Problem Solving Task 2.4

Dynamics

[ENN581] Robot Motion, Control and Planning

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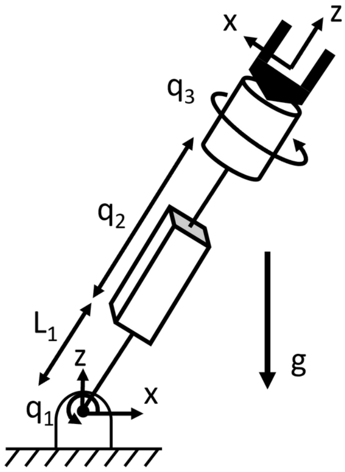

Figure 1: Robot Arm Schematic

1 Derive equations for the joint torques (or forces) in terms of the joint positions, velocities, and accelerations

The following derivation was found by appling the Lagrangian memthod, and implemented in MATLAB to simplify:

$$
\begin{aligned}
x_1 &= L_1\cos(q_1) & \dot{x}_1 &= -L_1\sin(q_1)\dot{q}_1 \\
y_1 &= L_1\sin(q_1) & \dot{y}_1 &= -L_1\cos(q_1)\dot{q}_1 \\
th_1 &= 0 & \dot{th}_1 &= 0 \\
\\
x_2 &= x_1 + q_2\cos(q_1) & \dot{x}_2 &= \dot{x}_1 + \dot{q}_2\cos(q_1) - q_2\sin(q1)\dot{q}_1 \\
y_2 &= y_1 + q_2\sin(q_1) & \dot{y}_2 &= \dot{y}_1 + \dot{q}_2\sin(q_1) - q_2\cos(q1)\dot{q}_1 \\
th_2 &= 0 & \dot{th}_2 &= 0 \\
\\
x_3 &= x_2 & \dot{x}_3 &= \dot{x}_2 \\
y_3 &= y_2 & \dot{y}_3 &= \dot{y}_2 \\
th_3 &= q_3 & \dot{th}_3 &= \dot{q}_3 \\
\\
\kappa_1 &= \frac{1}{2}m_1(x_1^2 + y_1^2) & P_1 &= m_1 y_1 g \\
\kappa_2 &= \frac{1}{2}m_2(x_2^2 + y_2^2) & P_2 &= m_2 y_2 g \\
\kappa_3 &= \frac{1}{2}m_3(x_3^2 + y_3^2) + \frac{1}{2}m_3R^2\dot{th}_3 & P_3 &= m_3 y_3 g
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}(q,\dot{q}) &= \sum_{i=1}^{3}(\kappa_i - P_i) \\
\tau_1 &= \frac{d}{dt}\left(\frac{\partial\mathcal{L}(q,\dot{q})}{\partial\dot{q}_1}\right) - \frac{\partial\mathcal{L}(q,\dot{q})}{\partial q_1} \\
\tau_2 &= \frac{d}{dt}\left(\frac{\partial\mathcal{L}(q,\dot{q})}{\partial\dot{q}_2}\right) - \frac{\partial\mathcal{L}(q,\dot{q})}{\partial q_2}
\end{aligned}
$$

Using `lagrange.m`, the following joint torque equations were derived:

$$\tau_1 = m_2 q_2^2 \ddot{q}_1 + m_3 q_2^2 \ddot{q}_1 + m_1 L_1^2 \ddot{q}_1 + m_2 L_1^2 \ddot{q}_1 + m_3 L_1^2 \ddot{q}_1 + 2m_2 q_2 \dot{q}_1 \dot{q}_2 + 2m_3 q_2 \dot{q}_1 \dot{q}_2 + 2m_2 L_1 q_2 \ddot{q}_1 + 2m_3 L_1 q_2 \ddot{q}_1$$
$$+ m_2 g \cos(q_1) q_2 + m_3 g \cos(q_1) q_2 + 2m_2 L_1 \dot{q}_1 \dot{q}_2 + 2m_3 L_1 \dot{q}_1 \dot{q}_2 + m_1 g L_1 \cos(q_1) + m_2 g L_1 \cos(q_1) + m_3 g L_1 \cos(q_1)$$
$$\tau_2 = m_2 \ddot{q}_2 + m_3 \ddot{q}_2 - m_2 L_1 \dot{q}_1^2 - m_3 L_1 \dot{q}_q^2 + m_2 g \sin(q_1) + m_3 g \sin(q_1) - m_2 q_2 \dot{q}_1^2 - m_3 q_2 \dot{q}_1^2$$
$$\tau_3 = 0$$

2 Determine the maximum torque (or force) in the joint during the motion of the manipulator. At what joint position does this occur?

Assuming no friction, given the mass, m_3, is a point mass, there is no work required to rotate it and thus experiences no torque, remains 0 for the entire trajectory. The other joints are as follows:

2.0.1 Joint 1

$$\tau_{1\text{max}} = 17.2211 \text{ Nm}$$
$$t = 0.66\text{s}$$
$$q = \begin{bmatrix} 0.3832 \text{ rad} \\ 0.3464 \text{ m} \\ 0.5748 \text{ rad} \end{bmatrix}$$

2.0.2 Joint 2

$$\tau_{2\text{max}} = 9.81 \text{ N}$$
$$t = 1\text{s}$$
$$q = \begin{bmatrix} 0.5236 \text{ rad} \\ 0.4 \text{ m} \\ 0.7854 \text{ rad} \end{bmatrix}$$