Solutions for HW3

2.18 (a) 4-path: The 4-path doesn't exist between p and q since $N_4(q)$ doesn't involve any vertices in V. 8-path: 4. m-path: 5. (b) 4-path: 6. 8-path: 4. m-path: 6.

3.4 (a) First convert the value of its pixels to binary. Then do AND operation with different masks respectively. (mask = 1, 10, 100, 1000, ...)

(b) 1st bit plane: $\begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{matrix}$, 2nd bit plane: $\begin{matrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{matrix}$,

3rd bit plane: $\begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{matrix}$, 4th bit plane: $\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{matrix}$

3.11 (a) $s = \begin{cases} \frac{r^2}{L-1}\ , & for\ 0 \le r \le L-1 \\ 0, & otherwise \end{cases}$

(b) $s = \begin{cases} \frac{z^3}{(L-1)^2}\ , & for\ 0 \le z \le L-1 \\ 0, & otherwise \end{cases}$

(c) $z = [r^2(L-1)]^{\frac{1}{3}}$

3.12 $z = (2r - r^2)^{\frac{1}{2}}$

3.18 (a) $w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 6 & 3 & 0 & 0 \\ 0 & 0 & 4 & 8 & 4 & 0 & 0 \\ 0 & 0 & 3 & 6 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (c) Same as (b)

3.20 (a) $w_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $w_2 = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ or $w_1 = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$, $w_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 6 & 3 & 0 & 0 \\ 0 & 0 & 4 & 8 & 4 & 0 & 0 \\ 0 & 0 & 3 & 6 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3.26 (a) The histograms of the blurred images are not equal. When images blurred, boundary points lead to different value. Since there are more boundary points in the second image, the histograms of the two blurred images will be different.

(b)

Value	Number of points	Value	Number of points
0	24	0	0
1/9	0	1/9	0
2/9	2	2/9	4
3/9	6	3/9	24
4/9	4	4/9	18
5/9	0	5/9	18
6/9	16	6/9	0
7/9	0	7/9	0
8/9	0	8/9	0
1	12	1	0

3.29 If we repeatedly filtering an image, ignoring border effects, all the pixels' value will be the same.

3.34 The summation of the bars' wide and separation is equal to the size of the box filter used in (b). When the mask moves form left to right one pixel per round, the result is the same. Hence, there's no clear separation in (b).

3.41 $\begin{bmatrix} -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \\ -\frac{1}{9} & \frac{17}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \end{bmatrix}$

4.7 (a) $\frac{1}{2}$ (b) 1 (c) > 1

4.9 (a) Use $cos(2\pi\mu_0 t) = \frac{1}{2}(e^{j2\pi\mu_0 t} + e^{-j2\pi\mu_0 t})$, and $\mathcal{F}\{e^{j2\pi\mu_0 t}\} = \delta(\mu - t_0)$ from hint.

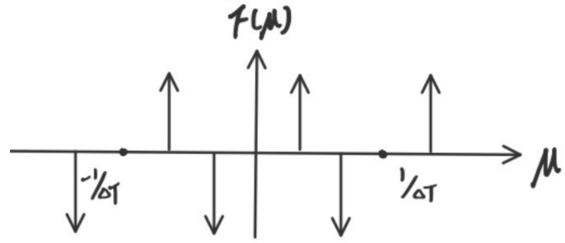
Then $\mathcal{F}\{cos(2\pi\mu_0 t)\} = \mathcal{F}\{\frac{1}{2}(e^{j2\pi\mu_0 t} + e^{-j2\pi\mu_0 t})\} = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]$

(b) Use $sin(2\pi\mu_0 t) = \frac{1}{2j}(e^{j2\pi\mu_0 t} - e^{-j2\pi\mu_0 t})$, and $\mathcal{F}\{e^{j2\pi\mu_0 t}\} = \delta(\mu - t_0)$ from hint.

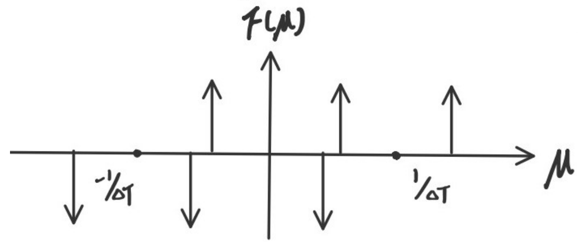
Then $\mathcal{F}\{sin(2\pi\mu_0 t)\} = \mathcal{F}\{\frac{1}{2j}(e^{j2\pi\mu_0 t} - e^{-j2\pi\mu_0 t})\} = \frac{1}{2j}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]$

4.10 (a) $\frac{1}{n}$ (b) n

(c)



(d)



(e)