2.18 Consider the image segment shown in the figure that follows.

(a)* As in Section 2.5, let $V = \{0,1\}$ be the set of intensity values used to define adjacency. Compute the lengths of the shortest 4-, 8-, and m-path between p and q in the following image. If a particular path does not exist between these two points, explain why.

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1	0	1	2

(b) Repeat (a) but using $V = \{1,2\}$.

3.4 Do the following:

(a) Propose a method for extracting the bit planes of an image based on converting the value of its pixels to binary.

(b) Find all the bit planes of the following 4-bit image:

0	1	8	6
2	2	1	1
1	15	14	12
3	6	9	10

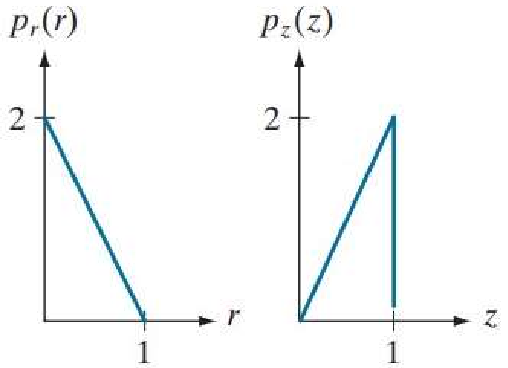
3.11 Assume continuous intensity values, and suppose that the intensity values of an image have the PDF $p_r(r) = 2r/(L-1)^2$ for $0 \le r \le L-1$, and $p_r(r) = 0$ for other values of r.

(a)* Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(b)* Find the transformation function that (when applied to the histogram-equalized intensities, s) will produce an image whose intensity PDF is $p_z(z) = 3z^2/(L-1)^3$ for $0 \le z \le L-1$ and $p_z(z) = 0$ for other values of z.

(c) Express the transformation function from (b) directly in terms of r, the intensities of the input image.

3.12 An image with intensities in the range [0,1] has the PDF, $p_r(r)$, shown in the following figure. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown in the figure. Assume continuous quantities, and find the transformation (expressed in terms of r and z) that will accomplish this.



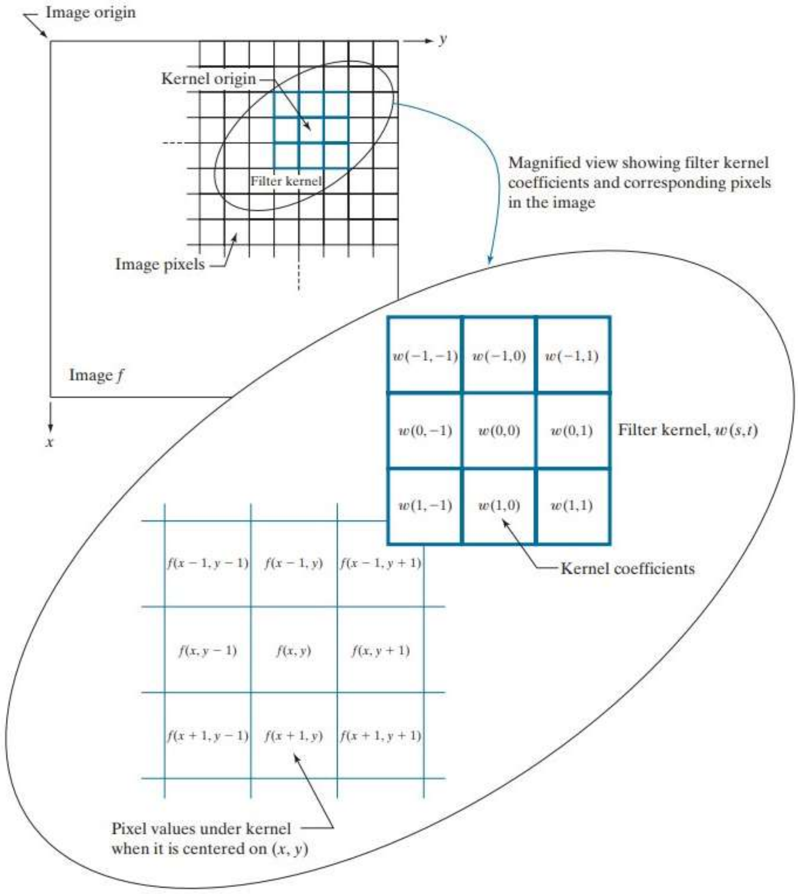


FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 kernel. The pixels are shown as squares to simplify the graphics. Note that the origin of the image is at the top left, but the origin of the kernel is at its center. Placing the origin at the center of spatially symmetric kernels simplifies writing expressions for linear filtering.

3.18 You are given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a)* Give a sketch of the area encircled by the large ellipse in Fig. 3.28 when the kernel is centered at point (2,3) (2nd row, 3rd col) of the image shown above. Show specific values of w and f.

(b)* Compute the convolution $w \star f$ using the *minimum* zero padding needed. Show the details of your computations when the kernel is centered on point (2,3) of f; and then show the final full convolution result.

(c) Repeat (b), but for correlation, $w \,☆\, f$.

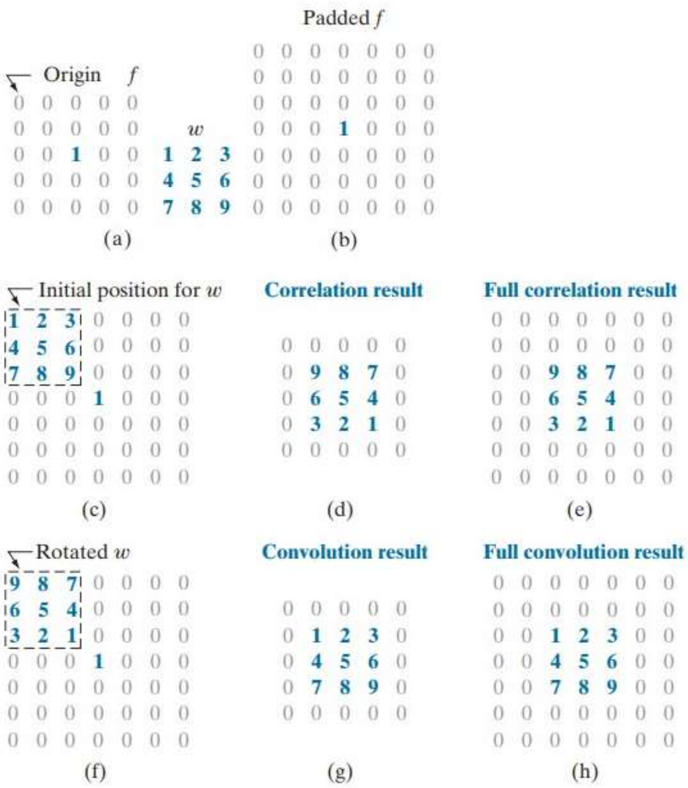


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D kernel with an image consisting of a discrete unit impulse. The 0's are shown in gray to simplify visual analysis. Note that correlation and convolution are functions of x and y. As these variable change, they *displace* one function with respect to the other. See the discussion of Eqs. (3-36) and (3-37) regarding full correlation and convolution.

3.20 The kernel, w, in Problem 3.18 is separable.

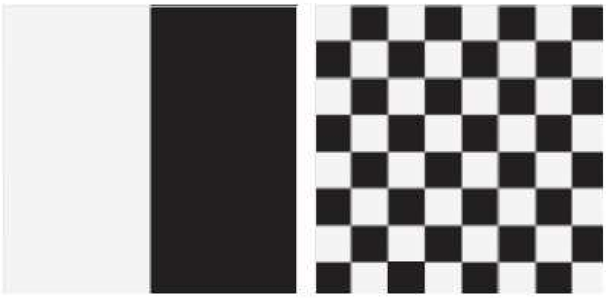
(a)* By inspection, find two kernels, w_1 and w_2 so that $w = w_1 \star w_2$.

(b) Using the image in Problem 3.18, compute $w_1 \star f$ using the *minimum* zero padding (see Fig. 3.30). Show the details of your computation when the kernel is centered at point (2,3) (2nd row, 3rd col) of f and then show the full convolution.

(c) Compute the convolution of w_2 with the result from (b). Show the details of your computation when the kernel is centered at point (3,3) of the result from (b), and then show the full convolution. Compare with the result in Problem 3.18(b).

3.26 The two images shown in the following figure are quite different, but their histograms are the same. Suppose that each image is blurred using a 3×3 box kernel.

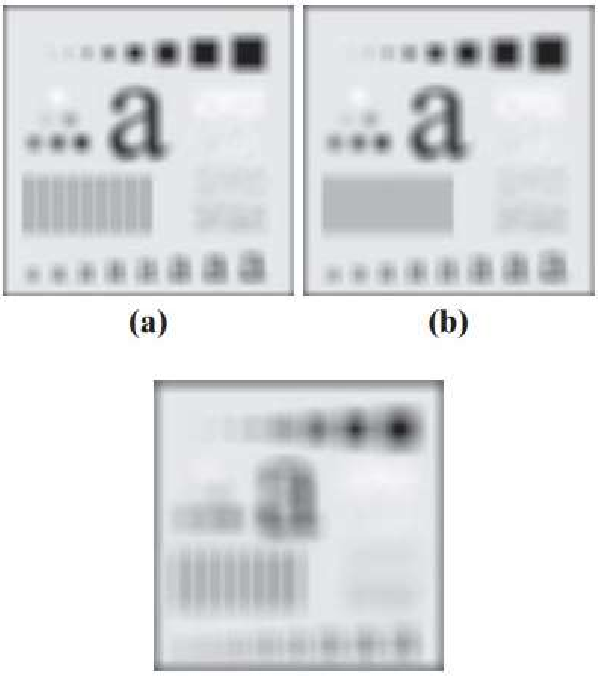
(a)* Would the histograms of the blurred images still be equal? Explain.



(b) If your answer is no, either sketch the two histograms or give two tables detailing the histogram components.

3.29* Discuss the limiting effect of repeatedly filtering an image with a 3×3 lowpass filter kernel. You may ignore border effects.

3.34 In the original image used to generate the three blurred images shown, the vertical bars are 5 pixels wide, 100 pixels high, and their separation is 20 pixels. The image was blurred using square box kernels of sizes 23, 25, and 45 elements on the side, respectively. The vertical bars on the left, lower part of (a) and (c) are blurred, but a clear separation exists between them.



(a) (b) (c)

However, the bars have merged in image (b), despite the fact that the kernel used to generate this image is much smaller than the kernel that produced image (c). Explain the reason for this.

3.41* Give a 3×3 kernel for performing unsharp masking in a single pass through an image. Assume that the average image is obtained using a box filter of size 3×3.

4.7 A function, $f(t)$, is formed by the sum of three functions, $f_1(t) = A\sin(\pi t)$, $f_2(t) = B\sin(4\pi t)$, and $f_3(t) = C\cos(8\pi t)$.

(a) Assuming that the functions extend to infinity in both directions, what is the highest frequency of $f(t)$? (*Hint:* Start by finding the period of the sum of the three functions.)

(b)* What is the Nyquist rate corresponding to your result in (a)? (Give a numerical answer.)

(c) At what rate would you sample $f(t)$ so that perfect recovery of the function from its samples is possible?

4.9 Show that the following expressions are true. (*Hint:* Make use of the solution to Problem 4.8):

(a)* $\Im\{\cos(2\pi\mu_0 t)\} = \frac{1}{2}[\delta(\mu - \mu_0) + \delta(\mu + \mu_0)]$

(b) $\Im\{\sin(2\pi\mu_0 t)\} = \frac{1}{2j}[\delta(\mu - \mu_0) - \delta(\mu + \mu_0)]$

Hint: **4.8*** Show that $\Im\{e^{j2\pi t_0 t}\} = \delta(\mu - t_0)$, where t_0 is a constant. (*Hint:* Study Example 4.2.)

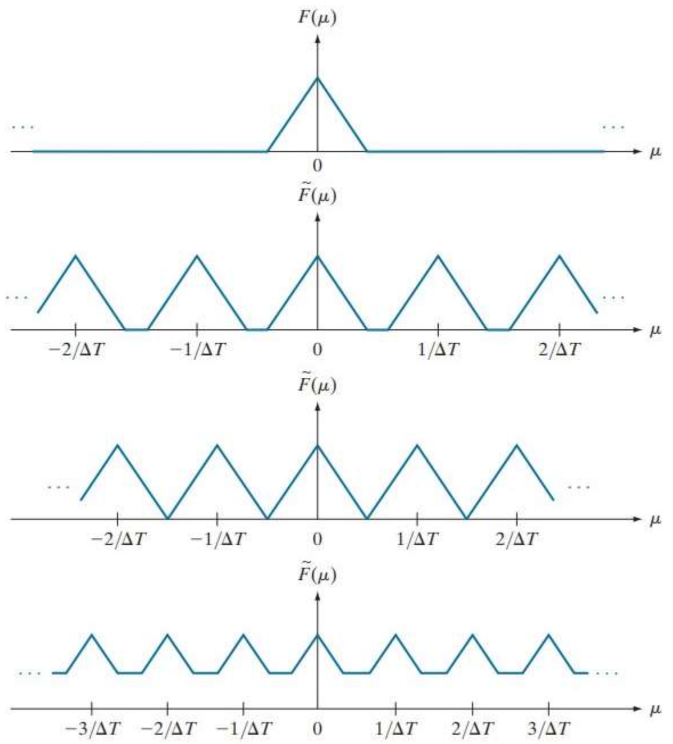


FIGURE 4.6 (a) Illustrative sketch of the Fourier transform of a band-limited function. (b)–(d) Transforms of the corresponding sampled functions under the conditions of over-sampling, critically sampling, and under-sampling, respectively.

4.10 Consider the function $f(t) = \sin(2\pi n t)$, where n is an integer. Its Fourier transform, $F(\mu)$, is purely imaginary (see Problem 4.9). Because the transform, $\tilde{F}(\mu)$, of sampled data consists of periodic copies of $F(\mu)$, it follows that $\tilde{F}(\mu)$ will also be purely imaginary. Draw a diagram similar to Fig. 4.6, and answer the following questions based on your diagram (assume that sampling starts at $t = 0$).

(a)* What is the period of $f(t)$?

(b)* What is the frequency of $f(t)$?

(c)* What would the sampled function and its Fourier transform look like in general if $f(t)$ is sampled at a rate higher than the Nyquist rate?

(d) What would the sampled function look like in general if $f(t)$ is sampled at a rate lower than the Nyquist rate?

(e) What would the sampled function look like if $f(t)$ is sampled at the Nyquist rate, with samples taken at $t = 0, \pm\Delta T, \pm 2\Delta T, \ldots$?