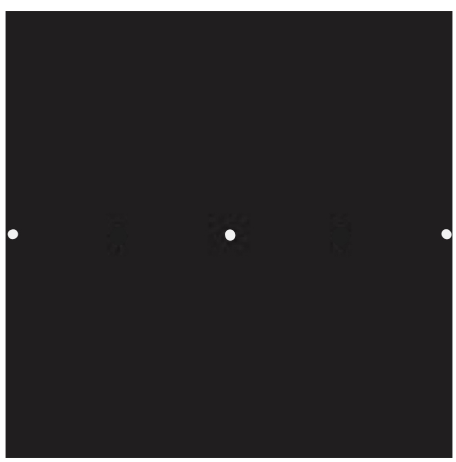
Solutions for HW5

4.21 (a)

(b) Because there is no change in the vertical axis, the image contains only horizontal axis frequency components.

(c)



(d) Dc terms are the sum of intensity values in the image. Since the number of black and white pixels in (a) and (c) are approximately equal (depends on the image size), we can say the dc terms are the same.

4.28 (a)

$$\Im\{\delta(x,y)\} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\delta(x,y)e^{-j2\pi(ux/M+vy/N)}$$
$$= e^{-j2\pi(u[0]/M+v[0]/N)} = 1$$

Because we used direct substitution into Eq. 4-67, and this equation and Eq. 4-68 are a Fourier transform pair, it must follows that the left side of the double arrow is the IDFT of right: $\Im^{-1}\{1\} = \delta(x,y)$.

(b) $\Im^{-1}\{MN\delta(u,v)\} = \frac{MN}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\delta(u,v)e^{j2\pi(ux/M+vy/N)}$
$$= e^{j2\pi(0x/M+0y/N)} = 1$$

Because we used direct substitution into Eq. 4-68, and this equation and Eq. 4-67 are a Fourier transform pair, it must follows that the right side of the double arrow is the DFT of left: $\Im\{1\} = MN\delta(u,v)$.

(c) $\Im\{\delta(x-x_0,y-y_0)\} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\delta(x-x_0,y-y_0)e^{-j2\pi(ux/M+vy/N)}$
$$= e^{-j2\pi(ux_0/M+vy_0/N)}$$

Because we used direct substitution into Eq. 4-67, and this equation and Eq. 4-68 are a Fourier transform pair, it must follows that the left side of the double arrow is the IDFT of right: $\Im^{-1}\{e^{-j2\pi(ux_0/M+vy_0/N)}\} = \delta(x-x_0,y-y_0)$.

(d) $\Im^{-1}\{MN\delta(u-u_0,v-v_0)\} = \frac{MN}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\delta(u-u_0,v-v_0)e^{j2\pi(ux/M+vy/N)}$
$$= e^{j2\pi(u_0x/M+v_0y/N)}$$

Because we used direct substitution into Eq. 4-68, and this equation and Eq. 4-67 are a Fourier transform pair, it must follows that the right side of the double arrow is the DFT of left: $\Im\{e^{j2\pi(u_0x/M+v_0y/N)}\} = MN\delta(u-u_0,v-v_0)$.

(e) $\Im\{cos(2\pi\mu_0x/M+2\pi v_0y/N)\}$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\frac{e^{j2\pi(\mu_0x/M+v_0y/N)}+e^{-j2\pi(\mu_0x/M+v_0y/N)}}{2}\right]e^{-j2\pi(\mu x/M+vy/N)}$$
$$= \frac{1}{2}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}e^{j2\pi(\mu_0x/M+v_0y/N)}e^{-j2\pi(\mu x/M+vy/N)}$$
$$+\frac{1}{2}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}e^{-j2\pi(\mu_0x/M+v_0y/N)}e^{-j2\pi(\mu x/M+vy/N)}$$
$$= \frac{MN}{2}[\delta(u-u_0,v-v_0)+\delta(u+u_0,v+v_0)]$$
$$= \frac{MN}{2}[\delta(u+u_0,v+v_0)+\delta(u-u_0,v-v_0)]$$

(f) $\Im\{sin(2\pi\mu_0x/M+2\pi v_0y/N)\}$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\frac{e^{j2\pi(\mu_0x/M+v_0y/N)}-e^{-j2\pi(\mu_0x/M+v_0y/N)}}{2j}\right]e^{-j2\pi(\mu x/M+vy/N)}$$
$$= \frac{1}{2j}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}e^{j2\pi(\mu_0x/M+v_0y/N)}e^{-j2\pi(\mu x/M+vy/N)}$$
$$-\frac{1}{2j}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}e^{-j2\pi(\mu_0x/M+v_0y/N)}e^{-j2\pi(\mu x/M+vy/N)}$$

$= \frac{MN}{2j}[\delta(u-u_0, v-v_0) - \delta(u+u_0, v+v_0)]$
$= \frac{jMN}{2}[\delta(u+u_0, v+v_0) - \delta(u-u_0, v-v_0)]$

4.43 The complex conjugate simply changes j to -j in the inverse transform, so the image on the right is given by

$\Im^{-1}[F^*(u,v)] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)e^{-j2\pi(ux/M+vy/N)}$
$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)e^{j2\pi(u(-x)/M+v(-y)/N)}$
$= f(-x,-y)$

which simply mirrors $f(x,y)$ about the origin, thus producing the image on the right.

4.52 (a)

$\Im\{\nabla^2 f(t,z)\} = \Im\{\frac{\partial^2 f(t,z)}{\partial t^2} + \frac{\partial^2 f(t,z)}{\partial z^2}\}$
$= \Im\{\frac{\partial^2 f(t,z)}{\partial t^2}\} + \Im\{\frac{\partial^2 f(t,z)}{\partial z^2}\}$
$= (j2\pi\mu)^2 F(\mu,v) + (j2\pi v)^2 F(\mu,v)$
$= -4\pi^2(\mu^2+v^2)F(\mu,v)$

(b) Generate a $P \times Q$ array centered on $[P/2, Q/2]$:

$H(u,v) = -4\pi^2([u-P/2]^2 + [v-Q/2]^2)$

for $u = 0,1,2,...,P-1$ and $v = 0,1,2,...,Q-1$ where P and Q are sizes to which the input image is padding prior to filtering. Then use $H(u,v)$ as any other filter transfer function.

(c) Laplacian computed in the frequency domain encompasses the entire image. Since the spatial kernel used in Fig.3.46(d) considers more pixels in the neighborhood compared with the kernel used in Fig.3.46(c), the result in Fig.3.46(d) is closer to the Laplacian result in the frequency domain.

5.10 The key to understanding the behavior of the contra-harmonic filter is to think of the pixels in the neighborhood surrounding a noise impulse as being constant, with the impulse noise point being in the center of the neighborhood. For the noise spike to be visible, its value must be considerably larger than the value of its neighbors. Also keep in mind that the power in the numerator is 1 plus the power in the denominator.

(a) By definition, pepper noise is a low value (really 0). It is most visible when surrounded by light values. Then center pixel (the pepper noise), will have little influence in the sums. If the area spanned by the filter is approximately constant, the ratio will approach the value of the pixels in the neighborhood, thus reducing the effect of the low-value pixel.

(b) The reverse happens when the center point is large and its neighbors are small. The center pixel will now be the largest. However, the exponent is now negative, so the small numbers will dominate the result. The numerator can then be thought of a constant raised to the power Q + 1 and the denominator as a the same constant raised to the power Q. That constant is the value of the pixels in the neighborhood. So the ratio is just that value.

(c) When the wrong polarity is used the large numbers in the case of the salt noise will be raised to a positive power, thus the noise will overpower the result. For salt noise the image will become very light. The opposite is true for pepper noise - the image will become dark.

(d) When Q = -1, the value of the numerator becomes equal to the number of pixels in the neighborhood (m x n). The value of the denominator become sum values, each of which is 1 over the value of a pixel in the neighborhood. This is the same as the average of 1/A, where A is the image average.

5.18 The filtering function of system can be divided into two part. First part is a Gaussian lowpass filter, and second part is a Gaussian highpass filter with center at (50, 50).

5.21 The impulse response given by textbook is wrong. The correct impulse response is $h(x, y) = e^{-[x^2+y^2]}$.

Using the given $f(x, y) = \delta(x - a)$, we have $f(\alpha, \beta) = \delta(\alpha - a)$. Then, using the impulse response above,

$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\alpha - a)e^{-[(x-\alpha)^2+(y-\beta)^2]}d\alpha d\beta$
$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\alpha - a)e^{-[(x-\alpha)^2]}e^{-[(y-\beta)^2]}d\alpha d\beta$
$= \int_{-\infty}^{\infty} \delta(\alpha - a)e^{-[(x-\alpha)^2]}d\alpha \int_{-\infty}^{\infty} e^{-[(y-\beta)^2]}d\beta$
$= e^{-[(x-a)^2]} \int_{-\infty}^{\infty} e^{-[(y-\beta)^2]}d\beta$

and

$e^{-[(y-\beta)^2]} = e^{-[(\beta-y)^2]} = \sqrt{2\pi(1/2)}\left[\frac{1}{\sqrt{2\pi(1/2)}}e^{-(1/2)[\frac{(\beta-y)^2}{(1/2)}]}\right],$

where $\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi(1/2)}}e^{-(1/2)[\frac{(\beta-y)^2}{(1/2)}]}d\beta = 1$ (Gaussian).

So we have that

$g(x,y) = \sqrt{\pi}e^{-[(x-a)^2]}.$

5.27 From Eq.(5-74),

$H(u,v) = \int_0^T e^{-j2\pi u x_0(t)}dt = \int_0^T e^{-j2\pi u[at^2/2]}dt = \int_0^T e^{-j\pi u a t^2}dt$

Using Euler's formula we obtain

$\int_0^T e^{-j\pi uat^2}dt = \int_0^T[cos(\pi uat^2) - jsin(\pi uat^2)]dt = \sqrt{\frac{T^2}{2\pi uaT^2}}[C(\sqrt{\pi ua}T) - jS(\sqrt{\pi ua}T)]$

where the forms

$C(z) = \int_0^z cost^2 dt$ and $S(z) = \int_0^z sint^2 dt$

are Fresnel cosine and sine integrals.

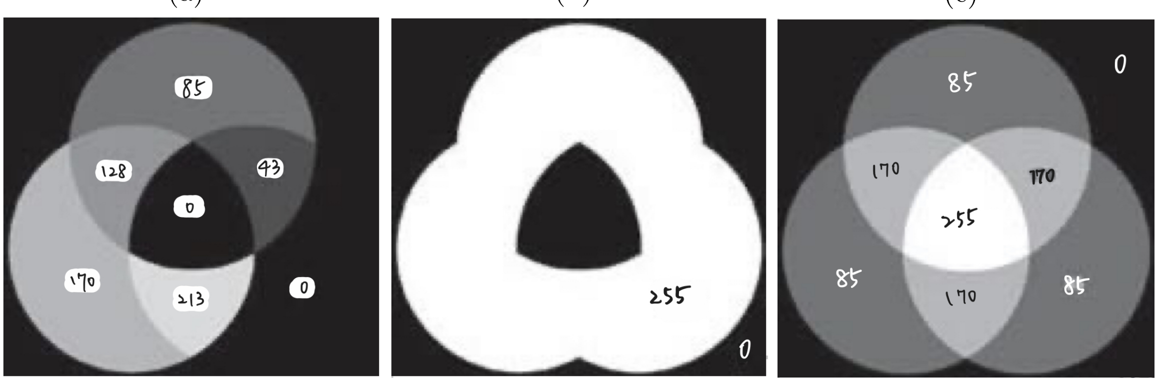
6.5 At the center point we have

$\frac{1}{2}R + G + \frac{1}{2}B = \frac{1}{2}(R+G+B) + \frac{1}{2}G = midgray + \frac{1}{2}G$

which looks to a viewer like pure green with a boost in intensity due to the additive gray component.

6.14 .

(a) (b) (c)



6.25 (a)

85	0
170	85

(a) Hue

255	255
255	255

(b) Saturation

85	85
85	85

(c) intensity

(b) The saturation image is constant, so smoothing it will produce the same constant value.

(c)

85	$0 \leftrightarrow 85$	0
$85 \leftrightarrow 170$	$0 \leftrightarrow 170$	$0 \leftrightarrow 85$
170	$85 \leftrightarrow 170$	85

6.26 (a) The cube is composed of 6 intersecting planes in RGB space. The general equation for such planes is $az_R + bz_G + cz_B + d = 0$ where a, b, c, and d are parameters and the z's are the components of any point (vector) z in RGB space lying on the plane. If an RGB point z does not lie on the plane, and its coordinates are substituted in the preceding equation, then equation will give either a positive or a negative valueu it will not yield zero. We say that z lies on the positive or negative side of the plane, depending on whether the result is positive or negative. We can change the positive side of a plane by multiplying its coeffcients (except d) by -1. Suppose that we test the point a given in the problem statement to see whether it is on the positive or negative side each of the six planes composing the box, and change the coeffcients of any plane for which the result is negative. Then, a will lie on the positive side of all planes composing the bounding box. In fact all points inside the bounding box will yield positive values when their coordinates are substituted in the equations of the planes. Points outside the box will give at least one negative or zero value. Thus, the method consists of substituting an unknown color point in the equations of all six planes. If all the results are positive, the point is inside the boxu otherwise it is outside the box.

(b) If the box is lined up with the RGB coordinate axes, then the planes intersect the RGB coordinate planes perpendicularly. The intersections of pairs of parallel planes establish a range of values along each of the RGB axis that must be checked to see if the if an unknown point lies inside the box or not. This can be done on an image per image basis (i.e., the three component images of an RGB image), designating by 1 a coordinate that is within its corresponding range and 0 otherwise. These will produce three binary images which, when ANDed, will give all the points inside the box