$$\sqrt{x^2+y^2} \leq z \leq \sqrt{1-x^2-y^2}$$

Ще използваме сферични координати:

$$x = \rho \sin\psi \cos\varphi$$

$$y = \rho \sin\psi \sin\varphi$$

$$z = \rho \cos\psi$$

$$0 \leq \psi \leq \pi, 0 \leq \varphi \leq 2\pi$$

Така, за първото ограничение имаме:

$$\sqrt{\rho^2 \sin^2\psi \cos^2\varphi + \rho^2 \sin^2\psi \sin^2\varphi} \leq \rho\cos\psi$$

$$\sqrt{\rho^2 \sin^2\psi (\cos^2\varphi + \sin^2\varphi)} \leq \rho\cos\psi$$

$$\sqrt{\rho^2 \sin^2\psi} \leq \rho\cos\psi$$

$$\rho\sin\psi \leq \rho\cos\varphi$$

$$\sin\psi \leq \cos\psi$$

$$\Rightarrow \psi \in [0; \frac{\pi}{4}]$$

За второто ограничение:

$$\rho\cos\psi \leq \sqrt{1-\rho^2 \sin^2\psi \cos^2\varphi - \rho^2 \sin^2\psi \sin^2\varphi}$$

$$\rho\cos\psi \leq \sqrt{1-\rho^2 \sin^2\psi (\cos^2\varphi + \sin^2\varphi)}$$

$$\rho\cos\psi \leq \sqrt{1-\rho^2 \sin^2\psi}$$

$$\rho^2\cos^2\psi \leq 1-\rho^2\sin^2\psi$$

$$1-\rho^2\sin^2\psi - \rho^2\cos^2\psi \geq 0$$

$$1-\rho^2(\sin^2\psi + \cos^2\psi) \geq 0$$

$$1-\rho^2 \geq 0 \Rightarrow \rho^2 \leq 1$$

$$\Rightarrow \rho \in [-1;1], \text{но } \rho \geq 0 \Rightarrow \rho \in [0;1]$$

За Якобиана J имаме $|J| = \rho^2 \sin\psi$.

Пресмятаме интеграла:

$$\int\limits_0^1 \int\limits_0^{\frac{\pi}{4}} \int\limits_0^{2\pi} \left(\rho^2 \sin\psi.\rho\cos\psi\sqrt{\rho^2\sin^2\psi\cos^2\varphi+\rho^2\sin^2\psi\sin^2\varphi+\rho^2\cos^2\psi}\right) d\varphi d\psi d\rho =$$

$$= \int\limits_0^1 \int\limits_0^{\frac{\pi}{4}} \int\limits_0^{2\pi} \left(\rho^2 \sin\psi.\rho\cos\psi\sqrt{\rho^2\sin^2\psi(\cos^2\varphi+\sin^2\varphi)+\rho^2\cos^2\psi}\right) d\varphi d\psi d\rho =$$

$$= \int\limits_0^1 \int\limits_0^{\frac{\pi}{4}} \int\limits_0^{2\pi} \left(\rho^3 \sin\psi\cos\psi\sqrt{\rho^2(\sin^2\psi+\cos^2\psi)}\right) d\varphi d\psi d\rho =$$

$$= \int\limits_0^1 \int\limits_0^{\frac{\pi}{4}} \int\limits_0^{2\pi} \left(\rho^4 \sin\psi\cos\psi\right) d\varphi d\psi d\rho = 2\pi \int\limits_0^1 \rho^4\, d\psi \int\limits_0^{\frac{\pi}{4}} (\sin\psi\cos\psi)\, d\psi =$$

$$= 2\pi \cdot \frac{\rho^5}{5}\bigg|_0^1 \cdot \frac{1}{4} = 2\pi \cdot \frac{1}{5} \cdot \frac{1}{4} = \frac{2\pi}{20} = \frac{\pi}{10} \approx 0,31415$$