Problem Solving Task 2.1
Forward and Inverse Kinematics of Manipulators

[ENN581] Robot Motion, Control and Planning

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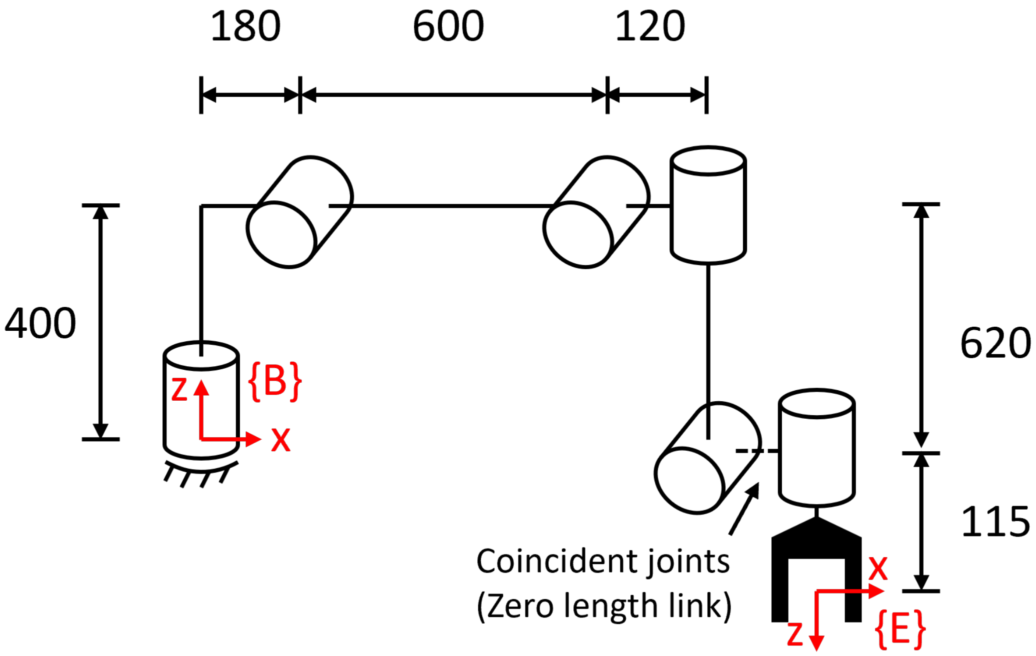

Figure 1: Robot Arm Schematic

1 What is the configuration space of this manipulator?

The 6DOF robot in Figure 1 has the following configuration space, assuming no collision:

$$\mathcal{C} \subset \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$$

$$(q_0, q_1, q_2, q_3, q_4, q_5) \in \mathcal{C} \text{ where } q_i \in [0, 2\pi)$$

2 Determine the kinematic relation between the end-effector frame {E} and the base frame {B}

Following the schematic in Figure 1, the ETS can easily be attained from a series of Elementary Transforms. A convention of joint rotation around z is used. The steps for each are outlined below, resulting in the ETS table in Table 1:

1. ${}^{B}T_0$ **BASE to joint0:** For the first joint, there is no translation or rotation required, as the joint rotates around the base z-axis at angle q_0
2. ${}^{0}T_1$ **joint0 to joint1:** From the first joint, there is a vertical translation of 400 along z, followed by a horizontal translation along x of 180. As this currently rotates around the y-axis, a rotation around x of 90 deg is applied, resulting in a new z direction, matching the negative y-axis. This joint then rotates about z_1 at an angle q_1.
3. ${}^{1}T_2$ **joint1 to joint2** The transform from joint1 to joint2 is a simple translation along joint1's x-axis by 600. joint2's rotation about z_2 is represented by q_2
4. ${}^{2}T_3$ **joint2 to joint3** A translation along joint2's x-axis will get the coordinate to the right position, but the orientation should be adjusted to keep the z-axis the axis of rotation. A position rotation about x (90 deg)was chosen to keep the next elementary transform positive. The result is a joint rotation about z_3 of q_3.
5. ${}^{3}T_4$ **joint3 to joint4** With the axis of rotation of joint3 being in-line with the link, the next transform is a simple positive-z translation by 620, followed by a -90 deg rotation about x to get the z-axis "out-of-page" again.

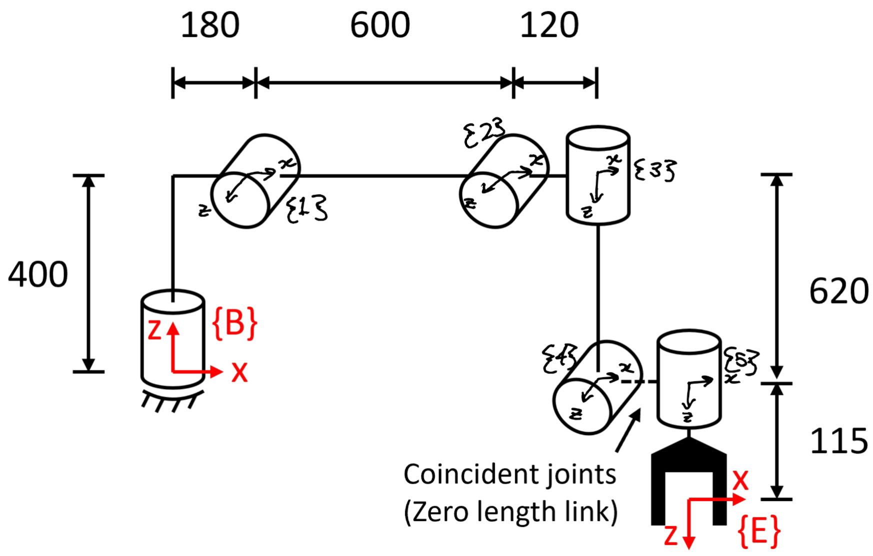

Figure 2: Coordinate Frame of Robot Joints

6. ${}^{4}T_5$ **joint4 to joint5** With the coincident joint, there is no translation required. A simple 90 deg rotation about x is needed to realign the coordinate frame with the next link in the chain and get a rotation q_4 about z_4
7. ${}^{5}T_E$ **joint5 to end-effector** The final transform to the end effector is a simple translation along z_4 to get the final coordinate frame.

link	link	joint	parent	ETS: parent to link
0	link0	0	BASE	$R_z(q_0)$
1	link1	1	link_1	$t_z(400) \oplus t_x(180) \oplus R_x(90\,\text{deg}) \oplus R_z(q_1)$
2	link2	2	link1	$t_x(600) \oplus R_z(q_2)$
3	link3	3	link2	$t_x(120) \oplus Rx(90\,\text{deg}) \oplus Rz(q_3)$
4	link4	4	link3	$t_z(620) \oplus R_x(-90\,\text{deg}) \oplus R_z(q_4)$
5	link5	5	link4	$R_x(90\,\text{deg}) \oplus R_z(q_5)$
6	@link6		link5	$tz(115)$

Table 1: ETS Table

5 Does the inverse kinematics problem have a closed-form solution? Is the solution unique?

The existence of a closed-form solution depends on the desired end-effector pose. If the desired pose, $\mathcal{X}$, lies within the dexterous workspace $\mathcal{W}_{\text{dex}}$, a solution exists. If there are no orientation constraints, $\mathcal{X}$ can lie anywhere within the robot's reachable workspace, $\mathcal{W}_{\text{reach}}$.
Given the last three joints can be modelled as a spherical joint, there will be a unique solution for ${}^{3}T_E$ (3 DOF to describe each angle of the desired $\mathcal{X}$), and, once q_0 is set correctly, the resulting problem becomes a planar RR to the desired $\{3\}$, which can have 2 solutions, depending on $\mathcal{X}$. Solving for position alone results in infinite number of solutions, as only 3 DOF is required for that solution, particularly when considering the continuous rotations about q_5. Therefore, the solution is not necessarily unique.

6 Give a sequence of steps to solve the inverse kinematics problem ie. given ${}^{B}T_E$, determine q_i.

The motion imposed by the end effector by joint angles q_3, q_4, q_5 can be modelled as a spherical wrist, due to the axial rotation of q_3 being applicable anywhere along the link (though physically it requires an interface per the schematic). This means the angles associated can be extracted using ZYZ RPY angles. The following steps can be followed to solve for the inverse kinematics solution analytically:

1. Solve for q_3, q_4, q_5 by extracting RPY angles from desired end effector pose, $\mathcal{X}$
2. Invert ${}^3T_E$ to find ${}^ET_3$, and use the resulting coordinate frame as the new goal position, $\mathcal{X}'$, for finding q_0, q_1, q_2. This transformation accounts for the additional translation of link3 that the spherical wrist model would otherwise not.
3. Find the angle q_0 required to create the plane containing joint2, joint3, $\mathcal{X}'$. This simplifies the problem to an RR planar robot.
4. Solve for $q2$, $q3$ per the planar RR case (see https://robotacademy.net.au/lesson/inverse-kinematics-for-a-2-joint-robot-arm-using-geometry/)