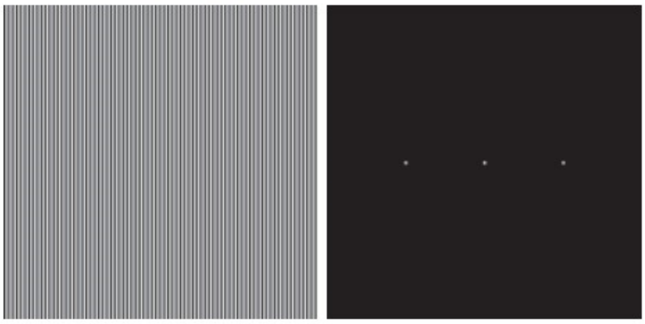
4.21 The image on the left in the figure below consists of alternating stripes of black/white, each stripe



being two pixels wide. The image on the right is the Fourier spectrum of the image on the left, showing the dc term and the frequency terms corresponding to the stripes. (Remember, the spectrum is symmetric so all components, other than the dc term, appear in two symmetric locations.)

(a)* Suppose that the stripes of an image of the same size are four pixels wide. Sketch what the spectrum of the image would look like, including only the dc term and the two highest-value frequency terms, which correspond to the two spikes in the spectrum above.

(b) Why are the components of the spectrum limited to the horizontal axis?

(c) What would the spectrum look like for an image of the same size but having stripes that are one pixel wide? Explain the reason for your answer.

(d) Are the dc terms in (a) and (c) the same, or are they different? Explain.

4.28 Show the validity of the following 2-D *discrete* Fourier transform pairs from Table 4.4:

(a)* $\delta(x,y) \Leftrightarrow 1$

(b)* $1 \Leftrightarrow MN\delta(u,v)$

(c) $\delta(x-x_0, y-y_0) \Leftrightarrow e^{-j2\pi(ux_0/M + vy_0/N)}$

(d)* $e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow MN\delta(u-u_0, v-v_0)$

(e) $\cos(2\pi\mu_0 x/M + 2\pi\nu_0 y/N) \Leftrightarrow$
$$(MN/2)\left[\delta(u+\mu_0, v+v_0) + \delta(u-u_0, v-v_0)\right]$$

(f)* $\sin(2\pi\mu_0 x/M + 2\pi\nu_0 y/N) \Leftrightarrow$
$$(jMN/2)\left[\delta(u+\mu_0, v+v_0) - \delta(u-u_0, v-v_0)\right]$$

TABLE 4.4
Summmary of DFT pairs. The closed-form expressions in 12 and 13 are valid only for continuous variables. They can be used with discrete variables by sampling the continuous expressions.

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x,y) + bf_2(x,y) \Leftrightarrow aF_1(u,v) + bF_2(u,v)$
3) Translation (general)	$f(x,y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$ $f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x,y)(-1)^{x+y} \Leftrightarrow F(u-M/2, v-N/2)$ $f(x-M/2, y-N/2) \Leftrightarrow F(u,v)(-1)^{u+v}$
5) Rotation	$f(r, \theta+\theta_0) \Leftrightarrow F(\omega, \varphi+\theta_0)$ $r = \sqrt{x^2+y^2} \quad \theta = \tan^{-1}(y/x) \quad \omega = \sqrt{u^2+v^2} \quad \varphi = \tan^{-1}(v/u)$
6) Convolution theorem†	$(f \star h)(x,y) \Leftrightarrow (F \bullet H)(u,v)$ $(f \bullet h)(x,y) \Leftrightarrow (1/MN)[(F \star H)(u,v)]$
7) Correlation theorem†	$(f \star h)(x,y) \Leftrightarrow (F^* \bullet H)(u,v)$ $(f^* \bullet h)(x,y) \Leftrightarrow (1/MN)[(F \star H)(u,v)]$
8) Discrete unit impulse	$\delta(x,y) \Leftrightarrow 1$ $1 \Leftrightarrow MN\delta(u,v)$
9) Rectangle	$\text{rec}[a,b] \Leftrightarrow ab\dfrac{\sin(\pi ua)}{(\pi ua)}\dfrac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0x/M + 2\pi v_0y/N) \Leftrightarrow \dfrac{jMN}{2}[\delta(u+u_0, v+v_0) - \delta(u-u_0, v-v_0)]$
11) Cosine	$\cos(2\pi u_0x/M + 2\pi v_0y/N) \Leftrightarrow \dfrac{1}{2}[\delta(u+u_0, v+v_0) + \delta(u-u_0, v-v_0)]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) Differentiation (the expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.	$\left(\dfrac{\partial}{\partial t}\right)^m \left(\dfrac{\partial}{\partial z}\right)^n f(t,z) \Leftrightarrow (j2\pi\mu)^m (j2\pi\nu)^n F(\mu,\nu)$ $\dfrac{\partial^m f(t,z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu,\nu); \quad \dfrac{\partial^n f(t,z)}{\partial z^m} \Leftrightarrow (j2\pi\nu)^n F(\mu,\nu)$
13) Gaussian	$A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

†Assumes that $f(x,y)$ and $h(x,y)$ have been properly padded. Convolution is associative, commutative, and distributive. Correlation is distributive (see Table 3.5). The products are elementwise products (see Section 2.6).

4.43 Consider the images shown. The image on the right was obtained by: (a) multiplying the image on the left by $(-1)^{x+y}$; (b) computing the DFT; (c) taking the complex conjugate of the transform; (d) computing the inverse DFT; and (e) multiplying the real part of the result by $(-1)^{x+y}$. Explain (mathematically) why the image on the right appears as it does.

4.52 Do the following:

(a) Show that the Laplacian of a continuous function $f(t,z)$ of two continuous variables, t and z, satisfies the following Fourier transform pair:

$$\nabla^2 f(t,z) \Leftrightarrow -4\pi^2(\mu^2 + \nu^2)F(\mu,\nu)$$

(*Hint:* See Eq. (3-50) and study entry 12 in Table 4.4.)

(b)* The result in (a) is valid only for continuous variables. How would you implement the continuous frequency domain transfer function $H(\mu,\nu) = -4\pi^2(\mu^2 + \nu^2)$ for discrete variables?

(c) As you saw in Example 4.21, the Laplacian result in the frequency domain was similar to the result in Fig. 3.46(d), which was obtained using a spatial kernel with a center coefficient equal to -8. Explain why the frequency domain result was not similar instead to the result in Fig. 3.46(c), which was obtained using a kernel with a center coefficient of -4.

(a) Hint:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \qquad (3\text{-}50)$$

a	b
c	d

FIGURE 3.46 (a) Blurred image of the North Pole of the moon. (b) Laplacian image obtained using the kernel in Fig. 3.45(a). (c) Image sharpened using Eq. (3-54) with $c = -1$. (d) Image sharpened using the same procedure, but with the kernel in Fig. 3.45(b). (Original image courtesy of NASA.)

5.10 In answering the following, refer to the contraharmonic filter in Eq. (5-26) :

(a)* Explain why the filter is effective in eliminating pepper noise when Q is positive.

(b) Explain why the filter is effective in eliminating salt noise when Q is negative.

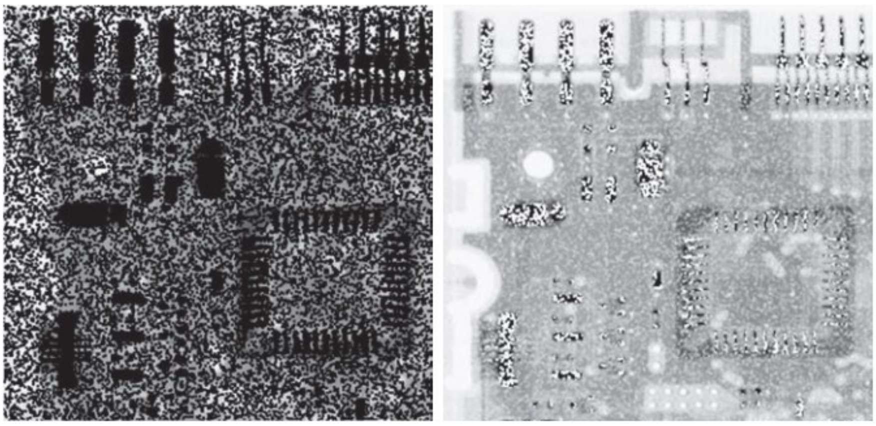
(c)* Explain why the filter gives poor results (such as the results in Fig. 5.9) when the wrong polarity is chosen for Q.

(d) Discuss the expected behavior of the filter when $Q = -1$.

$$\hat{f}(x,y) = \frac{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}} \tag{5-26}$$

a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering Fig. 5.8(b) using $Q = 1.5$.



5.18 An industrial plant manager has been promoted to a new position. His first responsibility is to characterize an image filtering system left by his predecessor. In reading the documentation, the manager discovers that his predecessor established that the system is linear and position invariant. Furthermore, he learns that experiments conducted under negligible-noise conditions resulted in an impulse response that could be expressed analytically in the frequency domain as

$$H(u,v) = e^{-[u^2/150\, +\, v^2/150]} + 1 - e^{-[(u\, -\, 50)^2/150\, +\, (v\, -\, 50)^2/150]}$$

The manager is not a technical person, so he employs you as a consultant to determine what, if anything, he needs to do to complete the characterization of the system. He also wants to know the function that the system performs. What (if anything) does the manager need to do to complete the characterization of his system? What filtering function does the system perform?

5.21* Consider a linear, position invariant image degradation system with impulse response

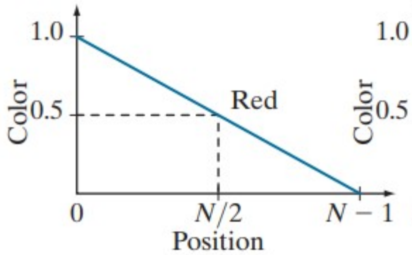
$$h(x,y) = e^{-\left[(x-\alpha)^2 + (y-\beta)^2\right]}$$

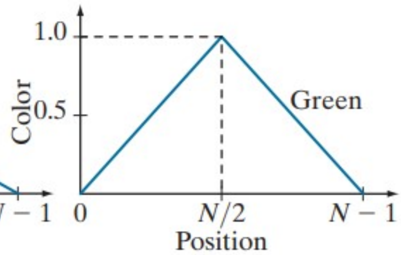
where x and y are continuous variables. Suppose that the input to the system is a binary image consisting of a white vertical line of infinitesimal width located at $x = a$, on a black background. Such an image can be modeled as $f(x,y) = \delta(x-a)$. Assume negligible noise and use Eq. (5-61) to find the output image, $g(x,y)$.

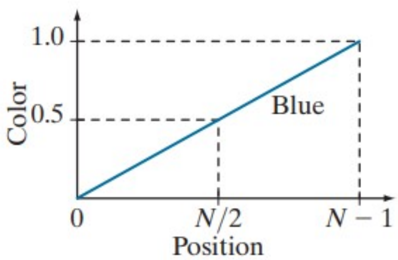
$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)\,d\alpha\,d\beta \tag{5-61}$$

5.27* Consider image blurring caused by uniform acceleration in the x-direction. If the image is at rest at time $t = 0$ and accelerates with a uniform acceleration $x_0(t) = at^2/2$ for a time T, find the blurring function $H(u,v)$. You may assume that shutter opening and closing times are negligible.

6.5 The R, G, and B component images of an RGB image have the horizontal intensity profiles shown in the following diagram. What color would a person see in the middle column of this image?





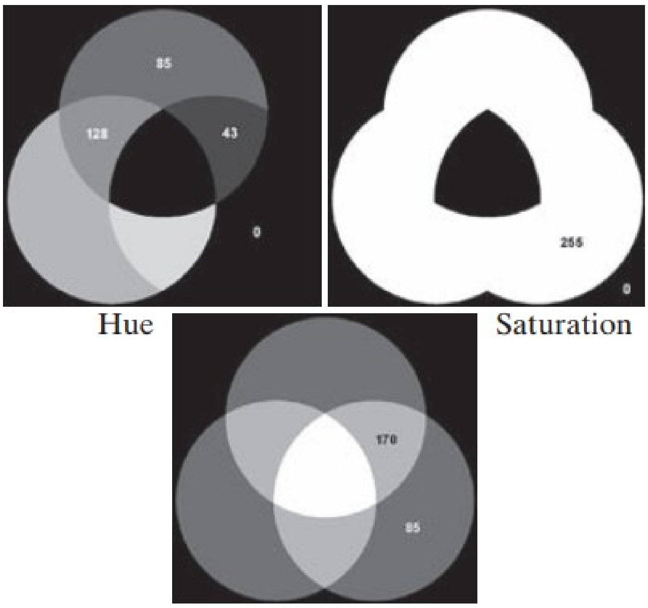


6.14 The following 8-bit images are the H, S, and I component images from Fig. 6.14. The numbers indicate gray-level values. Answer the following questions, explaining the basis for your answer in each. If it is not possible to answer a question based on the given information, state why you cannot do so.

(a)* Give the gray-level values of all regions in the hue image.

(b) Give the gray-level value of all regions in the saturation image.

(c) Give the gray-level values of all regions in the intensity image.

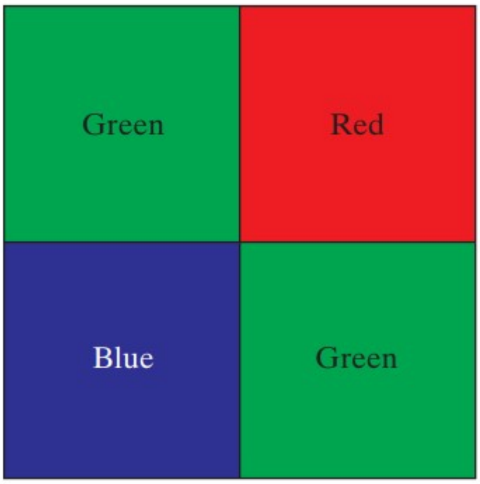


Hue

Saturation

Intensity

6.25 Consider the following 500×500 RGB image, in which the squares are fully saturated red, green, and blue, and each of the colors is at maximum intensity. An HSI image is generated from this image. Answer the following questions.



(a) Describe the appearance of each HSI component image.

(b)* The saturation component of the HSI image is smoothed using an averaging kernel of size 125×125. Describe the appearance of the result. (You may ignore image border effects in the filtering operation.)

(c) Repeat (b) for the hue image.

6.26 Answer the following.

(a)* Refer to the discussion in Section 6.7 about segmentation in the RGB color space. Give a procedure (in flow chart form) for determining whether a color vector (point) $\mathbf{z}$ is inside a cube with sides W, centered at an average color vector $\mathbf{a}$. Distance computations are not allowed.

(b) If the box is aligned with the axes this process also can be implemented on an image-by-image basis. Show how you would do it.