# Image Processing by a Discrete Reaction-Diffusion System

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## Abstract

We propose a new concept of image processing by use of self-organization mechanisms appeared in a discrete non-linear system. Edge detection, image segmentation, noise reduction and contrast enhancement can be achieved by use of a discrete reaction-diffusion model (Fitz-Hugh & Nagumo model) under the condition of Turing instability. Compared with the conventional method, the proposed one indicates a higher performance in processing for noisy image. Especially, by adding a suitable level of noise, the model works as contrast enhancement. It is confirmed that the non-linear effect of stochastic resonance (SR) brings a good performance in image processing.

## Key Words
Image Processing, Reaction-diffusion model, a discrete system, self-organization, stochastic resonance (SR)

## 1. Introduction

An increasing attention is focused on information processing by non-linear systems [1][2], in which spatio-temporal patterns are self-organized in nonequilibrium state [3][4]. Non-linear reaction-diffusion models can describe many natural phenomena such as propagation of active electrical pulse along a nerve axon [5] and biological pattern formation [6]. Recently, many applications have been tested to the engineering field such as information processing. We have proposed a new approach for image segmentation and edge detection based on a reaction-diffusion model (Fitz-Hugh & Nagumo (FHN) model) under the condition of Turing instability [7][8]. The Turing instability is known as a condition of self-organization like biological pattern formation [9]. However, it is usually hard to obtain a static structure through the instability. Choosing suitable parameter is sensitive to have a stable pattern. It is required to solve the problem to establish the engineering applications of the model.

In this paper, we clarify that it is possible to obtain a stable pattern on a discrete reaction-diffusion system, in which each element keeps a suitable distance. It is interesting to compare with the neural network having the discrete elements of nerves and axons. Next, we show a usefulness of this system in image processing; noise reduction and contrast enhancement utilizing a random noise. It has been reported that the noise acts effectively in the biological signal processing system such as sensors of living organisms including the visual system [1][10][11]. It is known as stochastic resonance (SR) observed only in non-linear system having a threshold for response. To our knowledge, there has been proposed no algorithm for image processing based on SR. We propose an algorithm using SR to recover a figure from low contrast images. A demonstration of real image processing is also carried out to confirm the practical use.

## 2. Behavior of Reaction-diffusion system

Let us explain the principle of self-organization in reaction-diffusion system. In this system, a number of elements are distributed on a lattice grid in experimental region. Each element is composed of two substances. One is activator which behaves as autocatalysis. The other is inhibitor which works to suppress the activator. At the stable state, the elements keep the balance of concentration of two substances. But, by adding the stimulus, they lose the balance. If the stimulus is over the threshold, the concentration of activator $u$ increases autocatalytically with time (firing). Once the element is fired, $u$ decreases gradually as time proceeds and finally the element returns to the stable steady state again. Such spontaneous state variation is observed on biological system such as the nerve cell.

Fitz-Hugh & Nagumo (FHN) equations are known as a model representing active electrical pulse propagation along a nerve axon. This model is given by,

$$\partial u/\partial t = D_u \nabla^2 u + (1/\varepsilon)\{u(1-u)(u-a)-v\}, \quad (1)$$

$$\partial v/\partial t = D_v \nabla^2 v + u - bv. \quad (2)$$

where $u$ and $v$ are concentrations of activator and inhibitor, $D_u$ and $D_v$ are their diffusion coefficients respectively, and $\varepsilon$ ($0<\varepsilon<<1$), $a$($0<a<0.5$) and $b$($b>0$) are constant parameters. Equation (1) and (2) describe time evolutions of $u$ and $v$.

## 2.1 Behavior of an element

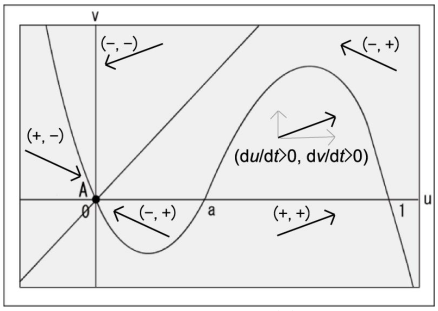
Now, we try to check a behavior of this model around stable state ($\partial u/\partial t=0$, $\partial v/\partial t=0$) with no effect of diffusions ($D_u=0$, $D_v=0$). Then we obtain nullclines as follows,

$$v = u(1-u)(u-a), \quad (3)$$

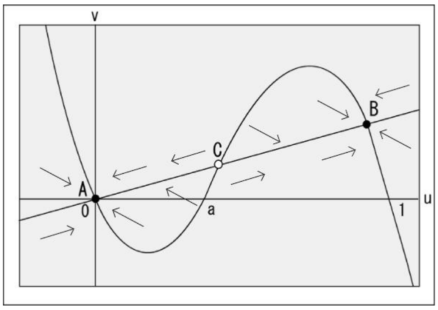
$$v = (1/b)u. \quad (4)$$

Figure 1 shows the nullclines of the model equations. The time evolutions of $u$ and $v$ are different in each domain separated by the lines(3) and (4), and they are shown by the arrows in Fig.1. The intersection of (3) and (4) shows a stable steady point in which time evolutions of $u$ and $v$ are static (zero) each other. Depending on the constants $a$ and $b$, the number of the stable point changes and the system shows different types. One of them is the mono-stable system having one stable steady point (Fig.1(a)). Another one is the bi-stable system having two stable steady points (Fig.1(b)). First, on the mono-stable system, although the state changes by adding the stimulus, it finally returns to the stable steady state along the arrows. On the other hand, the bi-stable system has three steady points (Fig.1(b) A,B,C). If the stimulus is over the threshold ($u>a$, $v=0.0$) the element goes to the state B, in the other case ($u<a$, $v=0.0$) the one goes to the state A. Incidentally, at the state C, the perturbation leads to break the steady state, so point C is called unstable steady point.

Figure 2 shows the difference of time evolutions added a different value of stimulus on the mono-stable system. In the case of $u>a$, $v=0.0$, the orbit goes to the line(1) (firing), and finally $u$ and $v$ reaches the state A. In the other case ($u<a$, $v=0.0$), the orbit goes to the line(2). Thus, parameter $a$ in eq.(1) works as a threshold value. For example, active electrical pulse propagation is observed on this system. Such behavior is one of the characteristics of non-liner system.




(a) mono-stable




(b) bi-stable

Fig.1 Null-cline (FHN model): Allows show the time evolutions in each domain separated by the lines(3) and (4). Along the arrows, the time evolutions changes as time proceeds.

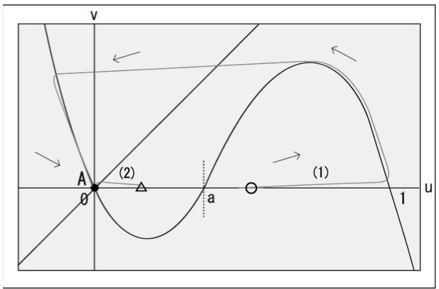



Fig.2 The difference of the time evolutions adding the different value of stimulus on the mono-stable system: The time evolutions added the different value of stimulus show different orbit. If the stimulus is over the threshold ($u>a$, $v=0.0$:firing), the orbit goes to the line (1). In the other case ($u<a$, $v=0.0$), the orbit goes to the line (2).

## 2.2 Pattern formation by the reaction-diffusion model

The substances (activator and inhibitor) are exchanged between the elements with the diffusion term. If an element is fired, that is, the concentrations of substances $u$ and $v$ increases (Fig.2 line(1)), it releases a great deal of substances to the neighboring elements. The neighboring elements receive the substances, and they are fired too when the concentrations is over the threshold. Thus, the trigger wave propagates across the system, and the pattern formation and the active electrical pulse propagation are archived.

Similar phenomenon is also observed in a chemical experiment. Belousov-Zhabotinsky (BZ) reaction is well known to self-organize the target pattern and the spiral pattern [12]. L.Kuhnert et al. proposed a chemical light sensitive system, a variant of BZ medium, in which chemical reaction front can be modified by light [2]. They reported that a light projection of a half-tone image on such a medium initiates a very complex response; they demonstrated contrast modification, discerning of contours and smoothing of partially degraded the picture. It attracts a great deal of attention on connecting to visual perception, in which image processing based on the self-organizing mechanism is realized. On the other hand, it seems important to have a stable pattern formation to guarantee a self-organized structure for example the pattern appearing on the skin [6]. The Turing instability mechanism ($D_u$<<$D_v$) is well known as a condition for obtaining the stable pattern formation [9]; the inhibitor propagates rapidly and restrains the propagation of trigger wave. We have proposed several algorithms for image processing such as edge detection and image segmentation under this condition ($D_u$<<$D_v$).

## 2.3 Image processing by the reaction-diffusion model

Figure3(a) shows a gray scale image; brightness of figure and ground are f(x,y)=178 and f(x,y)=0 respectively (256 gray levels), and size is 200 × 200pixels. Each pixel (element) has initial value provided in proportion to the brightness; namely $u$= - 0.05 ～ 1.0 corresponds to f(x,y) 0～255. That is, initial solution of $u$ ($u_0$ (x,y)) is provided by,

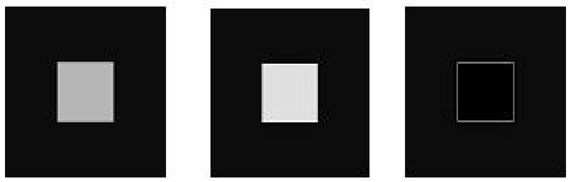
$$u_0\,(x,y)=\{f(x,y)/255.0\}\times(1.0+0.05) - 0.05. \qquad (5)$$

Thus initial value of Fig.3(a) is obtained; $u_0$=0.7 at figure region; $u_0$=0.0 at ground region. If $u$ is over or under the range, it is push into $u$=1.0 and $u$= - 0.05. Initial value of $v$($v_0$ (x,y)) is set on 0.0 in all pixels.

In the simulation, we set $D_u$<< $D_v$ to satisfy the condition of Turing instability. Parameter $a$ and $b$ are determined to chose a stable system, either mono-stable or bi-stable. Parameter ε controls the progress rate of $u$; $u$ grows more rapidly as ε decreases. Here, we set ε = 1.0× $10^{-3}$. Figure3(b) and (c) show the results of simulation after time fully passes; Fig.3(b) and (c) realized image segmentation by use of bi-stable and edge detection by use of mono-stable, respectively.

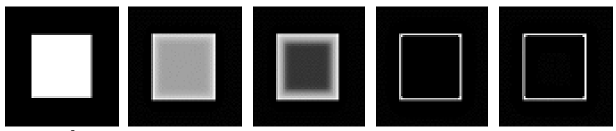
Thus, we have realized several methods for image processing based on the Turing instability [7][8], however, we did not optimize the parameters in the previous study. Figure4(a) and (b) show some results of numerical experiment. Both results are carried out under the same condition (parameters $D_u$, $D_v$, $a$, $b$ and ε are the same values respectively) excepting for the distance dx between each element and size of the image. Calculation in the numerical studies is carried out by Forward-Difference method. In this method, Δx and Δt are required; Δx is the grid spacing and Δt is the time step. In Fig.4(a), with long distance of dx=1.0 (discrete system) and the size of image is 128 × 128 pixels, stable pattern formation is realized. The stable pattern (the right top) corresponds to the edge of the given image (the left top). In Fig4(b), with short distance of dx=0.25 (continuous system) and the size of image is 512×512 pixels, the pattern spreads out as time proceeds. The result shows an important role of the distance parameter dx. Selecting a suitable distance dx leads to realize spontaneous image processing. Now, we recognize that changing the distance parameter corresponds to change the value of diffusion coefficients.

We carried out a detailed experiment of simulation (Fig.5). For convenience sake, we took two parameters $D_u$ and ε in one dimension system in space on the mono-stable. The system behaves like a discrete system as $D_u$ decreases, that is, the effect of diffusion is weak in the discrete system. Then the ratio of $D_u$ and $D_v$ is fixed 1:4 (the Turing instability condition). Figure5 shows several types of pattern formation in space (horizontal axis) – time (vertical axis) plot. The stable edge pattern is obtained in a restricted parameter domain in which ε and $D_u$ is moderately small (Fig.5: domain ①). Thus, in addition to the Turing instability condition, the distance dx between each element becomes an important control parameter to obtain a stable pattern formation. It seems interesting that the neural network has the discrete elements of neurons and axons. More detailed studies are required to establish the relationship between information processing in living organisms and the discrete system with non-linear reactor.

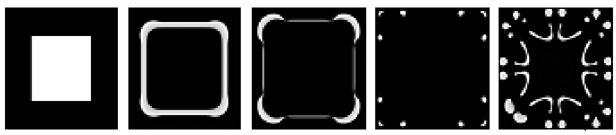


(a) initial image (b) final image (bi-stable) (c) final image (mono-stable)

Fig.3 Image processing by FHN model




(a) discrete system




(b) continuous system

Fig.4 The results in different conditions

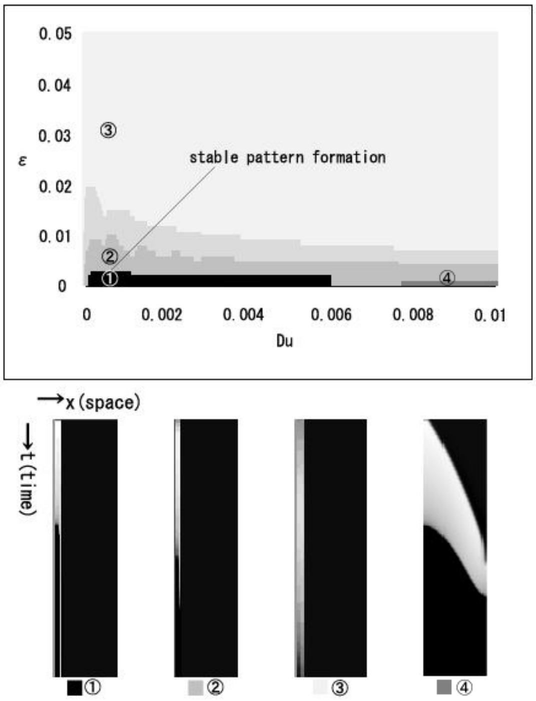



Fig.5 Investigation of the suitable value of parameters

The results of setting the parameter of each domain ①～④ shown by above figure correspond to below figure ■①, ■②, ③, ■④. The stable edge pattern is obtained in the parameter domain ① in which ε and $D_u$ is moderately small.

## 3. The performance for noisy image

### 3. 1 Noise reduction

Here, we show the usefulness of the proposed method for noisy image in processing. Figure6 is an image added noise to Fig3(a). The brightness of each pixel is given by,

$$u_0'(x,y)= u_0(x,y)+ c\times \mathrm{rnd}(x,y) \qquad (6)$$

where, rnd(x,y) is given randomly (-1 < rnd(x,y)<1) and *c* is a constant parameter determining intensity of noise (amplitude of noise); *c*=0.45 in Fig6.

Figure7 shows a result of image processing by conventional method (using the Median filter having 5×5 pixels window). The noise reduction is not enough and a shaky figure is obtained in the binarized process (Fig.7(b)). It seems impossible to detect a precise edge from this image (see Fig.7(c)). On the other hand, the precise figure-ground separation and edge detection are carried out by the reaction-diffusion model (Fig.8(a),(b)). A clear difference is recognized in enlarged figures (Fig.9). The proposed method based on the reaction-diffusion model brings high quality processing on noisy image. It is realized by reaction-diffusion mechanism with the Turing instability condition, which brings spontaneous pattern formation.

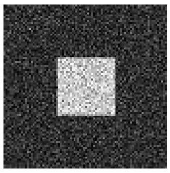
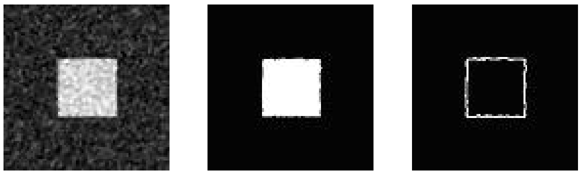
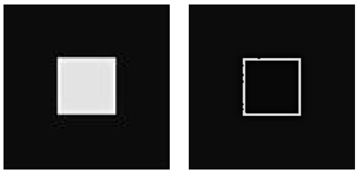


Fig.6 Noisy image



(a) Median filter (b) binarize process (c) edge detection

Fig.7 Results obtained by the conventional method



(a) bi-stable (b) mono-stable

Fig.8 The results obtained by the proposed reaction-diffusion model

(a) $D_u$=0.002,$D_v$=0.008,a=0.4,b=20.0, ε =0.001

(b) $D_u$=0.002,$D_v$=0.008,a=0.4,b=10.0, ε =0.001

(a) enlargement of Fig.6(b) (b) enlargement of Fig.7(a)

Fig.9 A detailed compression between the conventional Method and the proposed one

### 3. 2 Practical use of noise on low-contrast image

Unfortunately, the proposed model has a kind of defect. It is difficult to detect the image information if the brightness of the image isn't enough to exceed over a threshold. The threshold is determined by the parameters *a* and *b* in the model equations (see Eq.(1) and (2)). However, this threshold character has a great advantage in information processing under noise-rich environment. It has been reported that sensors of living organisms including the visual system make good use of noise to signal processing. This is known as stochastic resonance (SR), a mechanism of non-linear system, and it may contribute to the information processing. Here, we try to investigate the role of noise for image processing by utilizing the reaction-diffusion model.

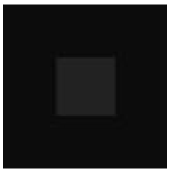
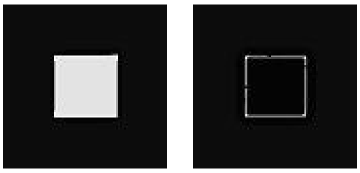


Fig.10 Low contrast image

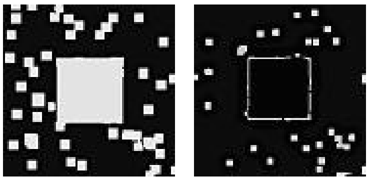
(a) (b)

c=0.0



(c) (d)

c=0.11



(e) (f)

c=0.15

Fig.11 The result of image processing under three types of noise amplitude

(L:bi-stable)$D_u$=0.002,$D_v$=0.008,a=0.1,b=12.0, $\varepsilon$ =0.001

(R:mono-stable)$D_u$=0.002,$D_v$=0.008,a=0.1,b=1.0, $\varepsilon$ =0.001

Figure10 shows a low contrast image. The init value $u_0$=0.1 and $u_0$=0.0 are given to the figure region and the ground region, respectively. We compare the results of the image processing adding three types of noise amplitude; (1) no noise(c=0.0), (2) a weak noise(c=0.11) and (3) a strong noise (c=0.15). Figure11 shows the results (left side is bi-stable system and right side is mono-stable one). Usually, it is impossible to detect the correct information under this condition because that the brightness of the image is under the threshold, (Fig.11 (a), (b)). On the other hand, by adding a moderate noise, the figure appears distinctly (Fig.11 (c), (d)); the noise supports to visualize the figure region by inducing the region to exceed over the threshold. Furthermore, the diffusion term fills the left pixels under the threshold. But if noise intensity is too strong, some of the pixels of the ground region are fired and the unnecessary noises remain in the image (see Fig.11 (e), (f)).

## 4. Discussion

We show that the proposed method based on the reaction-diffusion model realizes the high quality image processing on noisy image compared with the conventional method (Fig.7,8,9). Furthermore, we demonstrate that it is able to detect the correct information from the low contrast image by adding a moderate noise (Fig.11(c),(d)). Figure 12 shows a relationship between noise intensity and the precision of figure-ground separation. The precision is obtained as a ratio of pixels fitting to the original image; (ratio:100%=precision:1.0). A moderate noise increases the precision and realizes a good performance of image processing for the low contrast image. Such characteristic is known as SR, in which noise plays an important role. Generally, noise in signal acts as

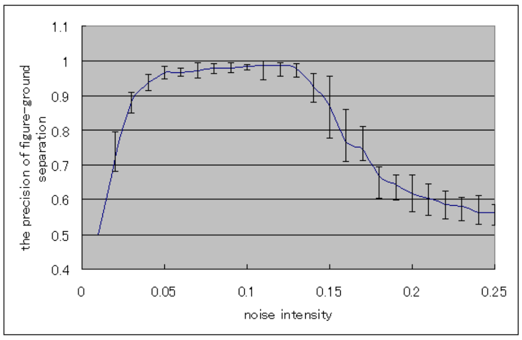
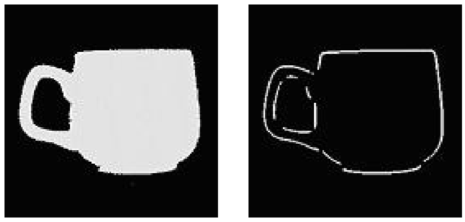



Fig.12 The relationship between noise intensity and the precision of figure-ground separation

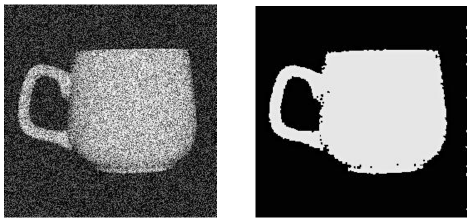
Fig.13 The real image(256 gray levels)



(a)figure-ground separation by use of bi-stable system

(b)edge detection by use of mono-stable system

Fig.14 Image processing for the real image

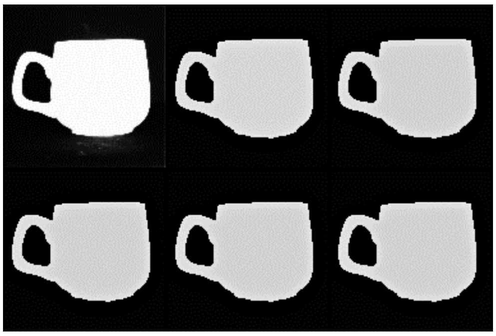


(a)image P4

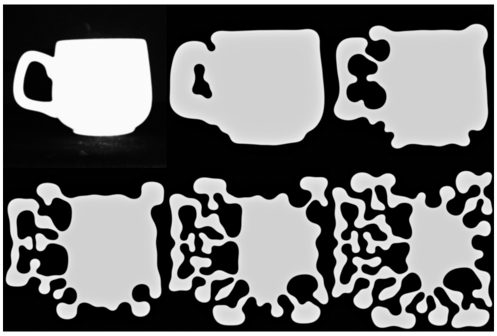
(b)the result of image segmentation

Fig.15 The image segmentation for noisy image

($D_u$=0.0025,$D_v$=0.01,a=0.38,b=20.0, $\varepsilon$ =0.001)



(a)The result of the discrete system



(b)The result of the continuous system

Fig.16 The comparison of the result on the different condition (discrete / continuous)

a kind of trouble maker in the information processing. However, our experiments clearly demonstrate a usefulness of noise. Noise can contribute to elevate the performance of the image processing.

We also demonstrate the image processing for real image (Fig.13(a) :256 gray levels). Figure14(a) and (b) show the results of the processing utilizing two systems; bi-stable system and mono-stable one. The proposed model realizes the image segmentation and the edge detection for the real image. Figure 15 shows the result of image segmentation for noisy image. The proposed method work to reduce the noise and obtain the figure correctly. The method of noise reduction is effective in the field such as the astronomical imaging and so on.

Furthermore, we confirm the difference of the results on the discrete system and the continuous system. The results are the same as the previous trial (Fig.4), in which the stable pattern is obtained under the discrete system.

These results suggest a good possibility for applying the proposed method to the practical use.

## 5. Conclusion

In this study, the following results are clarified.

1) The stable pattern formation is realized on a discrete reaction-diffusion system, in which each element has suitable distance (Fig.4).
2) The proposed method based on the reaction-diffusion model realizes a high quality processing especially on noisy image compared with the conventional method (fig.8,9).
3) By adding a moderate noise, the performance of the image processing increases, and the figure-ground separation from low contrast images realizes (Fig.11(c)(d)).

Thus we confirmed that our approach brought fruitful results for information processing. It is known that non-liner systems have many useful functions. For example, biological system makes good use of noise (SR). Such phenomenon can be observed in the visual system [1]. Recently, the researches of perception and recognition based on non-linear mechanism are carried out [13]. We hope that our approach (image processing based on non-linear system) is helpful to understand the mechanism of biological visual image processing.

## 6.Ackowledgement

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