Crypto Engineering Midterm Exam

April, 2023

Student Name : 范恩宇

Student ID : 109550135

Department : 資訊工程學系

For the following Questions, please write down your answers with explanations

Question 1 :

Let us see what goes wrong when a stream cipher key is used more than once. Below are eleven hex-encoded ciphertexts that are the result of encrypting eleven plaintexts with a stream cipher, all with the same stream cipher key. Your goal is to decrypt the last ciphertext, and submit the secret message within it as solution.

Hint: XOR the ciphertexts together, and consider what happens when a space is XORed with a character in [a-zA-Z].

Ciphertext #1 :

315c4eeaa8b5f8aaf9174145bf43e1784b8fa00dc71d885a804e5ee9fa40b
16349c146fb778cdf2d3aff021dfff5b403b510d0d0455468aeb98622b137
dae857553ccd8883a7bc37520e06e515d22c954eba5025b8cc57ee59418
ce7dc6bc41556bdb36bbca3e8774301fbcaa3b83b220809560987815f652
86764703de0f3d524400a19b159610b11ef3e

Ciphertext #2 :

234c02ecbbfbafa3ed18510abd11fa724fcda2018a1a8342cf064bbde548b
12b07df44ba7191d9606ef4081ffde5ad46a5069d9f7f543bedb9c861bf29
c7e205132eda9382b0bc2c5c4b45f919cf3a9f1cb74151f6d551f4480c82b
2cb24cc5b028aa76eb7b4ab24171ab3cdadb8356f

Ciphertext #3 :

32510ba9a7b2bba9b8005d43a304b5714cc0bb0c8a34884dd91304b8ad
40b62b07df44ba6e9d8a2368e51d04e0e7b207b70b9b8261112bacb6c86
6a232dfe257527dc29398f5f3251a0d47e503c66e935de81230b59b7afb5
f41afa8d661cb

Ciphertext #4 :

32510ba9aab2a8a4fd06414fb517b5605cc0aa0dc91a8908c2064ba8ad5e
a06a029056f47a8ad3306ef5021eafe1ac01a81197847a5c68a1b78769a3
7bc8f4575432c198ccb4ef63590256e305cd3a9544ee4160ead45aef5204
89e7da7d835402bca670bda8eb775200b8dabbba246b130f040d8ec6447
e2c767f3d30ed81ea2e4c1404e1315a1010e7229be6636aaa

Ciphertext #5 :

3f561ba9adb4b6ebec54424ba317b564418fac0dd35f8c08d31a1fe9e24fe
56808c213f17c81d9607cee021dafe1e001b21ade877a5e68bea88d61b9
3ac5ee0d562e8e9582f5ef375f0a4ae20ed86e935de81230b59b73fb4302
cd95d770c65b40aaa065f2a5e33a5a0bb5dcaba43722130f042f8ec85b7c
2070

Ciphertext #6 :

32510bfbacfbb9befd54415da243e1695ecabd58c519cd4bd2061bbde24e
b76a19d84aba34d8de287be84d07e7e9a30ee714979c7e1123a8bd9822
a33ecaf512472e8e8f8db3f9635c1949e640c621854eba0d79eccf52ff111
284b4cc61d11902aebc66f2b2e436434eacc0aba938220b084800c2ca4e6
93522643573b2c4ce35050b0cf774201f0fe52ac9f26d71b6cf61a711cc22
9f77ace7aa88a2f19983122b11be87a59c355d25f8e4

Ciphertext #7 :

32510bfbacfbb9befd54415da243e1695ecabd58c519cd4bd90f1fa6ea5ba
47b01c909ba7696cf606ef40c04afe1ac0aa8148dd066592ded9f8774b52
9c7ea125d298e8883f5e9305f4b44f915cb2bd05af51373fd9b4af511039f
a2d96f83414aaaf261bda2e97b170fb5cce2a53e675c154c0d9681596934
777e2275b381ce2e40582afe67650b13e72287ff2270abcf73bb02893283
6fbdecfecee0a3b894473c1bbeb6b4913a536ce4f9b13f1efff71ea313c866
1dd9a4ce

Ciphertext #8 :

315c4eeaa8b5f8bffd11155ea506b56041c6a00c8a08854dd21a4bbde54c
e56801d943ba708b8a3574f40c00fff9e00fa1439fd0654327a3bfc860b92f
89ee04132ecb9298f5fd2d5e4b45e40ecc3b9d59e9417df7c95bba410e9a
a2ca24c5474da2f276baa3ac325918b2daada43d6712150441c2e04f6565
517f317da9d3

Ciphertext #9 :

271946f9bbb2aeadec111841a81abc300ecaa01bd8069d5cc91005e9fe4a
ad6e04d513e96d99de2569bc5e50eeeca709b50a8a987f4264edb6896fb
537d0a716132ddc938fb0f836480e06ed0fcd6e9759f40462f9cf57f45641
86a2c1778f1543efa270bda5e933421cbe88a4a52222190f471e9bd15f65
2b653b7071aec59a2705081ffe72651d08f822c9ed6d76e48b63ab15d020
8573a7eef027

Ciphertext #10 :

466d06ece998b7a2fb1d464fed2ced7641ddaa3cc31c9941cf110abbf409e
d39598005b3399ccfafb61d0315fca0a314be138a9f32503bedac8067f03a
dbf3575c3b8edc9ba7f537530541ab0f9f3cd04ff50d66f1d559ba520e89a
2cb2a83

Target Ciphertext (decrypt this one) :

32510ba9babebbbefd001547a810e67149caee11d945cd7fc81a05e9f85a
ac650e9052ba6a8cd8257bf14d13e6f0a803b54fde9e77472dbff89d71b5
7bddef121336cb85ccb8f3315f4b52e301d16e9f52f904

For completeness, here is the python2 script used to generate the ciphertexts. (it doesn't matter if you can't read this)

```
import sys

MSGS = ( ---    11 secret messages    --- )

def strxor(a, b):          # xor two strings (trims the longer input)
    return "".join([chr(ord(x) ^ ord(y)) for (x, y) in zip(a, b)])

def random(size=16):
    return open("/dev/urandom").read(size)

def encrypt(key, msg):
    c = strxor(key, msg)
    print
    print c.encode('hex')
    return c

def main():
    key = random(1024)
    ciphertexts = [encrypt(key, msg) for msg in MSGS]
```

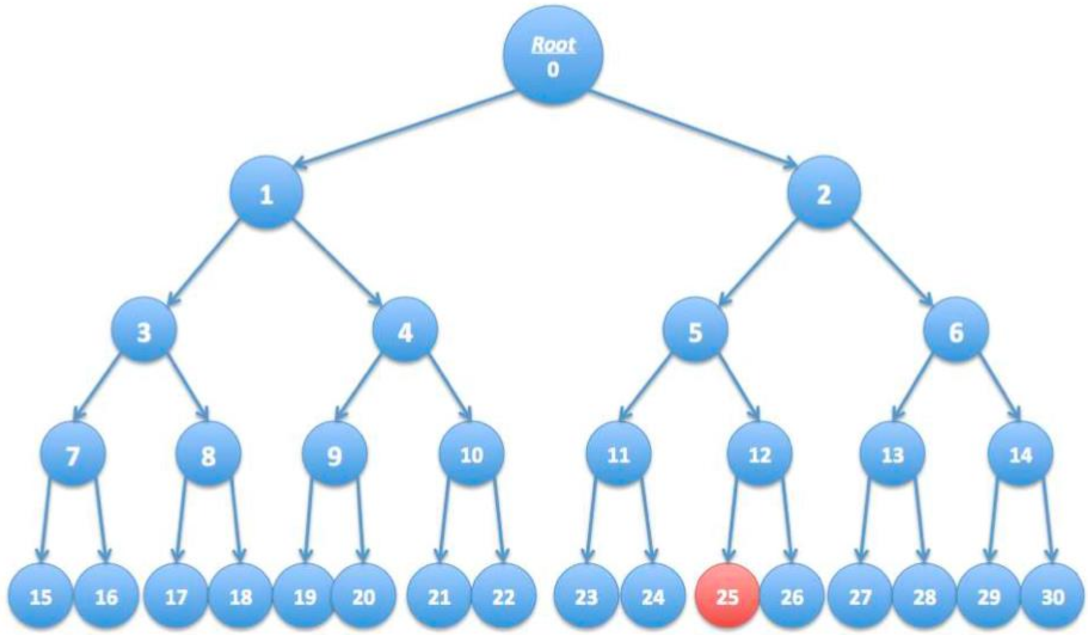
Question 2 :

Suppose you are told that the one time pad encryption of the message “attack at dawn” is “09e1c5f70a65ac519458e7e53f36” (the plaintext letters are encoded as 8-bit ASCII and the given ciphertext is written in hex). What would be the one time pad encryption of the message “attack at dusk” under the same OTP key?

Question 3 :

The movie industry wants to protect digital content distributed on DVD’s. We develop a variant of a method used to protect Blu-ray disks called AACS.
Suppose there are at most a total of n DVD players in the world (e.g. $\mathrm{n} = 2^{32}$). We view these n players as the leaves of a binary tree of height $log_2\mathrm{n}$. Each node in this binary tree contains an AES key k_i. These keys are kept secret from consumers and are fixed for all time. At manufacturing time each DVD player assigned a serial number $\mathrm{i} \in [0, \mathrm{n} - 1]$. Consider the set of nodes S_i along the path from the root to leaf number i in the binary tree. The manufacturer of the DVD player embeds in player number i the keys associated with the nodes in the set S_i. A DVD movie m is encrypted as $E(k_{root}, k) \parallel E(k, m)$ where k is a random AES key called a content-key and k_{root} is the key associated with the root of the tree. Since all DVD players have the key k_{root} all players can decrypt the movie m. We refer to $E(k_{root}, k)$ as the header and $E(k, m)$ as the body. In what follows the DVD header may contain multiple ciphertexts which each ciphertext is the encryption of the content-key k under some key k_i in the binary tree.
Suppose the keys embedded in DVD player number r are exposed by hackers and published on the Internet. In this problem we show that when the movie industry distributes a new DVD movie, they can encrypt the contents of the DVD using a slightly larger header (containing about $log_2 n$ keys) so that all DVD players, except for player number r, can decrypt the movie. In effect, the movie industry disables player number r without affecting other players.

As shown below, consider a tree with $n = 16$ leaves. Suppose the leaf node labeled 25 corresponds to an exposed DVD player key. Check the set of keys below under which to encrypt the key k so that every player other than player 25 can decrypt the DVD. Only four keys are needed.



(A) 14 (B) 28 (C) 11 (D) 4 (E) 6 (F) 2 (G) 26 (H) 1

Question 4 :

Continuing with the previous question, if there are n DVD players, what is the number of keys under which the content key k must be encrypted if exactly one DVD player's key needs to be revoked?

(A) 2 (B) n-1 (C) log_2n (D) n/2 (E) $\sqrt{n}$

Question 5 :

In the following let p be a prime. The set $Z_p = \{\, x \text{ integer},$ $\text{such that } 0 \le \mathrm{x} < \mathrm{p}\}$ is a group with respect to addition modulo p (i.e. every element x in Z_p has an inverse $-x \in Z_p$ such that $x + (-x) = 0 \; mod \; p$. The set $Z_p^* = \{\, x \text{ integer}, \text{ such that } 0 < \mathrm{x} < \mathrm{p}\}$ is a group with respect to multiplication modulo p (i.e. every element x in Z_p^* has an inverse $x^{-1} \in Z_p^*$ such that $xx^{-1} = 1 \; mod \; p$.

Another cipher with perfect secrecy. Consider the following cipher.

Let Z_p^* be the message space, the key space and the ciphertext space.

Alice and Bob share a key $k \in Z_p^*$ uniformly chosen at random. To send a message $m \in Z_p^*$ to Bob, Alice computes the ciphertext $c = mk \; mod \; p$.

1. Prove that this cipher provides **Perfect Secrecy** using the criterium we proved in class.

2. Why one-time pad are **Perfect Secrecy** and also **Semantic Secu**re?

3. Is the use of one-time pads susceptible to statistical analysis (especially if it is known that the plaintext is in American English)?

4. Did public-key encryption scheme provide Perfect Secrecy? We assume there is a public-key encryption scheme (KeyGen, Enc, Dec) with perfect correctness (i.e., for all messages M and valid key-pairs (PK, SK), we have $DEC_{sk}(Enc_{pk}$(M)) = M).

Question 6 :

Predicting generators. Consider the following *congruential generator*. It uses constants $a, b \in Z_p^*$. The seed is a value $x_0 \in Z_p^*$. The i^{th} value generated is computed as $x_i = ax_{i-1} + b \; mod \; p$.

The sequence output by the generator is $S = x_0, x_1, x_2, \ldots$ Assume that an attacker knows p and witness the sequence.

1. Prove that after a short prefix (i.e. a few of the values x_i's) the attacker is able to predict the rest of the sequence (i.e. the rest of the x_j's).

2. What does this say about the security of using the congruential generator as the keystream generator for a stream cipher?

3. If an attacker knows constants $a, b \in Z_p^*$ and p. How many output bits $S = x_0, x_1, x_2, \ldots$ did the attacker to know to rest sequences.

4. However, If an attacker knows p but know nothing about constants $a, b \in Z_p^*$. In this case, How many output bits $S = x_0, x_1, x_2, \ldots$ did the attacker need to know to recover the rest sequence?

Question 7 :

In standard RSA the modulus N is a product of two distinct primes. Suppose we choose the modulus so that it is a product of three distinct primes, namely $\mathrm{N} = \mathrm{pqr}$. Given an exponent e relatively prime to $\varphi(N)$ we can derive the secret key as $d = e^{-1}\, mod\ \varphi(N)$. The public key (N, e) and secret key (N, d) work as before. What is $\varphi(N)$ when N is a product of three distinct primes?

(A) $\varphi(N) = pqr - 1$

(B) $\varphi(N) = (p-1)(q-1)(r+1)$

(C) $\varphi(N) = (p+1)(q+1)(r+1)$

(D) $\varphi(N) = (p-1)(q-1)(r-1)$

Question 8 :

Suppose we choose the modulus so that it is a product of three distinct primes, namely N = 105. Given an encryption key is 13 which is co-prime to $\varphi(N)$. Please find the secret key as $d = e^{-1} mod\ \varphi(N)$ using Extended Euclidean Algorithm.

Question 9 :

An attacker intercepts the following ciphertext (hex encoded) :

20814804c1767293b99f1d9cab3bc3e7ac1e37bfb15599e5f40eef805488
281d

He knows that the plaintext is the ASCII encoding of the message "Pay Bob 100$" (excluding the quotes). He also knows that the cipher used is CBC encryption with a random IV using AES as the underlying block cipher. Show that the attacker can change the ciphertext so that it will decrypt to "Pay Bob 500$". What is the resulting ciphertext (hex encoded)? This shows that CBC provides no integrity.

Question 10 :

Let G be a finite cyclic group (e.g. $G = \mathbb{Z}_p^*$) with generator g. Suppose the Diffie-Hellman function $DH_g(g^x, g^y) = g^{xy}$ is difficult to compute in G. Which of the following functions is also difficult to compute :

As usual, identify the f below for which the contra-positive holds :

if $f(\cdot, \cdot)$ is easy to compute then so is $DH_g(\cdot, \cdot)$.

If you can show that then it will follow that if DH_g is hard to compute in G then so must be f.

(A) $f(g^x, g^y) = g^{2xy}$

(B) $f(g^x, g^y) = (g^2)^{x+y}$

(C) $f(g^x, g^y) = \sqrt{g^{xy}}$

(D) $f(g^x, g^y) = g^{x-y}$