COMP 140: Computational Thinking

Breadth-First Search

Names:

The breadth-first search algorithm is given here for your reference.

Algorithm 1: Breadth-First Search

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Input: Undirected graph $graph$, start node $start$ in $graph$
Output: $dist_{node}$, for each $node$ in the graph: the number of steps between $start$ and $node$ in
        $graph$; $parent_{node}$, for each node $node$ in $graph$: the parent of $node$ in the exploration of
        $graph$ starting from $start$

1  Initialize $queue$ to an empty queue;
2  Initialize $dist$ to an empty mapping;
3  Initialize $parent$ to an empty mapping;
4  foreach $node$ in $graph$ do
5      $dist_{node} \leftarrow \infty$;
6      $parent_{node} \leftarrow null$;
7  $dist_{start} \leftarrow 0$;
8  push $start$ onto $queue$;
9  while $queue$ is not empty do
10     pop $node$ off of $queue$;
11     foreach neighbor $nbr$ of $node$ do
12         if $dist_{nbr} = \infty$ then
13             $dist_{nbr} \leftarrow dist_{node} + 1$;
14             $parent_{nbr} \leftarrow node$;
15             push $nbr$ onto $queue$;
16 return $dist$, $parent$;
```

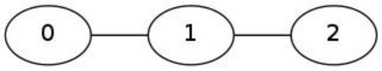
For each of the following four graphs, walk through BFS by hand. For graphs 1-3, you should use 0 as your start node; for graph 4 you should use Kevin Bacon.

As you go, you will perform two forms of bookkeeping. First, trace the edges as you traverse them with colored pencils, using a different color for each distance from the *start* node. Second, keep track of the states of the data structures as you progress through the algorithm. In particular, for each iteration of the while loop that you walk through, write down the state of *queue* (prior to the pop on line 10), *dist*, and *parent*. Also record the final states of these data structures upon reaching the return statement. This will be a lot of writing! So, you may take a few shortcuts. First, after the first iteration, you need not record the states of unvisited nodes (those with a distance of infinity) in either the *dist* or *parent* data structure. Second, if a data structure remains entirely unchanged since the previous iteration, you can simply write "no change".

You should write the state of the queue as a sequence of values $q_0, q_1, ..., q_{n-1}$, where q_0 is the head of the queue and q_{n-1} is the tail.

For Graph 4, you do not need to record the states of the data structures at every step. Instead, you may just trace the edges with colored pencils and record the final (upon return) states.

For example, consider the following graph:



Performing BFS on this graph using node 0 as the *start* node should be recorded as follows:

Iteration 0:

queue: 0

dist:

$0 \rightarrow 0$

$1, 2 \rightarrow \infty$

parent:

$0, 1, 2 \rightarrow null$

Iteration 1:

queue: 1

dist:

$0 \rightarrow 0$

$1 \rightarrow 1$

parent:

$0 \rightarrow null$

$1 \rightarrow 0$

Iteration 2:

queue: 2

dist:

$0 \rightarrow 0$

$1 \rightarrow 1$

$2 \rightarrow 2$

parent:

$0 \rightarrow null$

$1 \rightarrow 0$

$2 \rightarrow 1$

Return:

queue: $empty$

dist:

$0 \rightarrow 0$

$1 \rightarrow 1$

$2 \rightarrow 2$

parent:

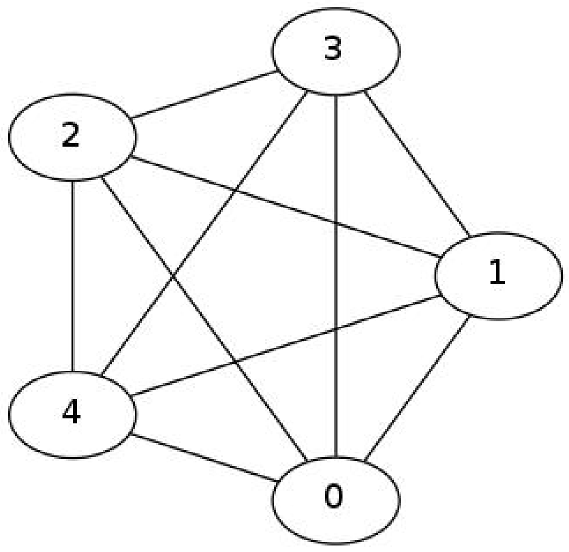
$0 \rightarrow null$

$1 \rightarrow 0$

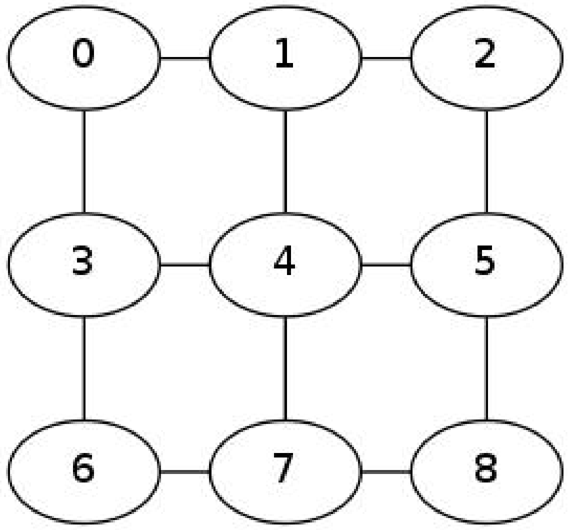
$2 \rightarrow 1$

This should result in edges (0, 1) and (1, 2) each having a unique color, since (0, 1) is 1 hop from the *start* node whereas (1, 2) is 2. Note that if you had performed BFS starting from node 1, both edges would have been given the same color as they are in the same “layer”. Note also that in some graphs, not all edges will be colored, as it will be possible to reach every node from the *start* node without traversing every edge in the graph.

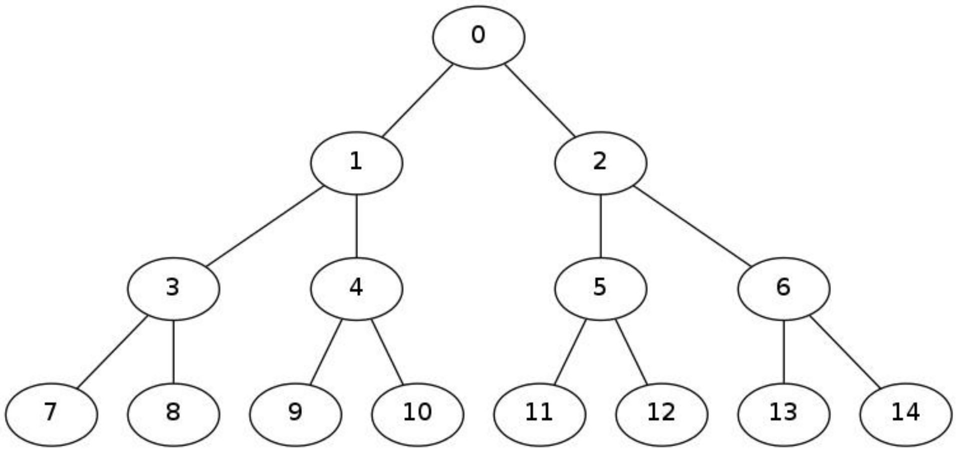
1 Graph 1



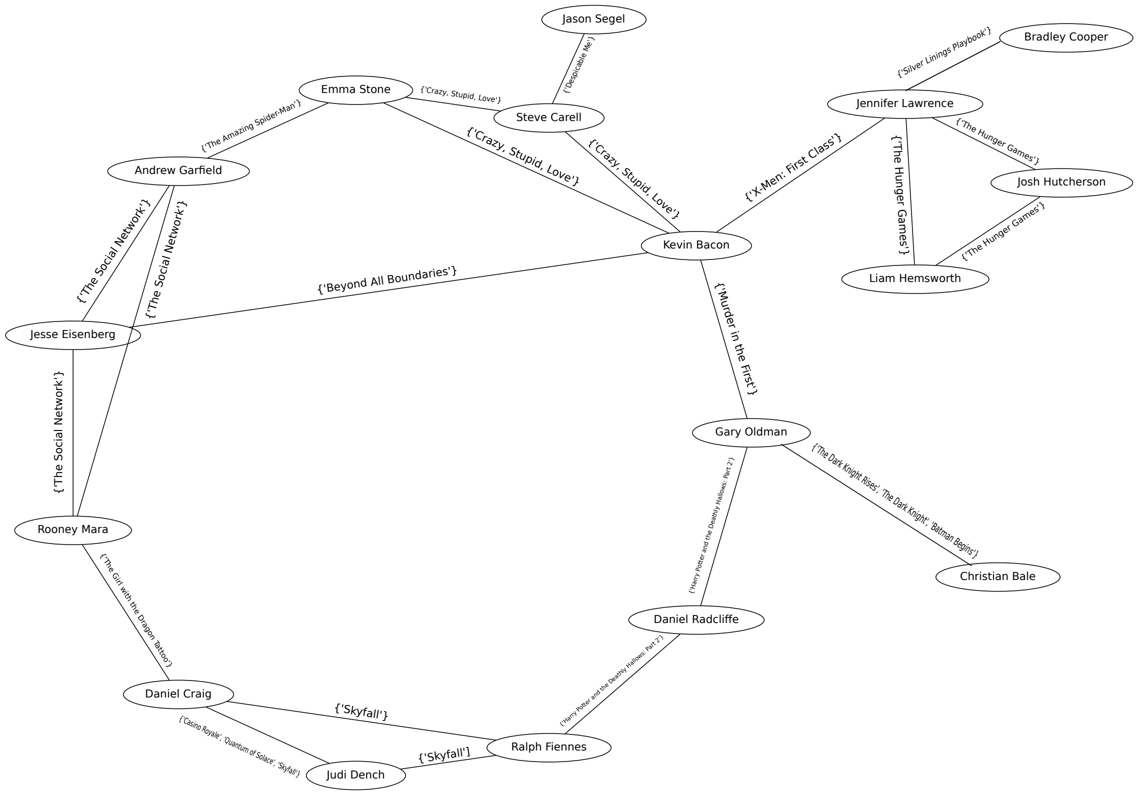
2 Graph 2



3 Graph 3



4 Graph 4


Jason Segel
{'Despicable Me'}
Bradley Cooper
{'Silver Linings Playbook'}
Emma Stone
{'Crazy, Stupid, Love'}
Steve Carell
Jennifer Lawrence
{'The Amazing Spider-Man'}
{'Crazy, Stupid, Love'}
{'Crazy, Stupid, Love'}
{'X-Men: First Class'}
{'The Hunger Games'}
{'The Hunger Games'}
Andrew Garfield
Josh Hutcherson
{'The Hunger Games'}
Kevin Bacon
{'The Social Network'}
{'The Social Network'}
Liam Hemsworth
{'Beyond All Boundaries'}
{'Murder in the First'}
Jesse Eisenberg
{'The Social Network'}
Gary Oldman
{'The Dark Knight Rises', 'The Dark Knight', 'Batman Begins'}
{'Harry Potter and the Deathly Hallows: Part 2'}
Rooney Mara
Christian Bale
{'The Girl with the Dragon Tattoo'}
Daniel Radcliffe
{'Harry Potter and the Deathly Hallows: Part 2'}
Daniel Craig
{'Skyfall'}
{'Casino Royale', 'Quantum of Solace', 'Skyfall'}
Ralph Fiennes
{'Skyfall']
Judi Dench