Problem Solving Task

[EGH586] Decisions and Control

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1 Task 1: System Identification

1.1 Data Generation

In Figure 1, results of the noisy measurement sequence generator is shown. It shows an appropriate generation process, with random sampling for the measurements centered around the constant ground truth values.

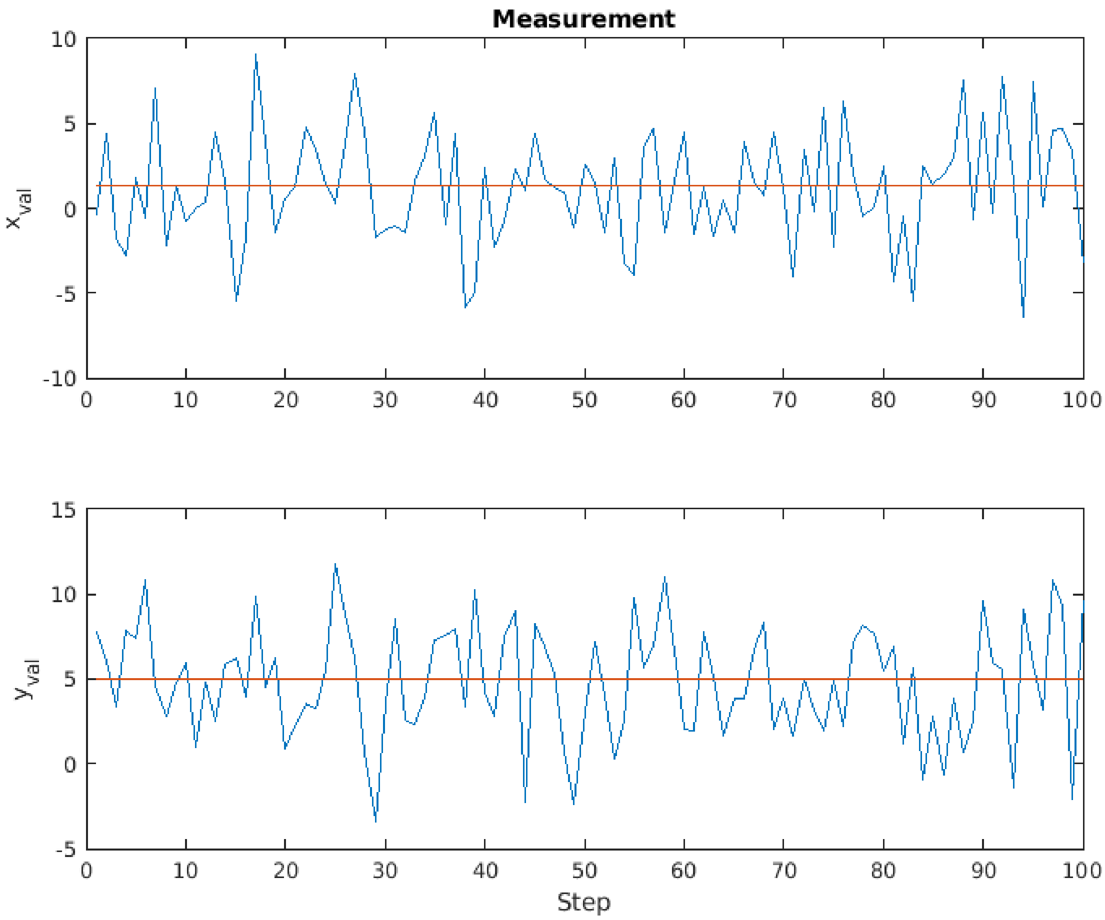


Figure 1: (1.1) Noisy Measurement Generation

1.2 Recursive Estimation

Figure 2 shows a simple parameter estimation in aciton. Despite the variance found within the noise in Figure 1, the estimated values maintain relatively low error after a short adjustment period.

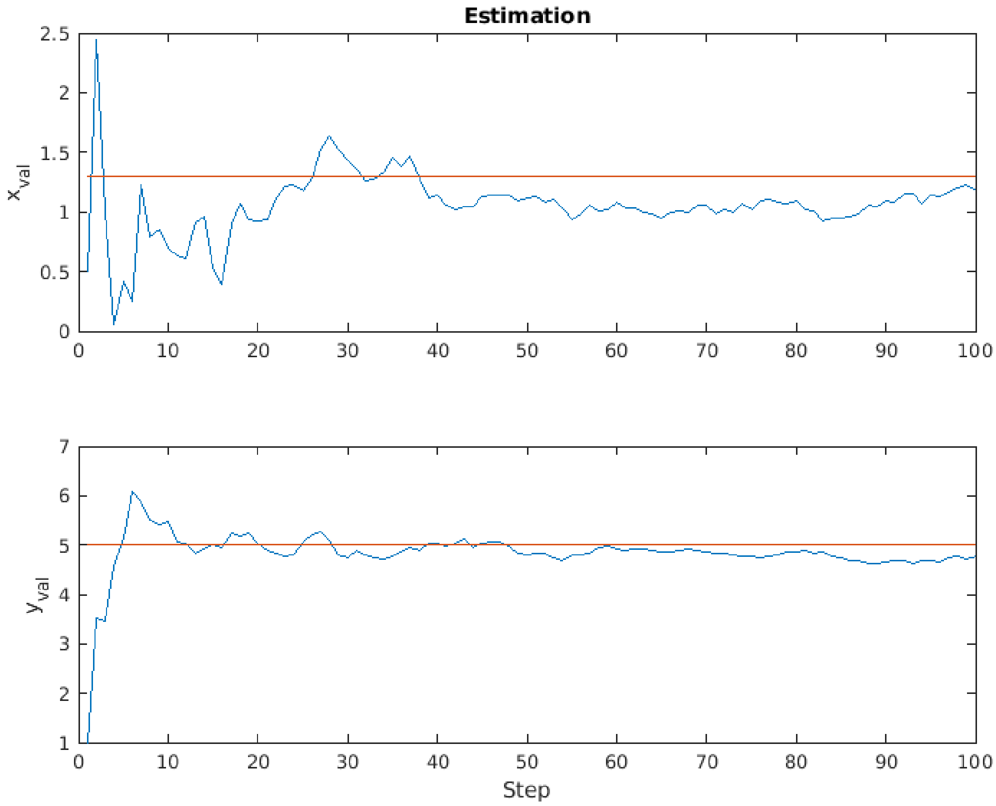


Figure 2: (1.2) Recursive Estimation

The error can be seen to quickly converge to a reasonably low error, shown in Figure 3. As reflected in Figure 2, the x error appears to take longer to settle, finding a value around step 20, though seemingly has a smaller amplitude. The y error, on the other hand, converges quicker, at around step 10.

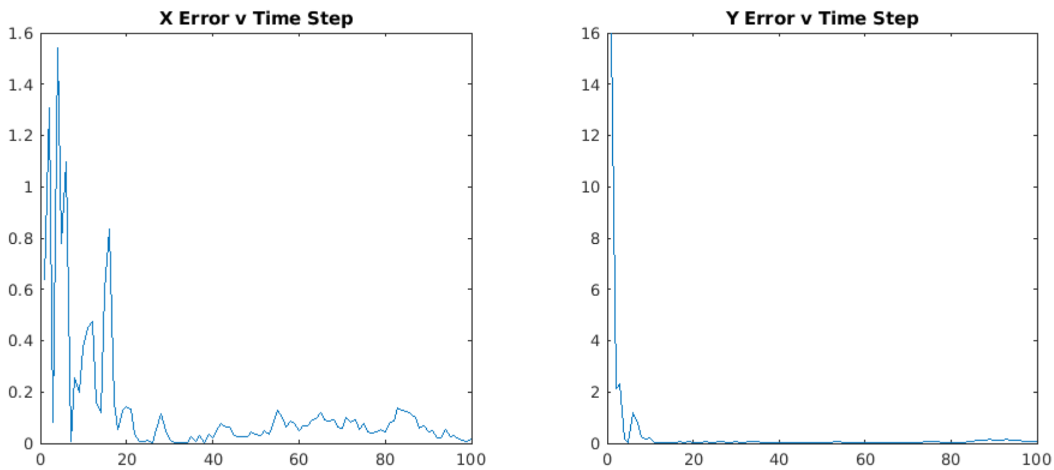


Figure 3: (1.2)RMSE

1.3 Parametric Study (Results)

		R Val			
		0.1	1	10	100
C_1 Val	[0.5, 1]	[0.1492, 0.5028]	[0.2436, 0.5849]	[0.4693, 0.8838]	[1.7415, 1.5160]
	[1.3, 5]	[0.0929, 0.0825]	[0.1649, 0.2214]	[0.5661, 1.0815]	[2.1498, 1.8979]
	[100, 100]	[12.6040, 12.1566]	[12.4606, 12.1967]	[12.7574, 12.0957]	[11.5909, 12.9027]

Table 1: (1.3) RMS values of different R / C_1 combinations

1.4 Parametric Study (Analysis)

(a) What is the role of R and/or $\hat{C}_1$ on estimation error?

As R defines the variance of the noise in the measurement, a larger value of R results in poorer identification performance, and a higher error. It can be seen in Table 3 that there is a certain robustness with small error differences between 0.1 $\rightarrow$ 10, despite the magnitude increase, but there is still a point which performance cannot be sustained, as reflected in the transition 10 $\rightarrow$ 100.

$\hat{C}_1$, being the initial estimate, can impact the time to convergence and overall performance of the identification. In the extreme case, we can see that even with low noise the RMS is excessively high. This is the nature of RMS, however, as it is an average of the available points, and the identification cannot converge quick enough to mitigate the effects of the distant initial estimate. Figure 4 shows that, from about step 70, the error approaches error values comparable to some of the noisier sequences, but doesn't quite recover to the same order as equivalent sequences with closer initial estimates.

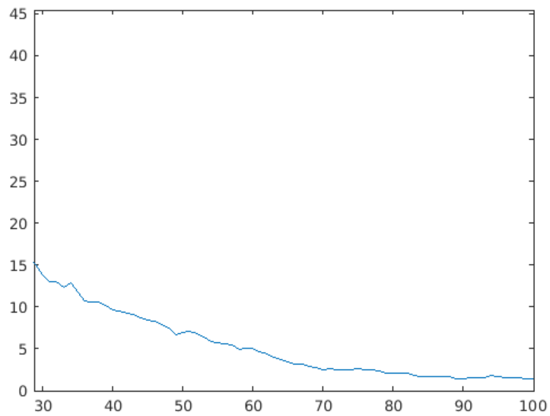


Figure 4: End of Error vs Step plot, with $\hat{C}_1$=[100,100], R=10

(b) What might happen if you had a longer/shorter data sequence?

With a longer data sequence, it would be expected that there would be some improvement over the RMS longer-term, but the performance is ultimately limited by the variance in measurement noise. Figure 5 shows that even in the large distance initial estimate case, the ongoing error is very small.

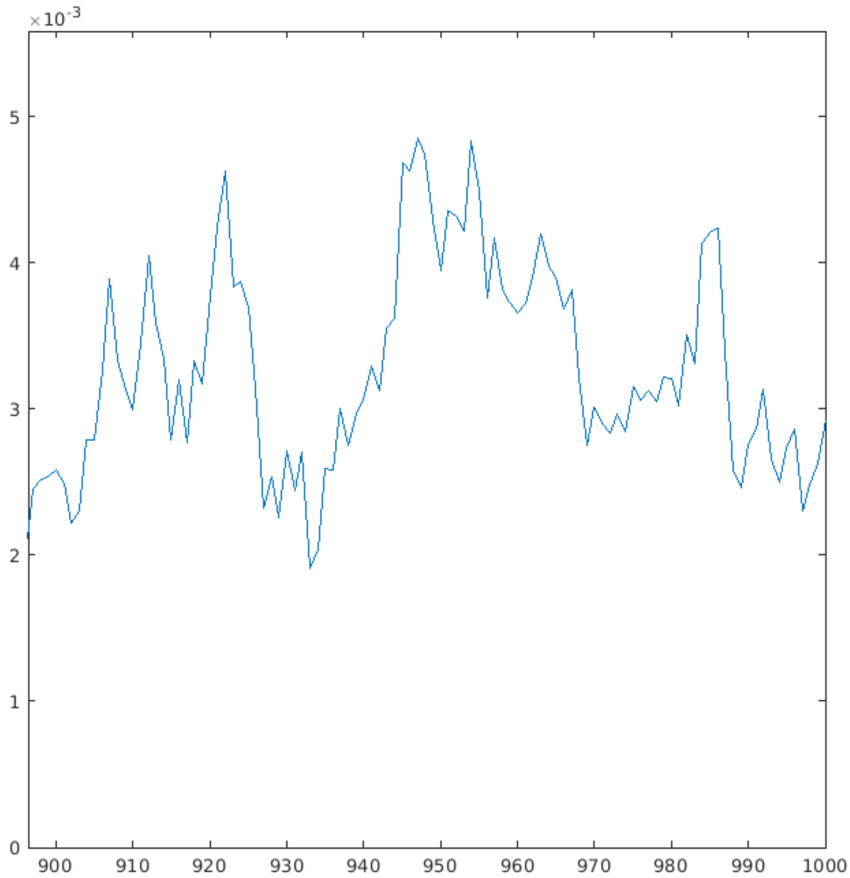


Figure 5: End of Error vs Step plot, with T=1000, $\hat{C}_1$=[100,100], R=10

In the case of a shorter data sequence, the estimate may not have time to converge, and would have significant error. This is exemplified by having $\frac{1}{3}$ the sequence length in the default case. The x-estimate does not converge sufficiently, and the resulting RMS error would be quite large.

(c) Summarise what you discovered (i.e. compare and contrast the impact of $\hat{C}_1$, R and data length on estimation error).

While large distances of $\hat{C}_1$ to the initial estimate can result in huge RMS errors, it doesn't quite have the same inherent impact on error as the sequence length or R value. With a sufficiently large sequence length, any distance can be mitigated long-term. The problem with $\hat{C}_1$ distance arises when constraints are imposed with the data length. The R value, however, will always be a limiting factor of performance, irrespective of the data length.

1.5 Importance of Parameter Estimation

(a) How would you choose to initialise your estimator when/if not provided an initial estimate? Why? What is important in your choice?

Initialising the estimator would depend on the system being identified. Domain knowledge can be helpful, where an assumption can be made about the initial state within reason. For example, calculating based on typical initial conditions, or even randomly sampling over a range of values, defined by reasonable upper and lower bounds, based on the system. This provides a ballpark estimate, and as seen with $\hat{C}_1$, can have any error mitigated under the right conditions. It is important to consider what the typical values are when applying this strategy, to ensure its on the same scale as the true value. While zeroing the estimate can also work, this strategy avoids issues on large nominal values, where a 0-value is atypical, and has a large distance to the true value.

(b) Give an example from previous units that you have studies where you used (or could have used) parameter estimation. Why useful?

Not a unit, but in the development of a micromouse robot it would have been very beneficial to apply parameter estimation. There were many struggles simply getting the robot to drive straight, but with parameter estimation, effects of wheel slip and varying friction could have been accounted for with proper application of parameter estimation.

(c) Give an example related to the context of this unit (i.e. related to autonomous systems) where the need for parameter estimation might arise? Why important?

One example is related to the control of a quadrotor. Even the simple act of hovering requires a suitable dynamics model for varying the speed of the rotors correctly. This model is dependent on a variety of parameters that are unknown, or that vary or degrade over time. To that end, parameter estimation provides information needed to account for this uncertainty, and improve stability. Without it, control behaviour may diverge from the designed controller, potentially creating a safety hazard.

2 Task 2: Kalman Filter

2.1 Data Generation

Figure 6 shows the data generation for a vehicle position, and the corresponding generated GPS measurements. The measurements very closely follow the trend of the ground truth, however, it can be seen that there is some noise which results in some minor error.

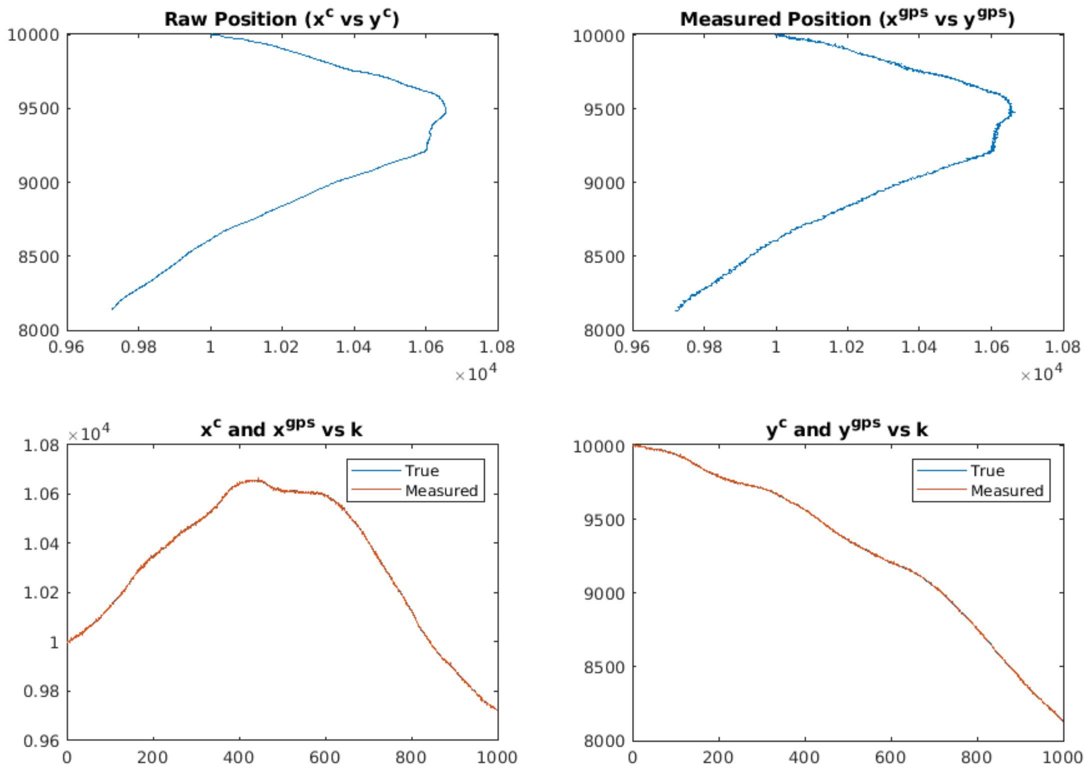


Figure 6: (2.1) Generated and Measured Position Data

2.2 Kalman Filter

Figure 7 shows the x and y values of the ground truth, kalman estimate, and noisy measurement. It is difficult to discern from the full plot alone, so figures 8a and 8b are also provided, revealing the initial conditions of the Kalman estimate, and accuracy respectively. Specifically, its shown in Figure 8a that the Kalman estimate immediately approaches a reasonable estimate, likely on account of a good initial estimate. Figure 8b then exemplifies the accuracy of the Kalman estimate, where it is more representative of the ground truth than the measurements are.

The following RMS value was realised with a seed value of 42:

rms =

3.4752 3.4293 1.6497 1.6273

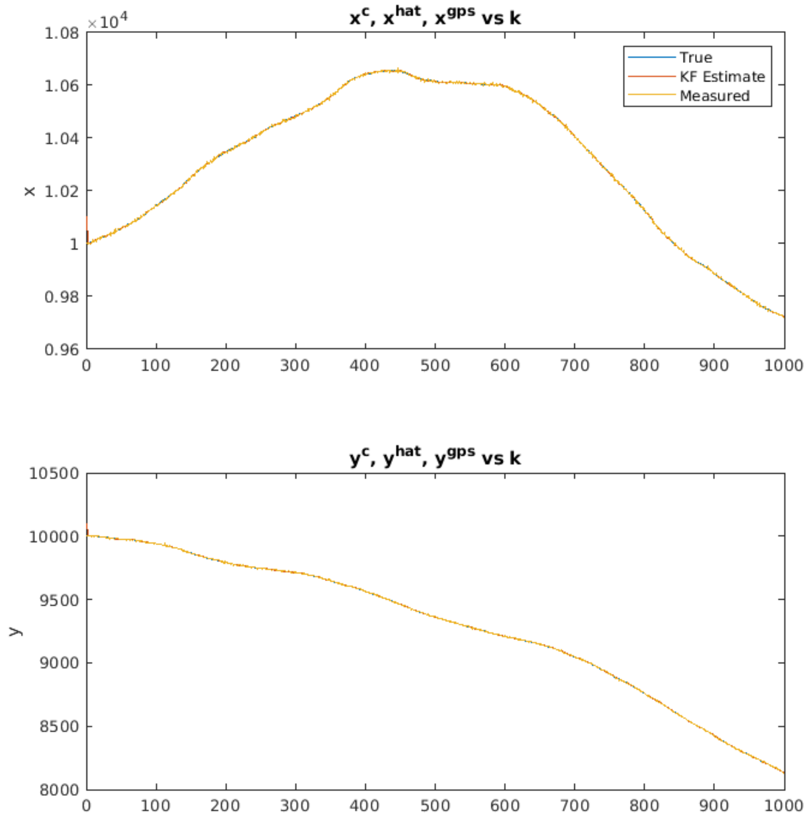
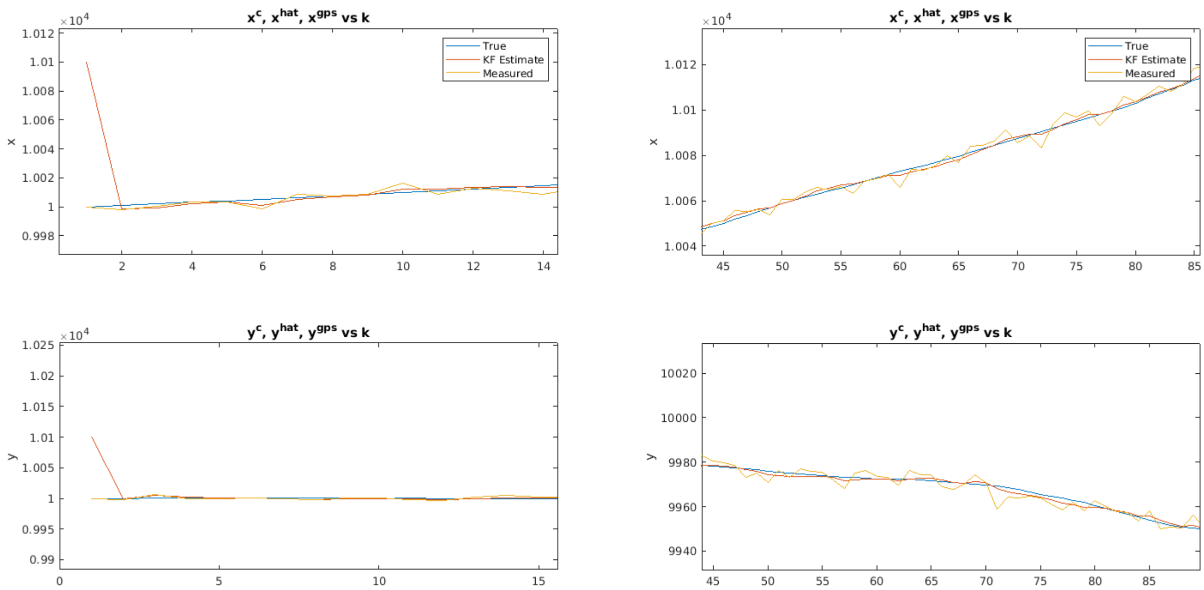


Figure 7: Full XY Position Plot w/ Kalman Estimate



(a) Start of XY Plot

(b) Accurate XY Plot

Figure 8: Finer regions of XY Position plot

2.3 Parametric Study (Results)

		RMS	P_{1000}	K_{1000}
R val	1	[3.215, 3.216, 1.602, 1.6]	[0.441, 0.441, 0.125, 0.125]	[0.361, 0.361, 0.079, 0.079]
	10	[3.475, 3.429, 1.649, 1.627]	[2.504, 2.504, 0.358, 0.358]	[0.131,0.131,0.009,0.009]
	100	[5.142, 5.087, 1.855, 1.929]	[14.124, 14.124, 1.073, 1.073]	[0.222, 0.222, 0.0278, 0.027]
	1000	[9.617, 10.977, 2.197, 2.113]	[79.495, 79.495, 3.290, 3.290]	[0.076, 0.076, 0.003, 0.003]
Q val	0.001	[3.361, 3.417, 1.628, 1.587]	[1.412, 1.412, 0.107, 0.107]	[0.131, 0.131, 0.009, 0.009]
	0.01	[3.475, 3.429, 1.649, 1.627]	[2.504, 2.504, 0.358, 0.358]	[0.222, 0.222, 0.0278, 0.027]
	0.1	[3.688, 3.735, 1.773, 1.684]	[4.416, 4.416, 1.251, 1.251]	[0.361, 0.361, 0.079, 0.079]
	1	[3.957, 3.921, 2.283, 2.318]	[7.644, 7.644, 4.730, 4.730]	[0.553, 0.553, 0.211, 0.211]
P_0 val	10	[3.485, 3.484, 1.676, 1.666]	[2.504, 2.504, 0.358, 0.358]	[0.222, 0.222, 0.027, 0.027]
	100	[3.551, 3.536, 1.697, 1.657]	[2.504, 2.504, 0.358, 0.358]	[0.222, 0.222, 0.027, 0.027]
	1000	[3.475, 3.429, 1.649, 1.627]	[2.504, 2.504, 0.358, 0.358]	[0.222, 0.222, 0.0278, 0.027]
	10000	[3.470, 3.481, 1.603, 1.604]	[2.504, 2.504, 0.358, 0.358]	[0.222, 0.222, 0.027, 0.027]

Table 2: Results of parametric study of R, Q and P_0. Each study uses the default case (defined by Task 2), with only the designated value changed.

2.4 Parametric Study (Analysis)

(a) In terms of RMS, P1000, and K1000, describe what happens for different choices of P0, R and Q? In your answer, include a description of the impact on each of RMS, P1000, and K1000 if P0 increases. Similar, if R increases. Similar, if Q increases.

Choices of the initial error covariance, P_0, does little to impact the overall performance of the filter, especially final value P_{1000}, K_{1000} and RMS, as the filter quickly forgets this initial choice.

With larger values of R, the RMS increases, as do the P_{1000} values, which is expected as the filter will be producing less reasonable estimates. K_{1000} values fluctuate between lower and higher values, though given these represent gain, its not necessarily indicative of performance, other than that low values represent high-noise signals, which is exemplified in the R=1000 case.

Increasing Q results in an upward trend for RMS error and P_{1000}, indicating less accurate estimates. However, K_{1000} values are also increasing, hinting at a rise in quality of measurements.

(b) Describe what happens if incorrect parameters are used in the KF equations. That is, data is generated with (slightly?) different model matrices than those used in the filter implementation. This situation is called model-mismatch and very commonly occurs in realistic problems. Hint: You may need to run a number of extra cases having a model mismatch.

When the model doesn't match, minor issues like degraded performance or delayed convergence can occur. In worse scenarios, the values can diverge completely. This can be harmful as decisions can be made that are destabilising. In Figure 9, an example of model mismatch where the A matrix passed into the Kalman filter model is A + 0.1A. Interestingly, in this scenario the overall trend of the data is maintained, but at an extreme offset.

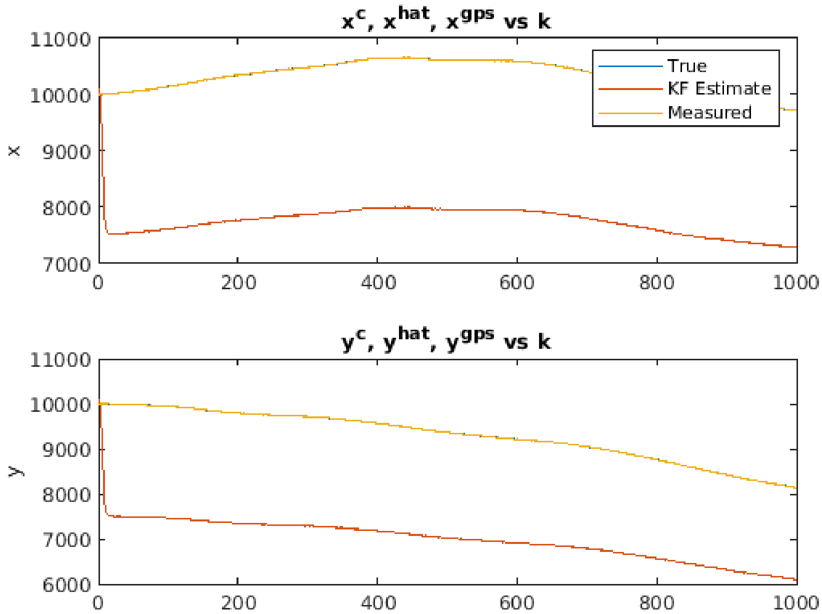


Figure 9: Model Mismatch in Kalman 'A' Matrix

2.5 Kalman Filters Explanation

(a) Give an example from previous units that you have studied where you could have used a Kalman filter. Why would this have been useful/better than what you did?

As with the parameter estimation, the micromouse model is a prime example of where a Kalman filter would have been beneficial. The nature of the Kalman filter enables the use of multiple sources of data at different timings. This would have enabled us to apply wheel encoders, IMUs and IR sensors to create a weighted estimate of the robot's position, as opposed to the unweighted fusion we went with.

(b) Consider a practical situation that has the possibility of model-mismatch (see the previous task for definition). Report what strategies might you use to manage, minimise or avoid the performance impact of model mismatch (this can include the addition of extra processing stages or implementation steps).

Any situation that applies sensors is prone to model-mismatch as a result of sensor degradation or breakages. For example in an INS-based positioning system, the sensor could malfunction, or even function correctly with sufficient drift, resulting in model mismatch. The application of redundant or complementary sensors can mitigate this risk, by cross-validating the data and enabling the filter to assign appropriate gains.

3 Task 3: Extended Kalman Filter

Figures 10 and 11 show the implementation of an Extended Kalman Filter with non-linear measurements. The power of the Kalman filter shines in this example, where, with a good model, almost incoherent, noisy data can provide good approximation in a Kalman estimate. The following implementation, seeded at 42, provided an RMS as follows:

rms =
    35.2595 39.0952 1.3037 1.2991

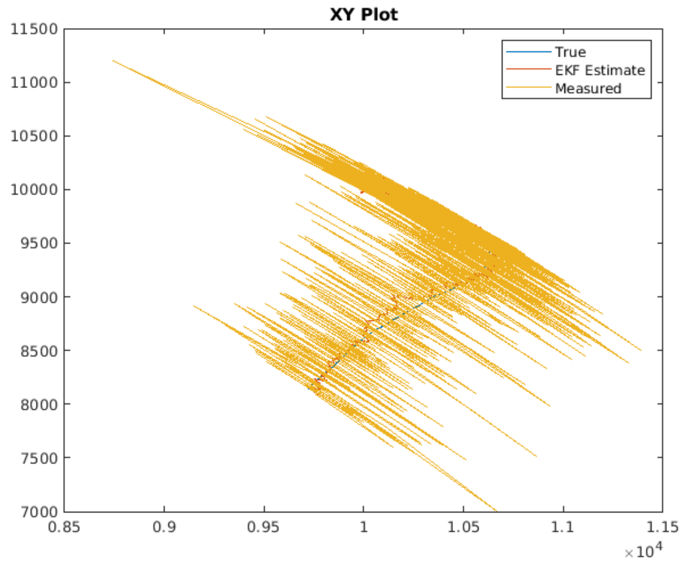


Figure 10: Full XY Position Plot w/ Extended Kalman Estimate

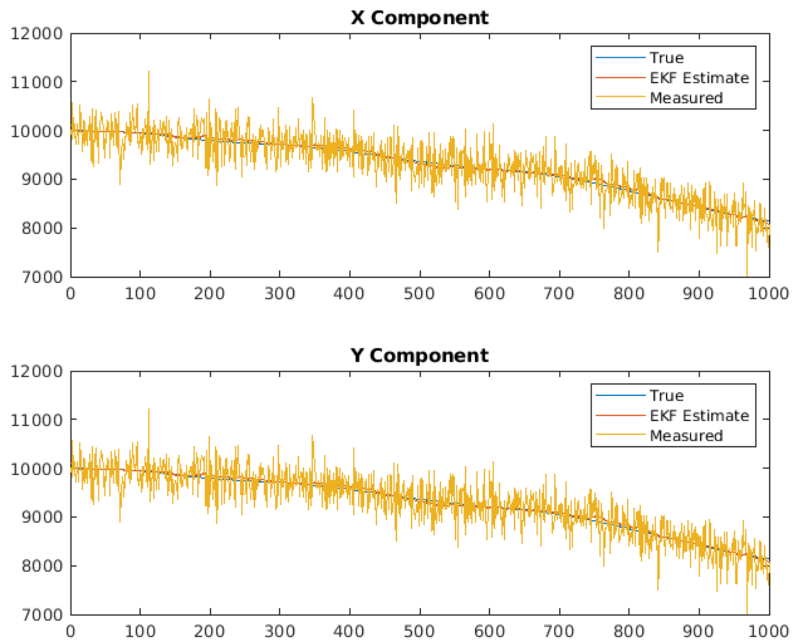


Figure 11: X & Y Components of Extended Kalman Estimate

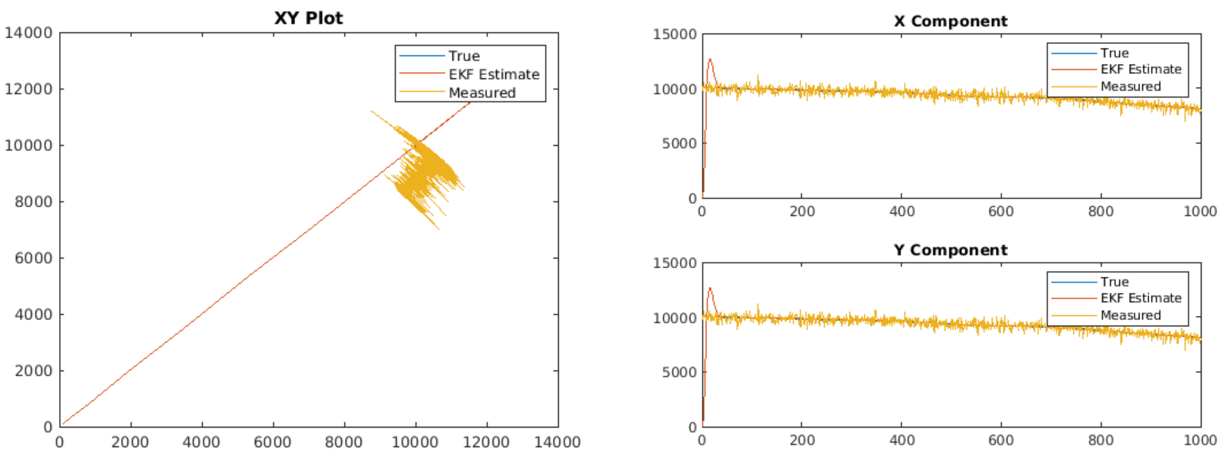
Parametric Study

		$\hat{x}_0$		
		[0, 0, 0, 0]	x_0	$-x_0$
P_0 Val	0.1I	[698.697, 698.552, 118.025, 117.870]	[35.259, 39.095, 1.303, 1.299]	[5.42k, 19.31k, 46, 29]
	100I	[353.820, 352.799, 162.107, 161.796]	[37.243, 40.912, 1.778, 1.787]	[3.67k. 19.22k, 64, 39]
	10000I	[315.498, 315.954, 156.427, 156.408]	[42.234, 45.450, 2.663, 2.991]	[1.901k, 3.590k, 276, 166]

Table 3: (3) RMS values of different $\hat{x}_0$ / P_0 combinations

Zeroed $\hat{x}_0$

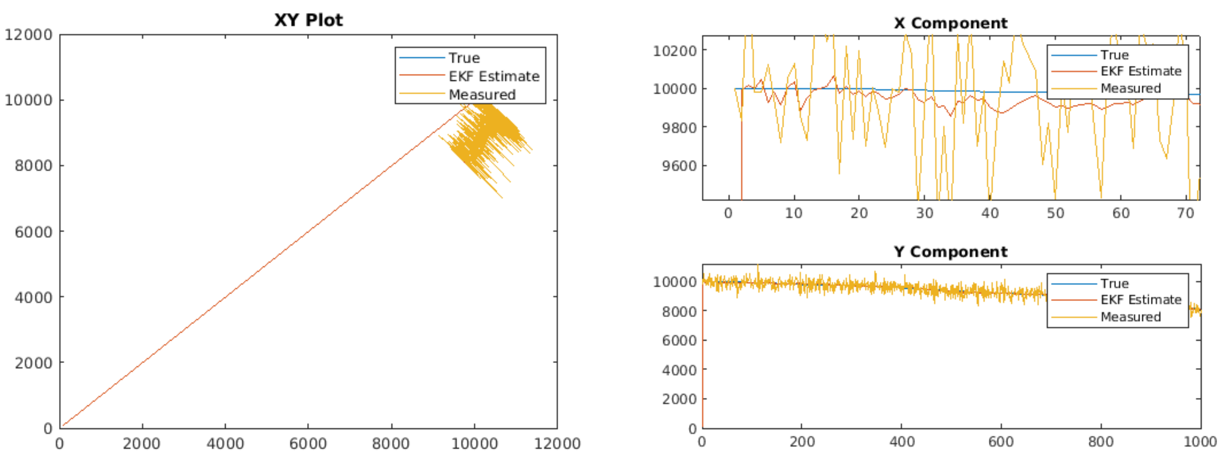
In the zeroed $\hat{x}_0$ case, we expect to see large errors as it readjusts for the significant difference in measurement. With a low P_0, the filter believes its error to be small, resulting in an overall larger error compared to the higher P_0 cases, as there is slower convergence due to overshoot when trying to correct to reasonable estimates.



(a) XY Plot

(b) Component Plot

Figure 12: Zero $\hat{x}_0$, P0=0.1I



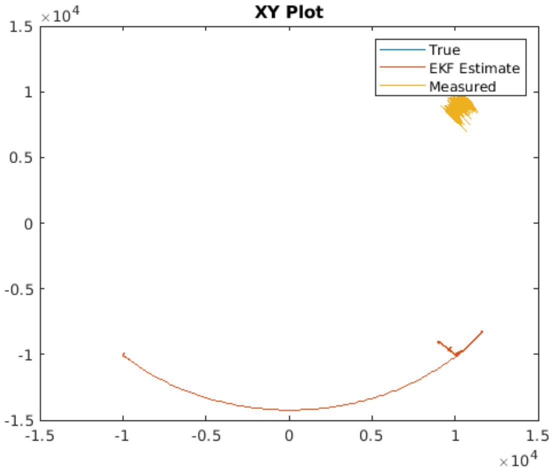
(a) XY Plot

(b) Component Plot

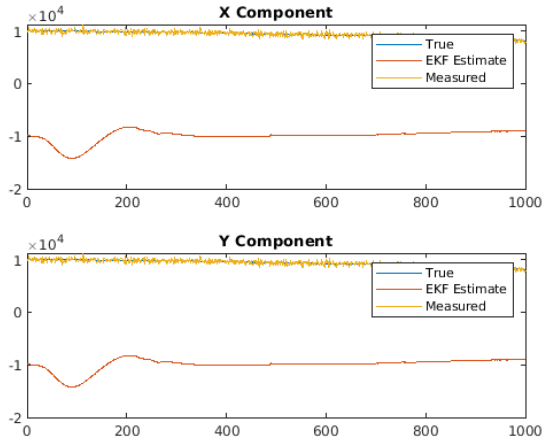
Figure 13: Zero $\hat{x}_0$, P0=10000I

Initial estimate of negative x_0

At twice the distance away from the zero case, the filter is unable to recover effectively. In the small P_0 case, the filter does not recover at all.

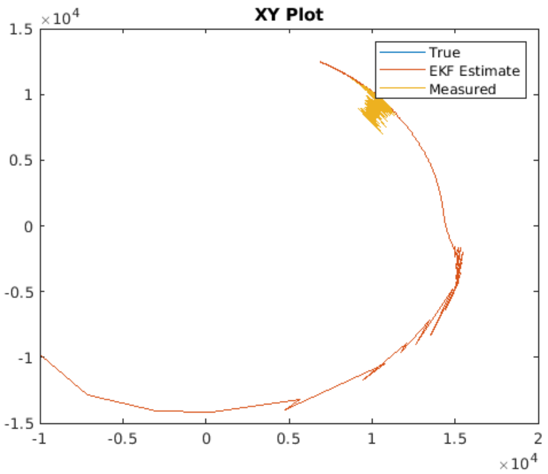


(a) XY Plot

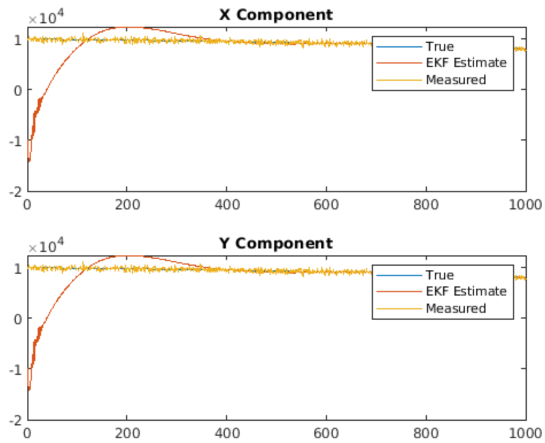


(b) Component Plot

Figure 14: $\hat{x}_0 = -x_0$, P0=0.1I



(a) XY Plot



(b) Component Plot

Figure 15: $\hat{x}_0 = -x_0$, P0=10000I

4 Task 4: Nonlinear Filter

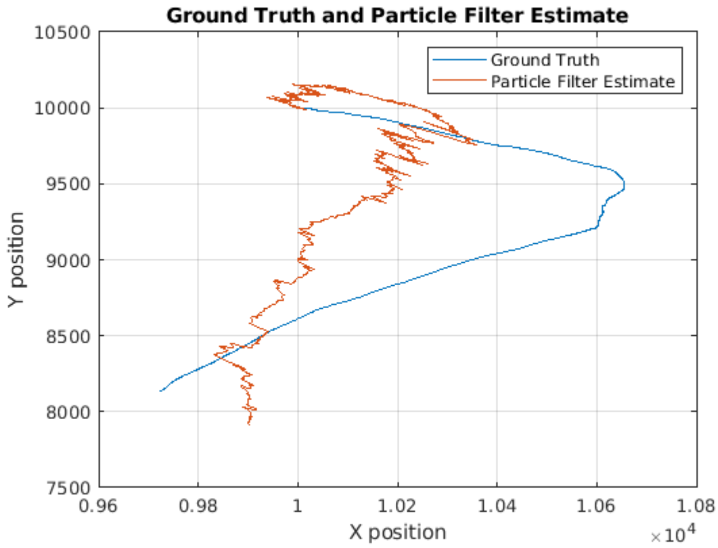


Figure 16: Particle Filter Estimation (N=1000)

Parametric Study

	N=1000	N=2000	N=5000
Q=0.01	[292.0489, 313.0406, 0.8619, 0.9321]	[233.0362, 250.1285, 0.9293, 1.0199]	[233.2129, 253.6993, 1.1026, 1.1727]
Q=0.1	[238.3932, 284.3770, 1.8977, 2.2909]	[313.3432, 347.6295, 2.4106, 2.7224]	[104.4820, 119.8921, 1.1573, 1.2713]
Q=1	[207.6268, 232.8224, 3.4667, 3.9097]	[221.8129, 233.3457, 2.7838, 3.0064]	[209.1561, 240.3754, 3.3433, 3.7281]

Table 4: RMS Values of Different Q / N combinations

A brief parametric study reveals that increasing the number of particles improves the model performance, when accompanied with an appropriate Q values to impose variety in particle realisations. The combination of proper diversity and quantity, like in much of statistics, results in a more accurate model estimate.

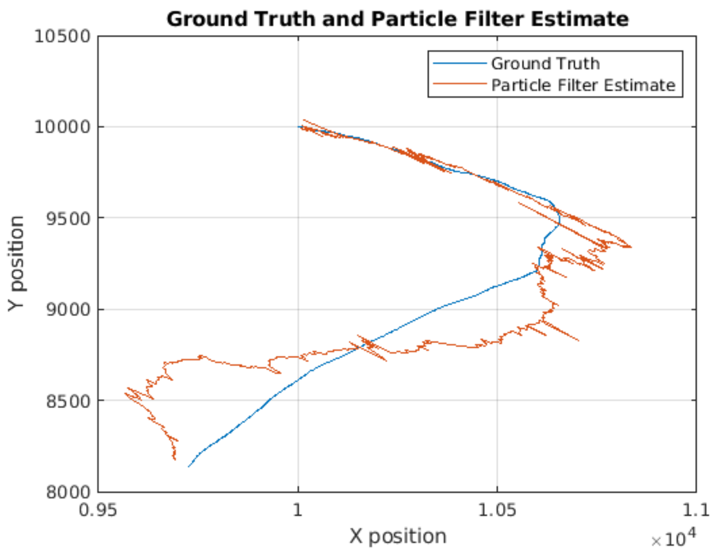


Figure 17: Particle Filter XY Plot (N=5000, Q=0.1)