each area so that the sum of expected profit is maximized.

## Assumptions

1. The expected profits are constants regarding each unit of bicycles that can be relocated to other areas. Therefore, it is a problem of deterministic optimization;
2. In this challenge, **only one area is the source of bicycles**. Therefore, there will be only relocations starting from this area.

The input is sourced as spreadsheets (**all in only one workbook file named BicyclesRelocationData.xlsx**), described below:

- A spreadsheet named “**Categories**” has each bicycle category as header, and respectively, **two rows** of data:
  1. the surplus of bicycles available in the source area for each category;
  2. and the constant for how much space each bicycle category occupies in the truck. As an example, if a bicycle category named “Child” has constant 1 and the category “Adult” has constant 1.5, then an “Adult” bicycle occupies 50% more space in the truck than a “Child” bicycle.
- **Spreadsheets with names starting with “ExpectedProfitsArea”** and ending with a number. These numbers indicate which destination area is the spreadsheet describing. Each of these spreadsheets has all bicycle categories as header and various rows of data. Each row presents how much profit it is expected to be earned by sending one **additional** bicycle to that destination area.

Characteristics of the expected profit:

- **The expected profits data is not accumulated.** As an example, the expected profit of the second rental is only about the second rental. If you need the expected profits, for example, for renting 15 bicycles of a category in a specific area, it is needed to sum all the first 15 values regarding this bicycle category and area;
- Since the bike sharing demand varies according to different factors, the profit earned by the first rental of a bicycle may be greater or lower than the profit earned by the second rental of the same bicycle in the same region;
- **The expected profit for relocating the  $n^{\text{th}}$  bicycle is only earned if all earlier bicycles of this same category and area are rented.** In other words, the 30<sup>th</sup> expected profit of a bicycle category and area is only earned if its earlier 29 bicycles were already relocated.
- **The only property the expected profit values have is that they are non-negative numbers.** Therefore, it is not possible to make other assumptions about the series of expected profits, such as linear or convex growth;

- The length of the expected profits data may be different than the surplus of bicycles of each category. In the case of not having expected profits data for all surplus bicycles, **the profit for relocating more bicycles than the category surplus must be considered as zero.**

This challenge is divided into minor tasks, described as follows.

## Tasks

1. Formulate and solve the optimization problem
  - a. Write a mathematical formulation for solving this optimization problem, considering that the total space capacity of the truck is represented by a constant  $T$ . **Define the constants and variables, and describe the constraints and objective function** to maximize the sum of expected profits.
  - b. Choose a programming language and a solver (such as CBC or GLPK) for optimizing the formulation that you propose at task 1 a). **Considering the capacity of the truck as  $T = 80$ .** Solve the problem and show your code, the optimal solution, and the optimal objective value.
  - c. Now the company RentalBike wants to evaluate the impact in the objective function by using a truck with different capacity. **Variate the capacity  $T$  from 100 to 4000, with steps of 100** (i.e.  $T = [100, 200, 300, \dots, 3900, 4000]$ ) and re-run your optimization model for each of these parameters. Plot a graph with the optimal objective values gotten for each  $T$ . The graph must show the capacity  $T$  as x-axis and the objective values as y-axis.
2. Solving with algorithms
  - a. Going back to task 1, imagine the same problem **but without the constraint(s) regarding the truck's capacity**. By doing so, the optimization may assume that the truck has an unlimited capacity and ignore checking it. Knowing this, implement an algorithm that finds a good solution for this problem. It is preferred (but not mandatory) that your algorithm always finds an optimal solution, having the lowest time and space complexity as possible. Show your code, the best found solution, and its objective value.
  - b. What is the time complexity of your algorithm? What is the auxiliary space complexity of your algorithm?

- c. The algorithm you proposed in letter a) will always find an optimal solution, regardless of the input? How?
  - d. By analyzing the input data, which preprocessing task can be made to this instance in order to make it faster or lighter to be solved?
3. Going further on task 2 (problem without the capacity constraint). **Here it is not necessary to code, only explain your answers**
- a. How can parallel processing be applied for reducing the run time of solving this problem?
  - b. If it is assured that the number of bicycles to be redistributed will always be at least 90% of the expected demand for all areas, can you adapt your algorithm to use this property for running faster?
  - c. Now the company RentalBike wants to avoid relocating to an area that is not prioritized, unless other regions are reasonably served. What must be changed or included in the formulation you proposed, at Task 1 a), to assure that the area number 1 will only receive a bicycle if at least 85% of the surplus of each category is already allocated to other areas?
  - d. Now RentalBike wants to balance the profits among areas. What must be changed in your formulation to maximize the expected profits of the region with lowest expected profits, instead of simply maximizing the summation of profits?
4. Going further on business. **Here it is not necessary to code, only explain your answers**
- a. Which other analysis, optimizations or policies do you suggest to RentalBike that could bring business impact?