Mathematical Programming in Advanced Analytics

Project 3 Deliverables

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Abstract

The purpose of this project is to analyze 60 periods of returns of 100 stocks and to optimize portfolio selection through MATLAB using quadratic programming. Task one performs the calculation to get an efficient frontier plotted with 30 optimal portfolios and the corresponding stock portion for each portfolio.

0.1 Task 1

What is the objective of this task:

To minimize risk of portfolio, which represented by stock variance and covariance, and to maximize the total expected return of the portfolio. Then plot an portfolio efficient frontier.

Why is this task important:

With the efficient frontier, we can make recommendation to clients who are interested in buying a portfolio with certain preferences. The efficient frontier allows us to know what is the highest return at a given point of risk, and what is the lowest risk as a given point of return. And we can then recommend the portfolio, which tells how much portion of the total cash to invest in each stock to get the desired return and risk.

How I did it:

First, I translate the mathematical representation of the objective function into code in MATLAB.

$$\text{Minimize } \lambda\left(\sum_{i=1}^{n}\sigma_i^2 x_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sigma_{i,j}x_i x_j\right) - (1-\lambda)\left(\sum_{i=1}^{n}\bar{r}_i x_i\right)$$

$$\text{s.t.}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0;\ \forall i$$

Figure 1: Mathematical Representation of the Objective Function

```
% The objective function
fun = @(x) lambda*(x * covMatrix * x') - (1-lambda)*sum(averageReturn.*x);
```

Figure 2: MATLAB Code of the Objective Function

Then I set up the constraint matrix, which guarantees I spend all the money and prevents short selling.

```
% No inequality constraints
A = [];
b = [];

% One equality constraint that all the money must be invested and it is
% not allowed to short a stock
Aeq = ones(1,100);
beq = 1;

% Upper bound and lower bound
lb = zeros(1,100);
ub(1:100) = 1;
```

Figure 3: Equality Constraint

After that, we set lambda which represents the emphasis between return and risk to 0 and 1. Then using fmincon to solve the quadratic function will gives us the return upper bound and the return lower bound. fmincon is a

MATLAB solver that finds minimum of constrained nonlinear multivariable function. With the return upper bound and lower bound, we divide the difference between the upper bound and lower bound into 28 steps, which gives us 28 required returns between the upper bound and lower bound. We then change the objective function to just minimize risk and change the equality constraints to include this required return and solve the problem to get the 28 portfolios.

```
fun = @(x) x * covMatrix * x';
requiredReturn = PortfolioReturnLB + (k * step);
Aeq = [ones(1,100);averageReturn];
beq = [1;requiredReturn];
```

Figure 4: New Objective Function in MATLAB Code

Result:

At last, we can plot the return and risk of these 30 portfolios onto the graph and plot the efficient frontier.

This is the Portfolio Efficient Frontier plotted with 30 portfolios:

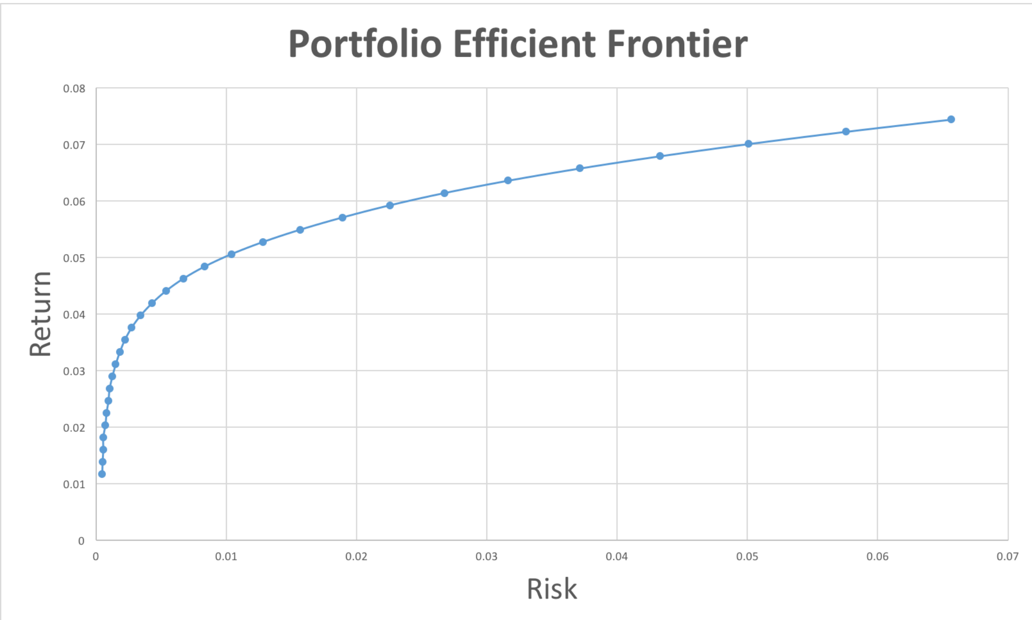

Figure 5: Portfolio Efficient Frontier

The following 30 graphs are the 30 portfolios' composition:

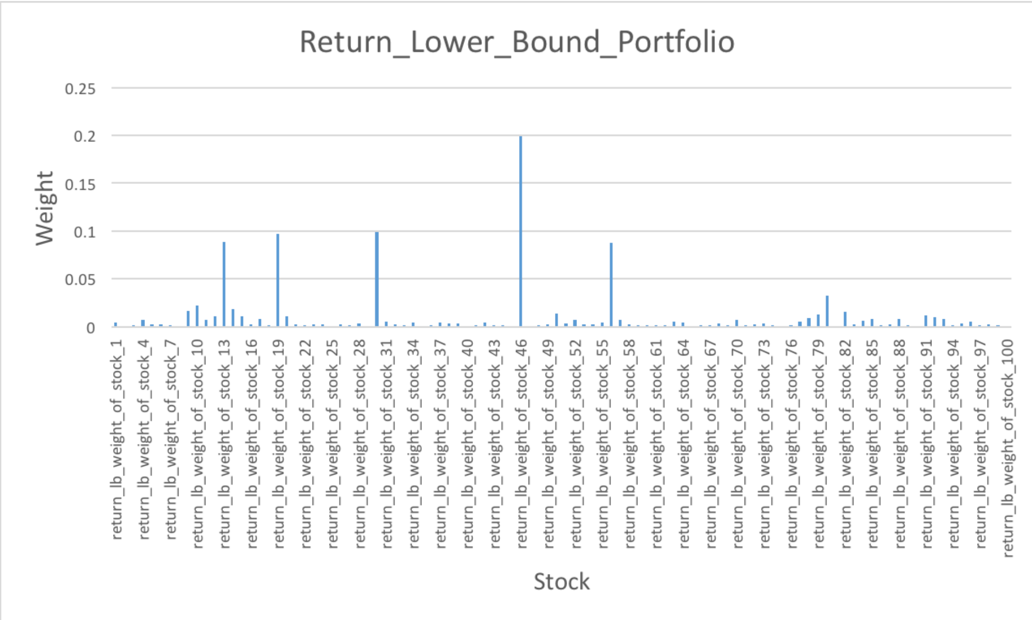

Figure 6: Return Lower Bound Portfolio

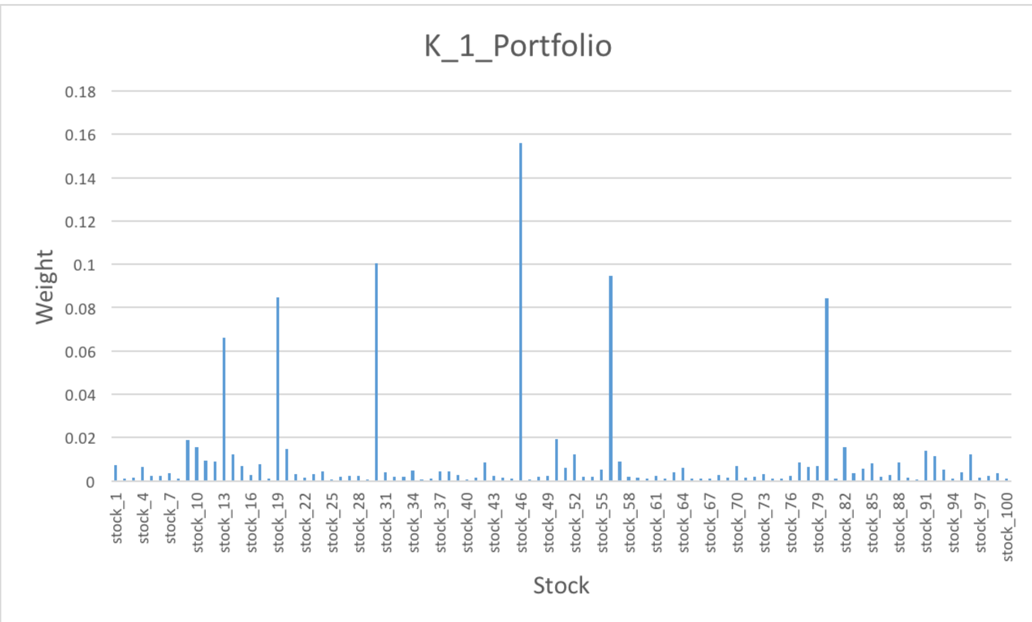

Figure 7: Return Lower Bound Plus 1 Step

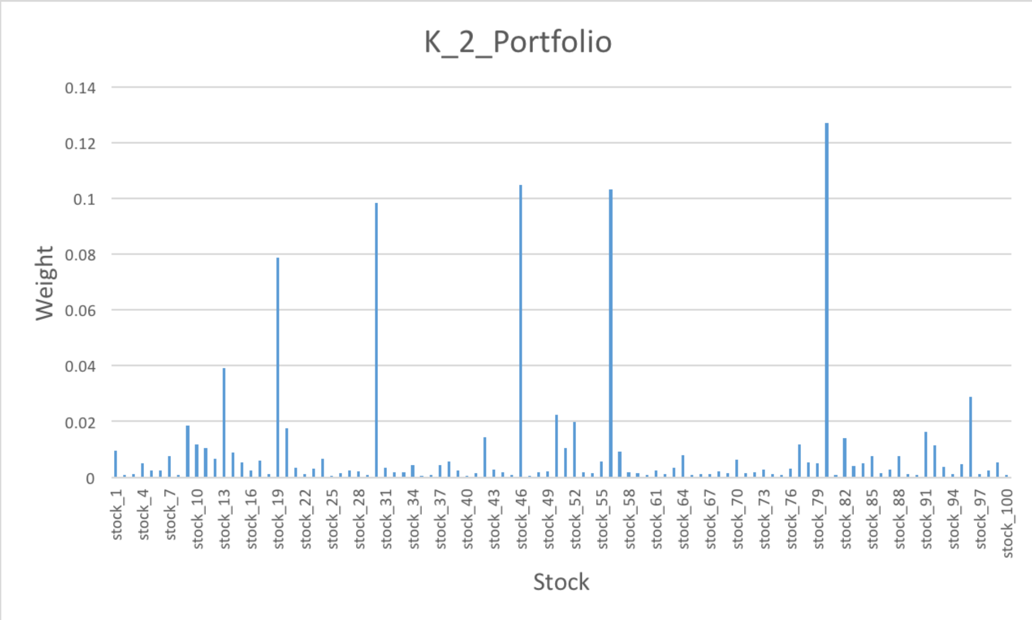

Figure 8: Return Lower Bound Plus 2 Step

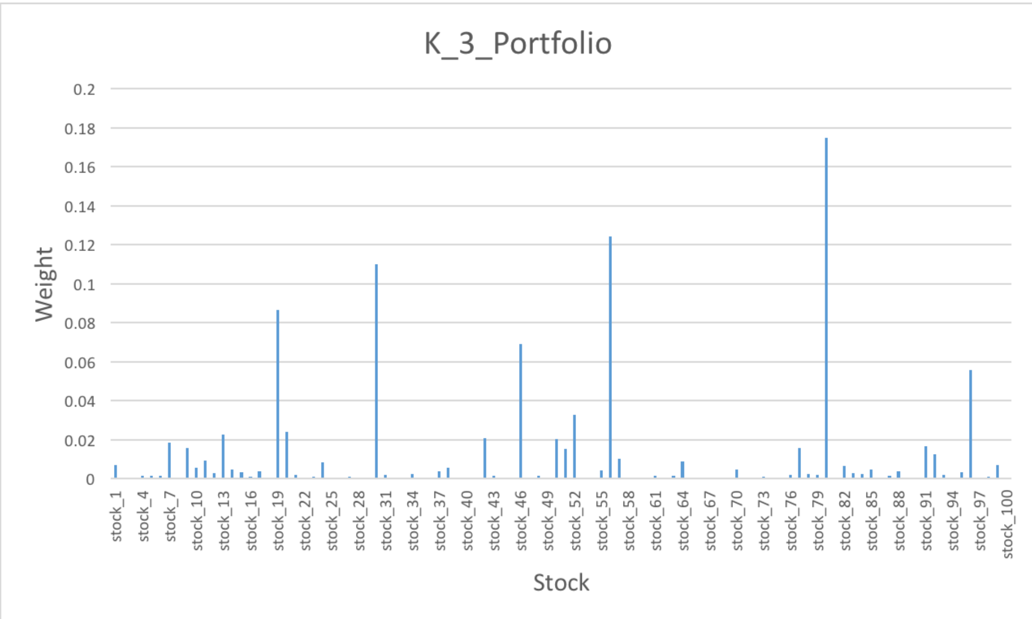

Figure 9: Return Lower Bound Plus 3 Step

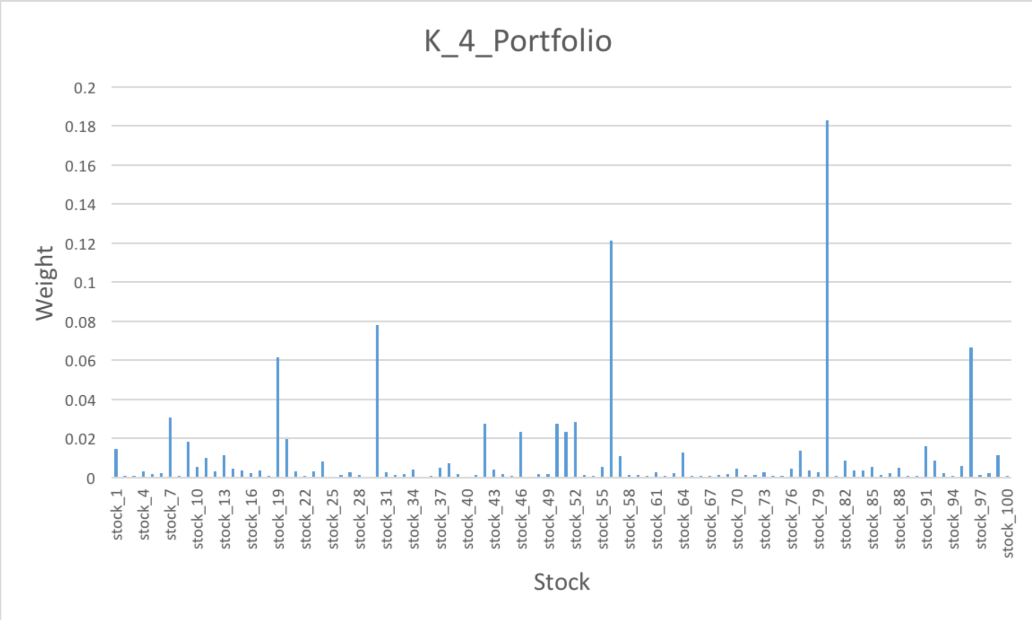

Figure 10: Return Lower Bound Plus 4 Step

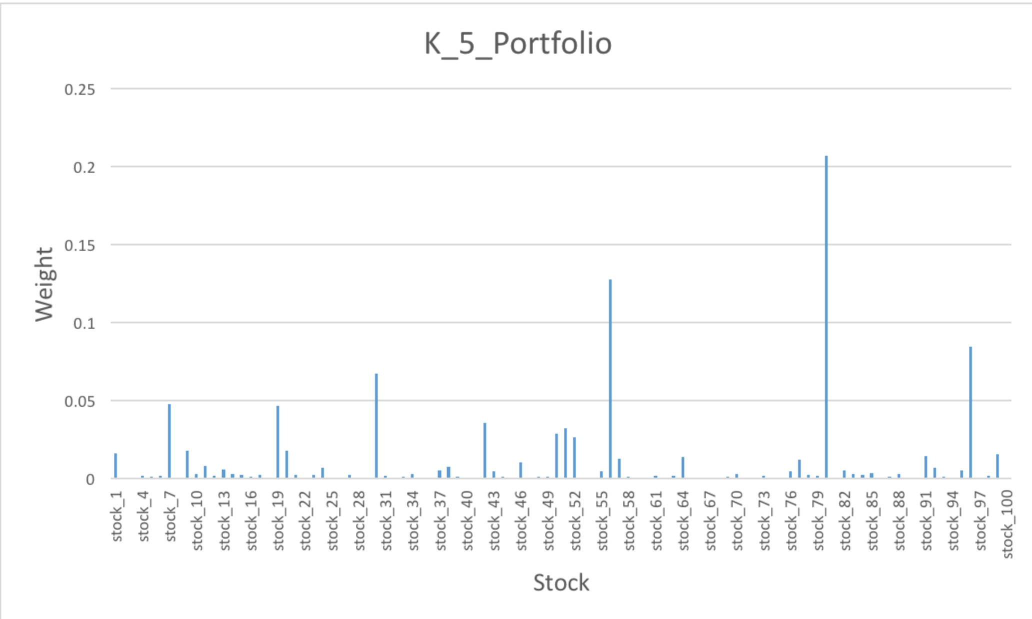

Figure 11: Return Lower Bound Plus 5 Step

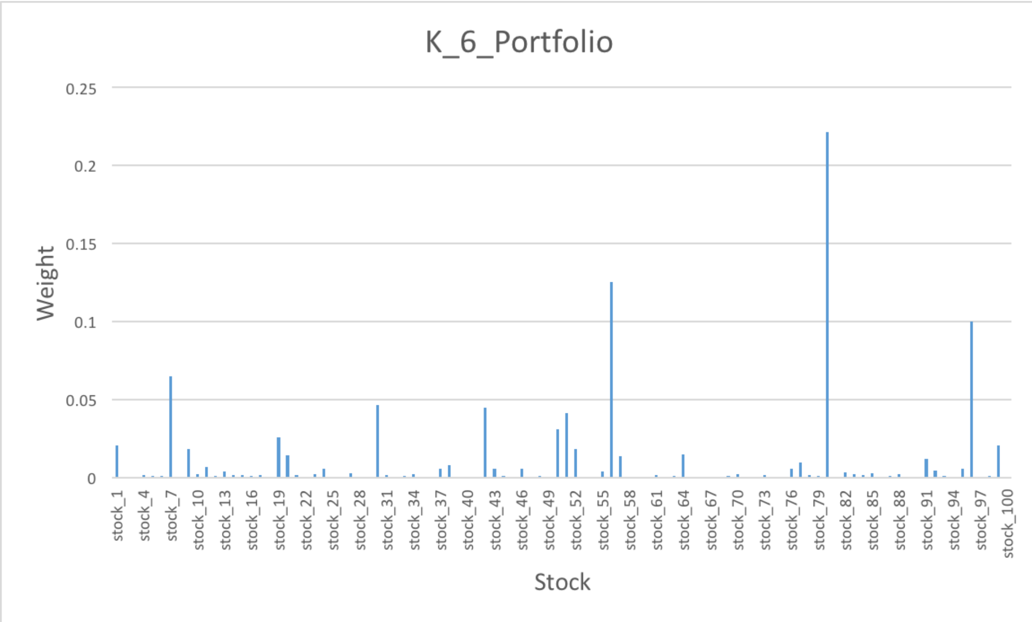

Figure 12: Return Lower Bound Plus 6 Step

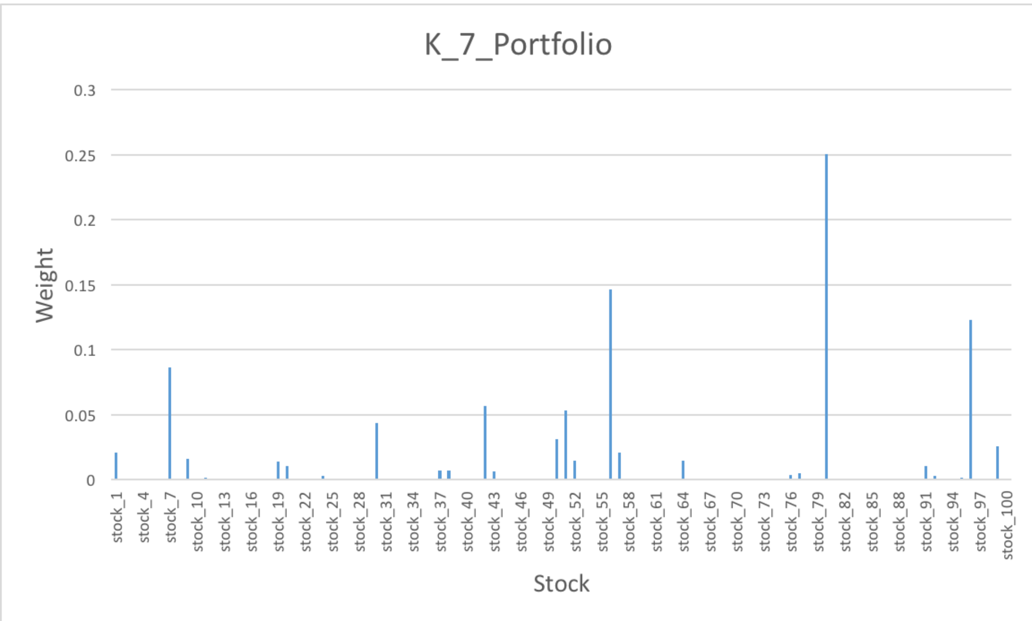

Figure 13: Return Lower Bound Plus 7 Step

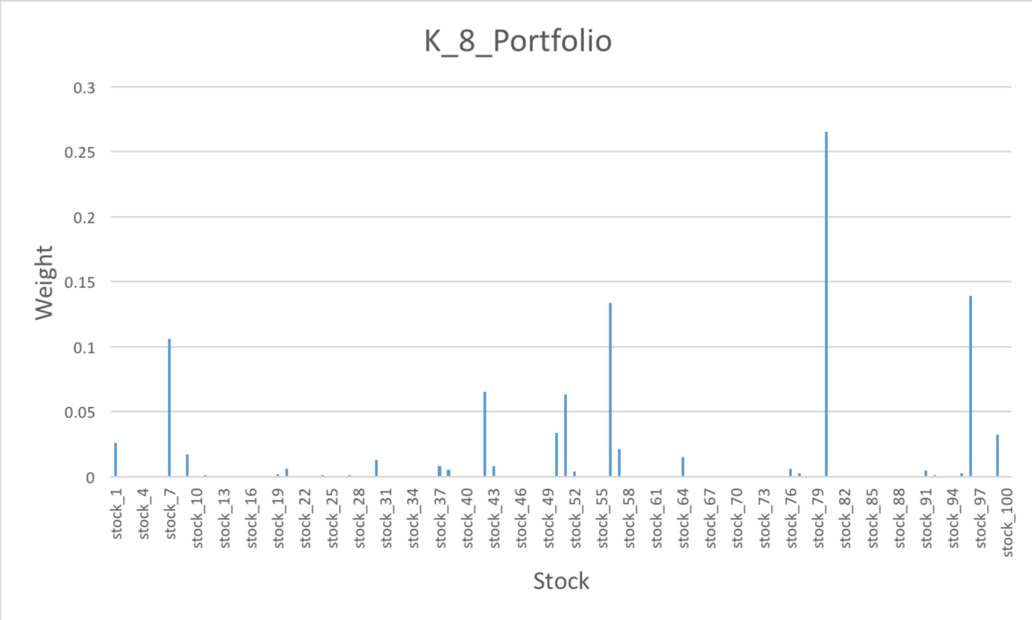

Figure 14: Return Lower Bound Plus 8 Step

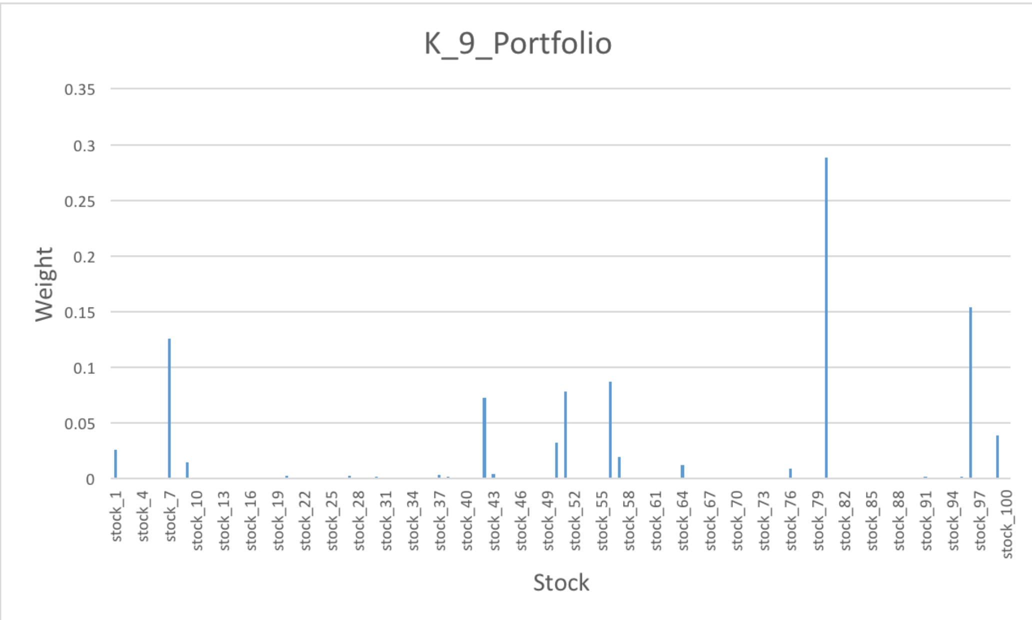

Figure 15: Return Lower Bound Plus 9 Step

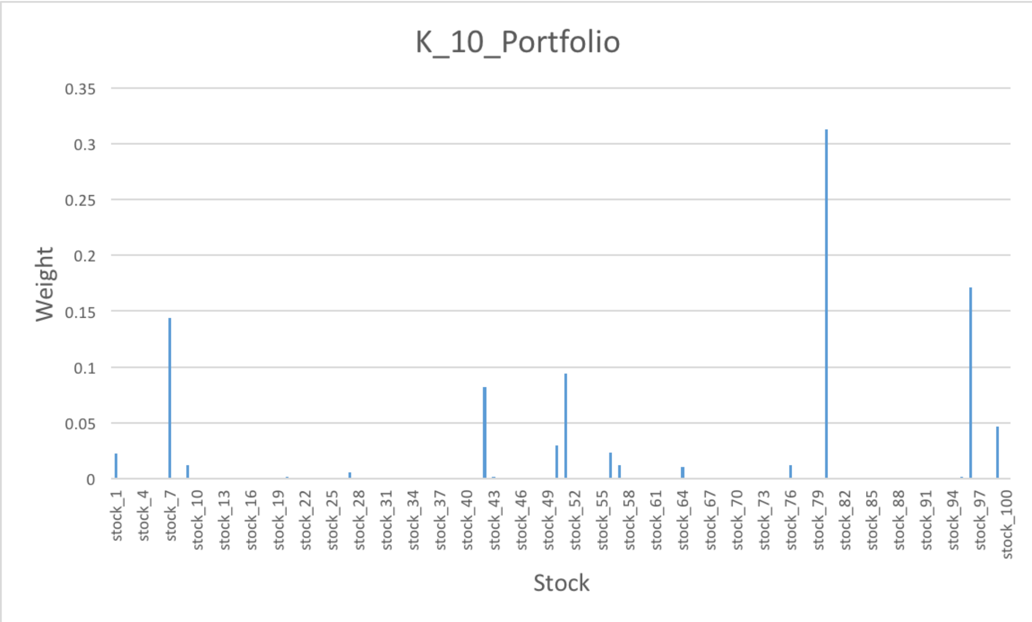

Figure 16: Return Lower Bound Plus 10 Step

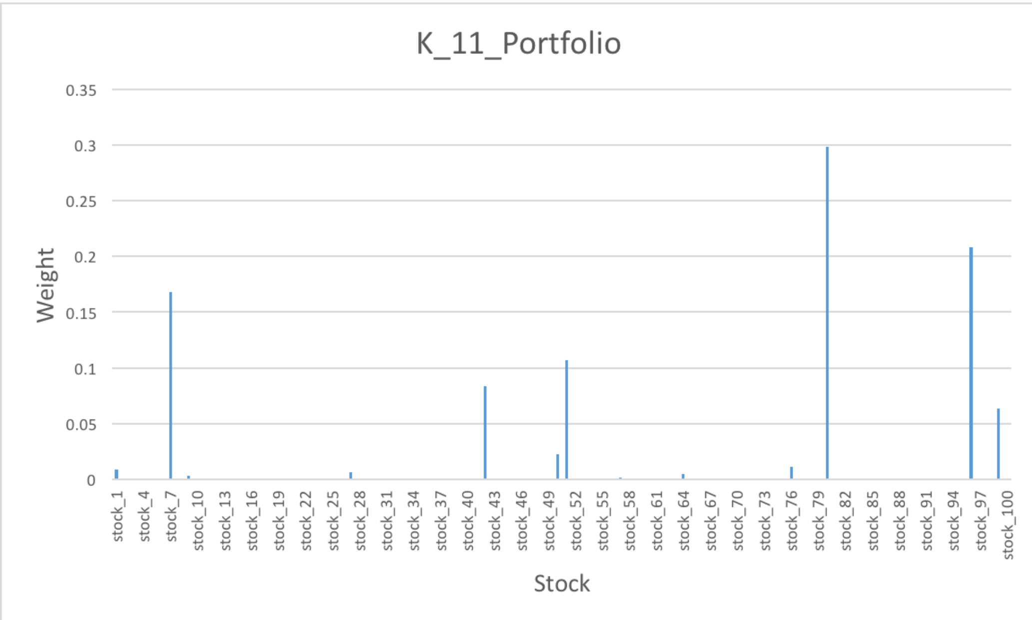

Figure 17: Return Lower Bound Plus 11 Step

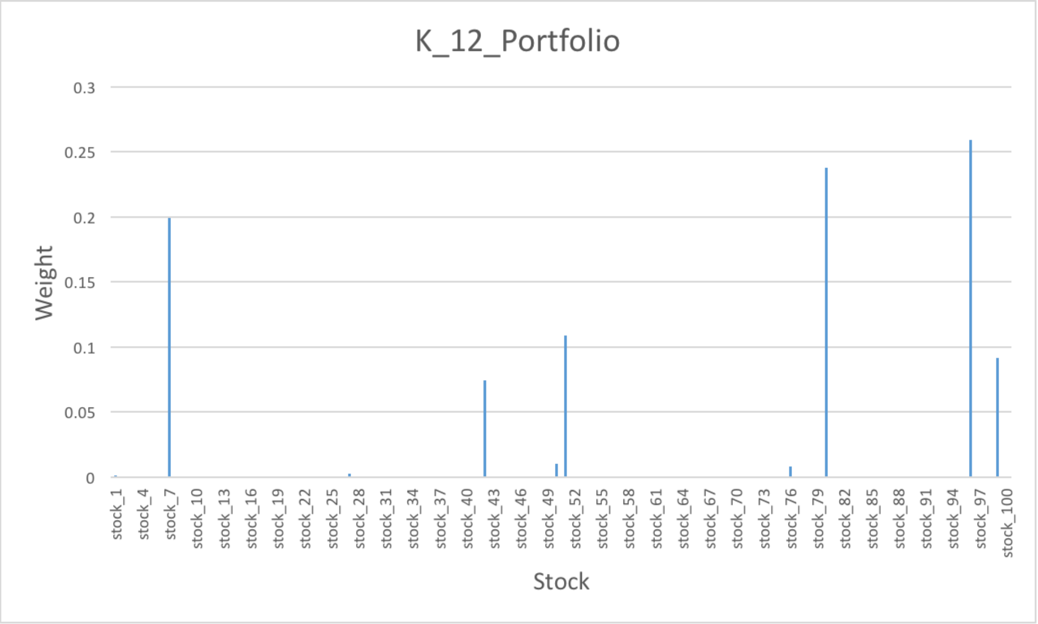

Figure 18: Return Lower Bound Plus 12 Step

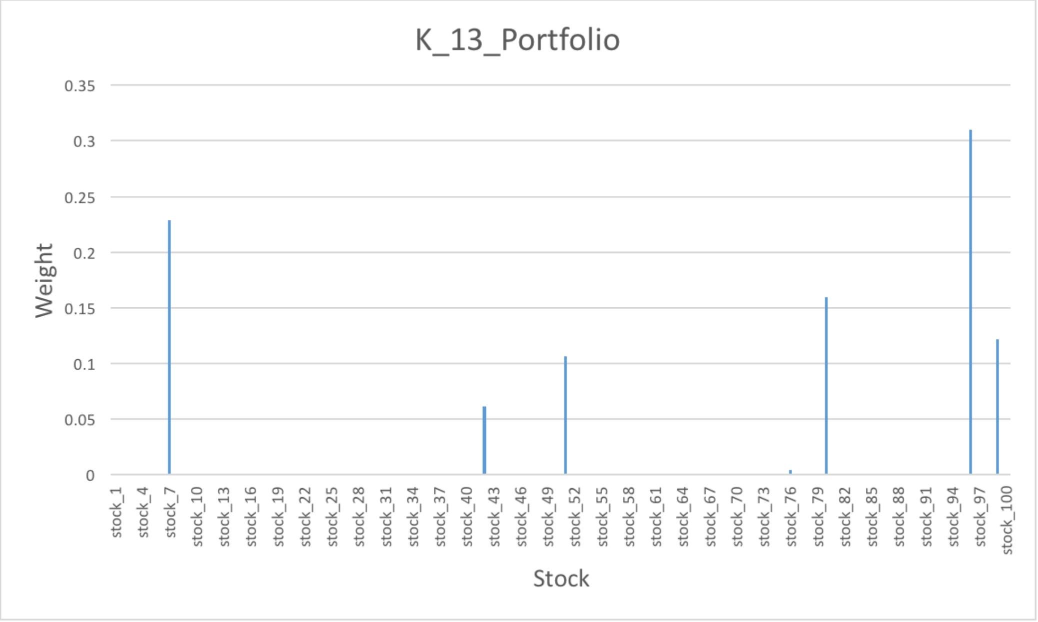

Figure 19: Return Lower Bound Plus 13 Step

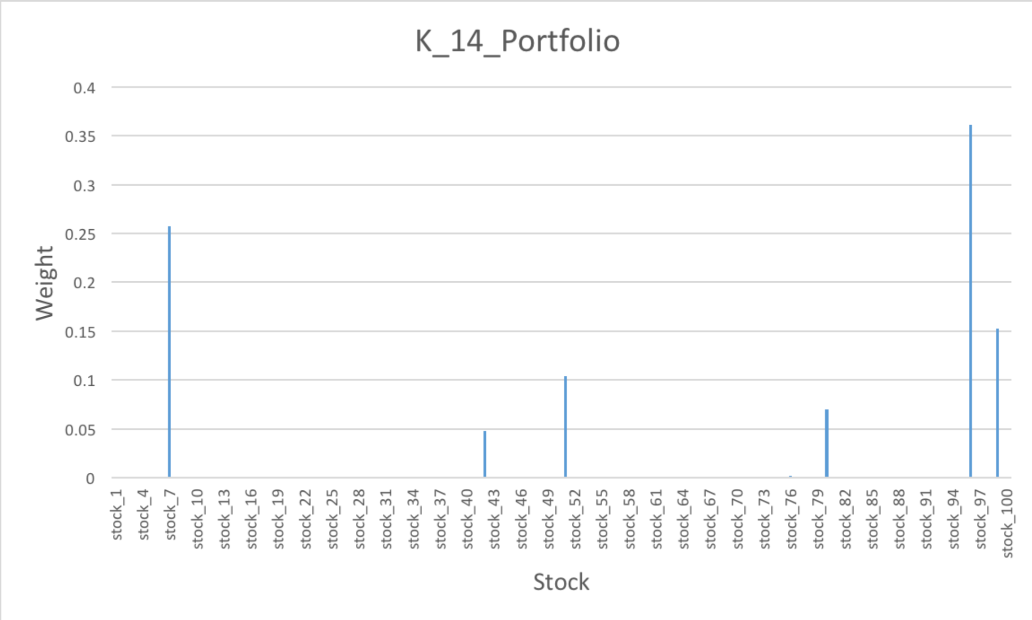

Figure 20: Return Lower Bound Plus 14 Step

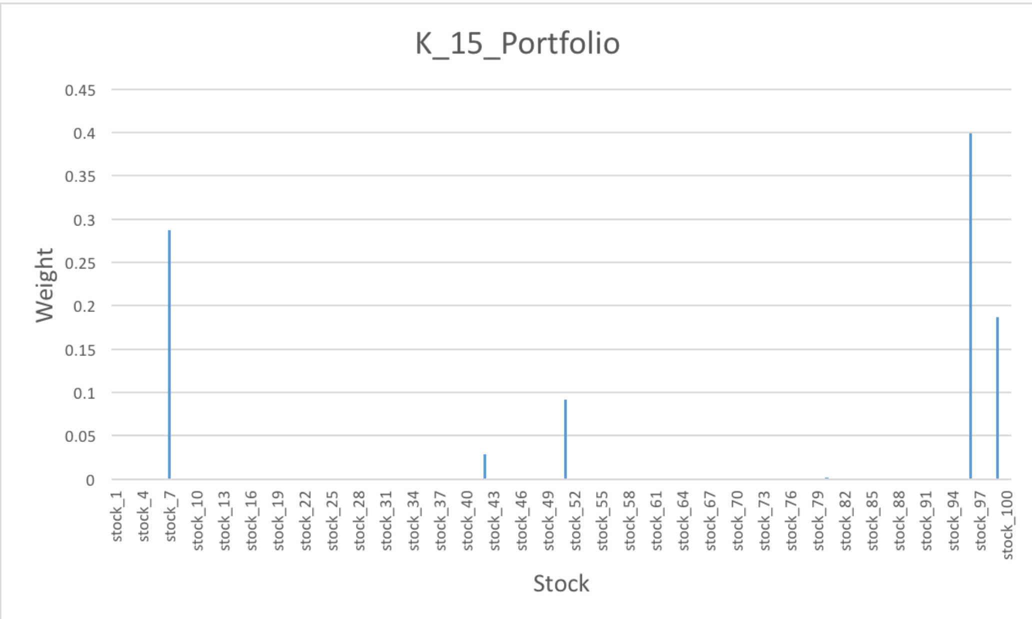

Figure 21: Return Lower Bound Plus 15 Step

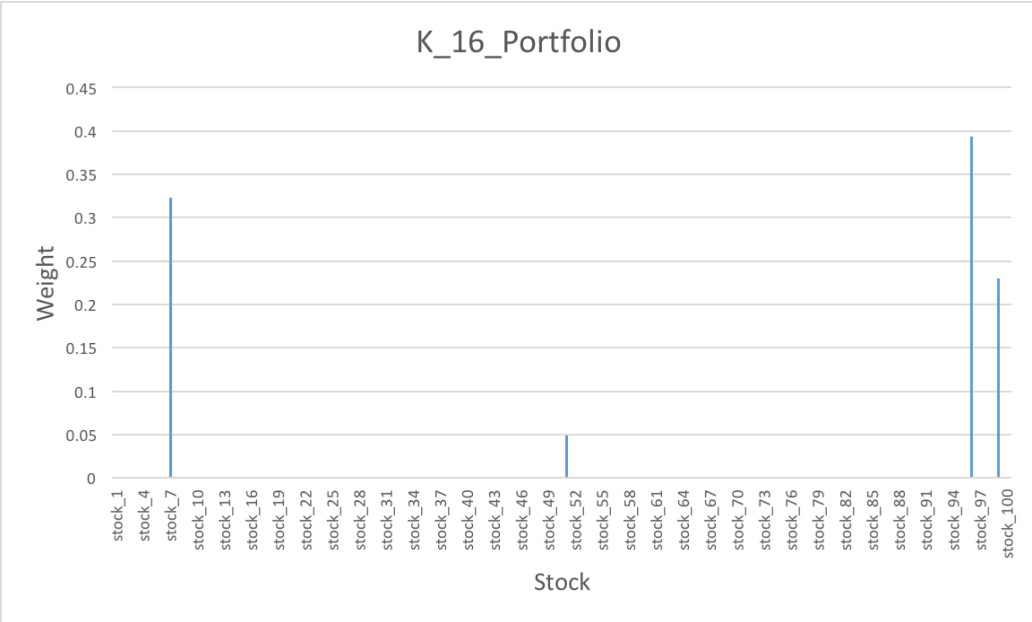

Figure 22: Return Lower Bound Plus 16 Step

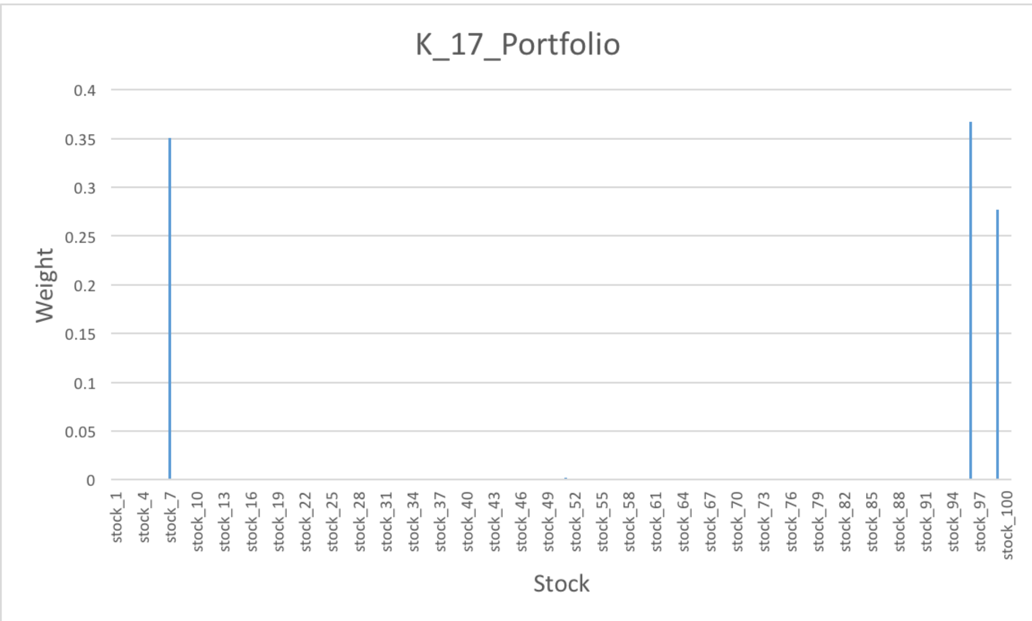

Figure 23: Return Lower Bound Plus 17 Step

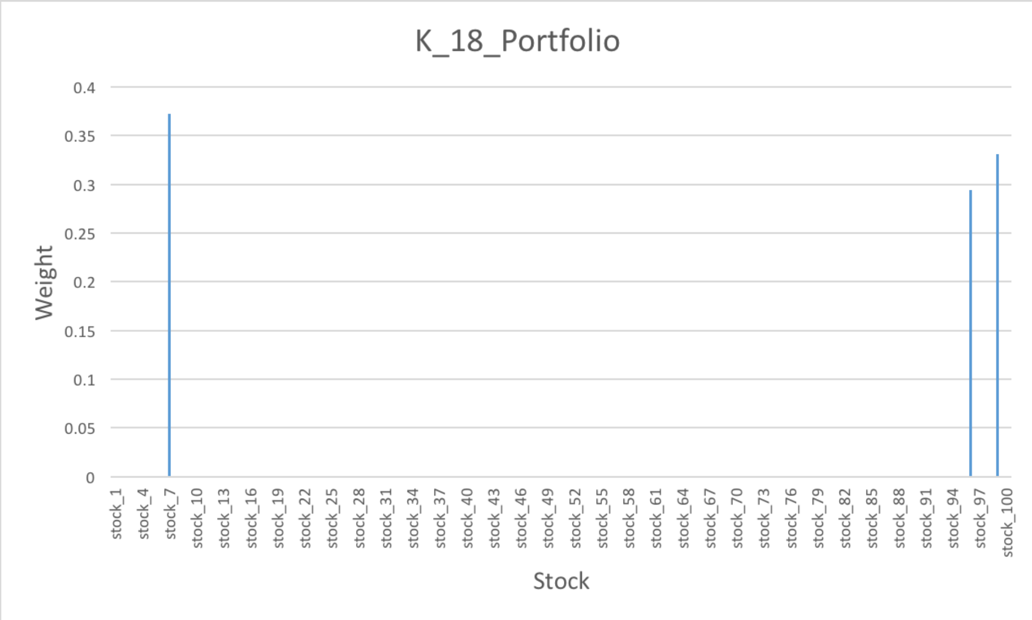

Figure 24: Return Lower Bound Plus 18 Step

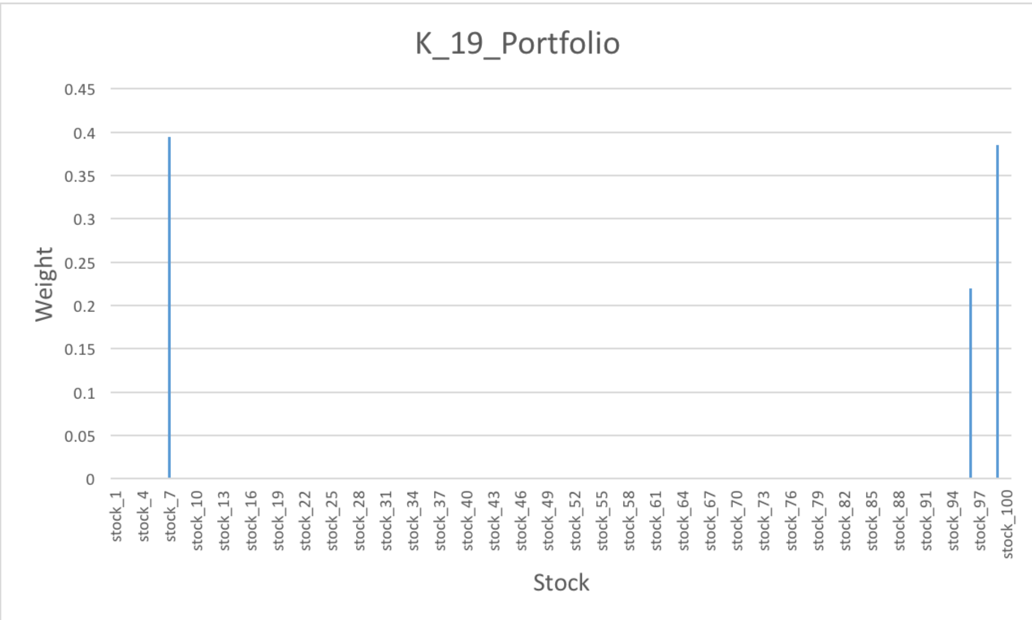

Figure 25: Return Lower Bound Plus 19 Step

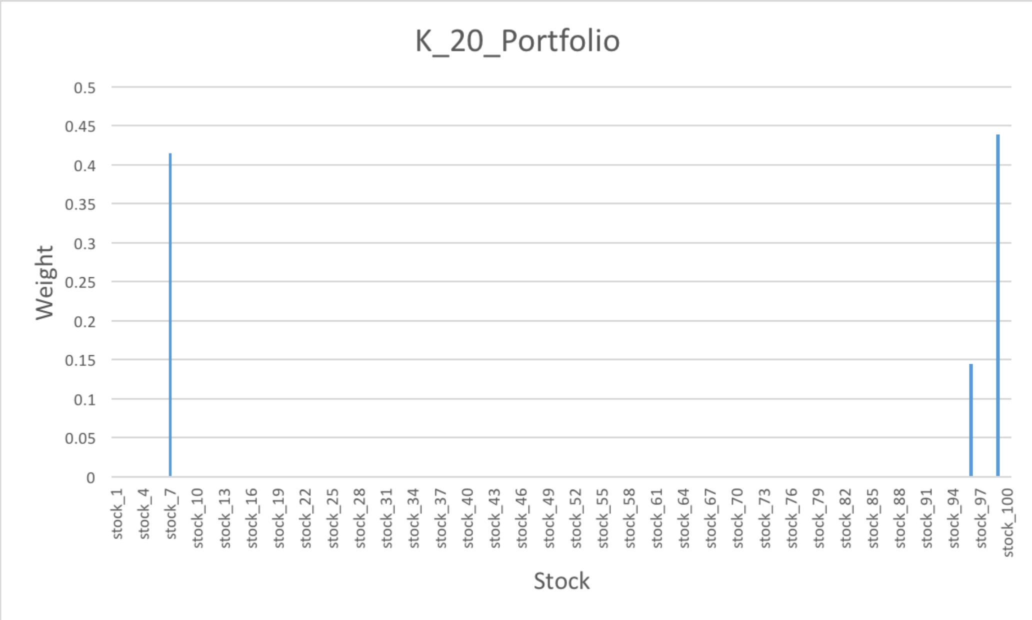

Figure 26: Return Lower Bound Plus 20 Step

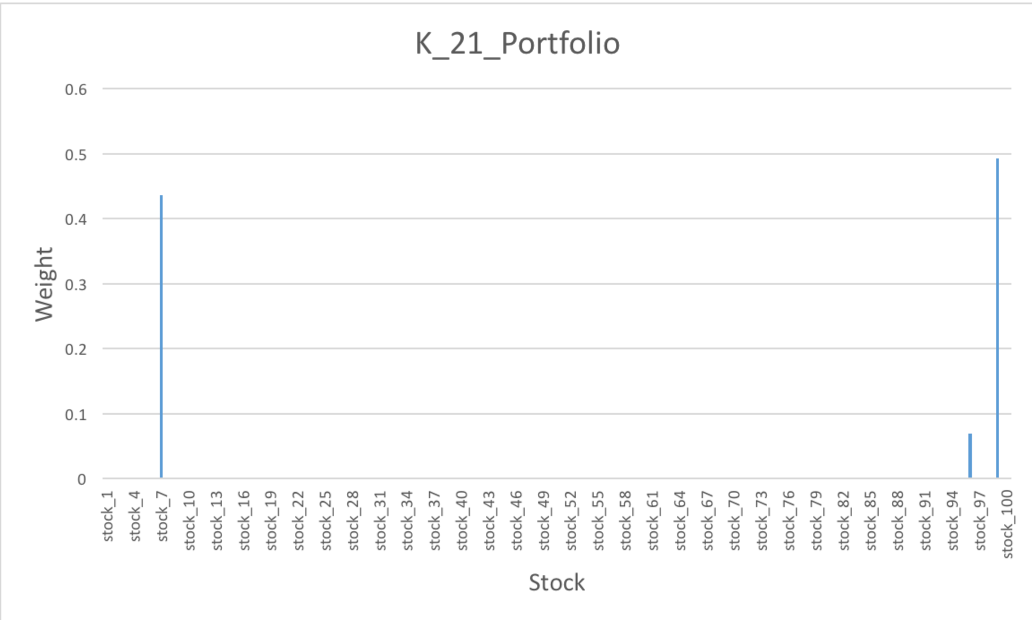

Figure 27: Return Lower Bound Plus 21 Step

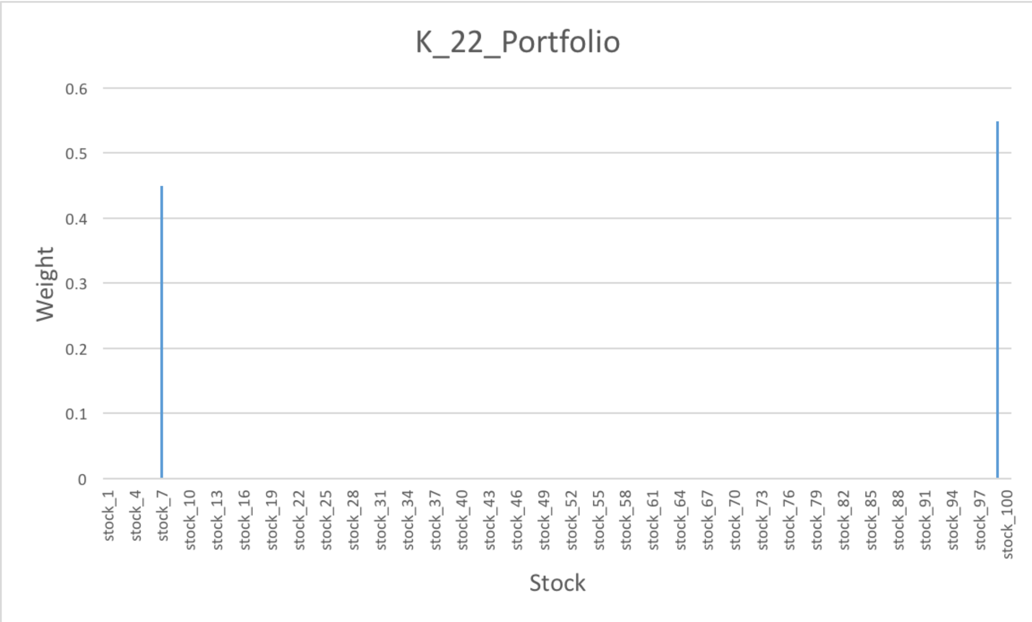

Figure 28: Return Lower Bound Plus 22 Step

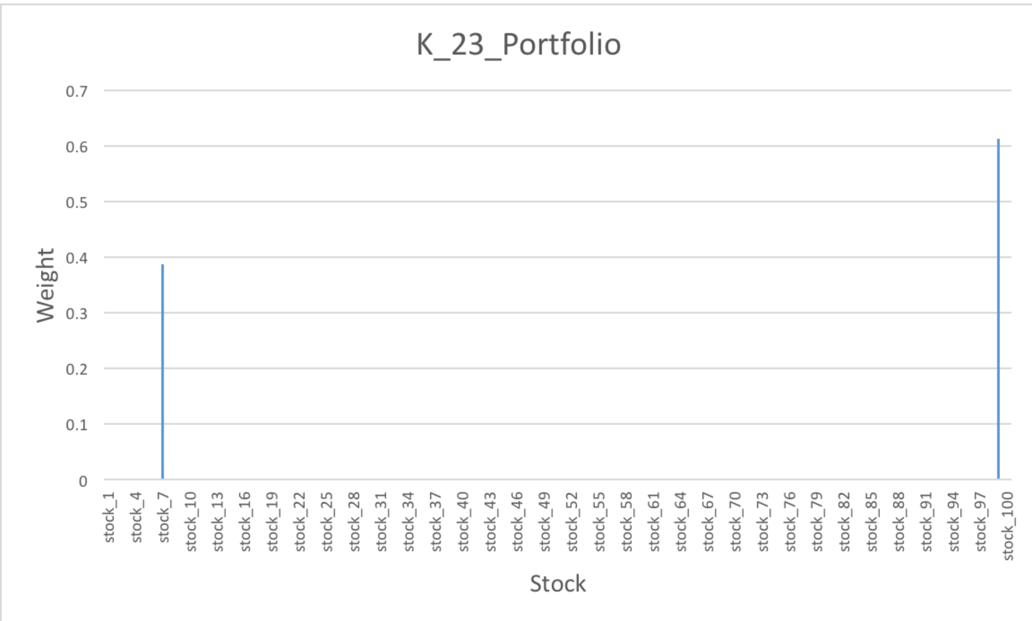

Figure 29: Return Lower Bound Plus 23 Step

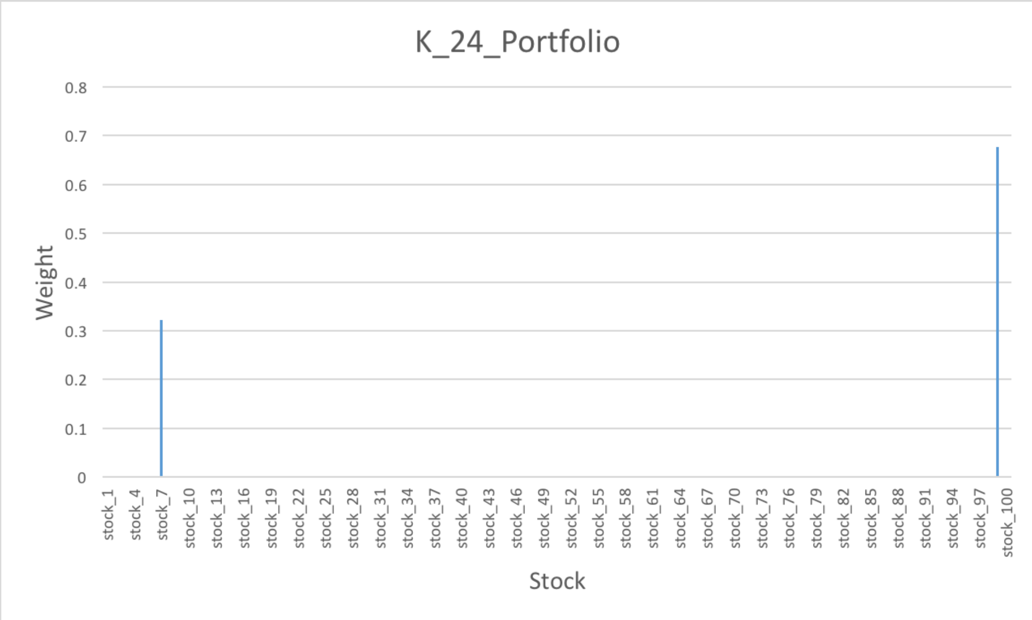

Figure 30: Return Lower Bound Plus 24 Step

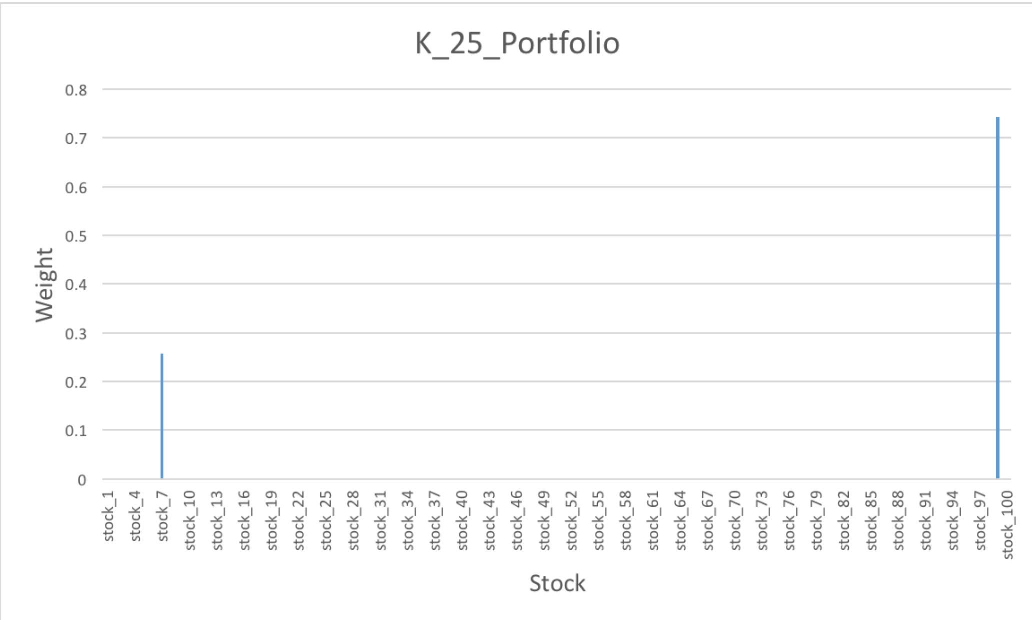

Figure 31: Return Lower Bound Plus 25 Step

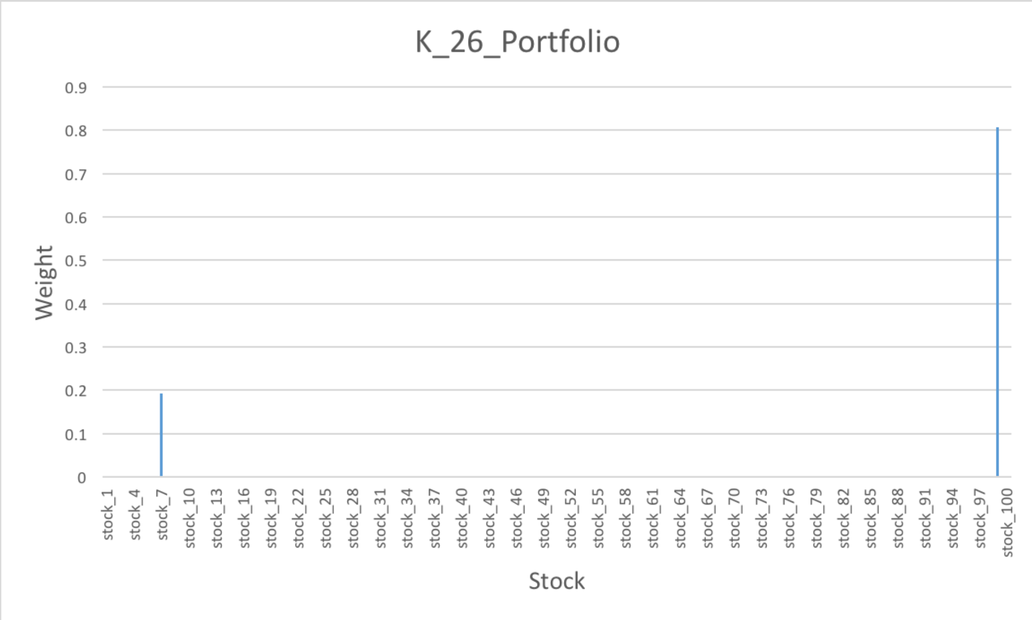

Figure 32: Return Lower Bound Plus 26 Step

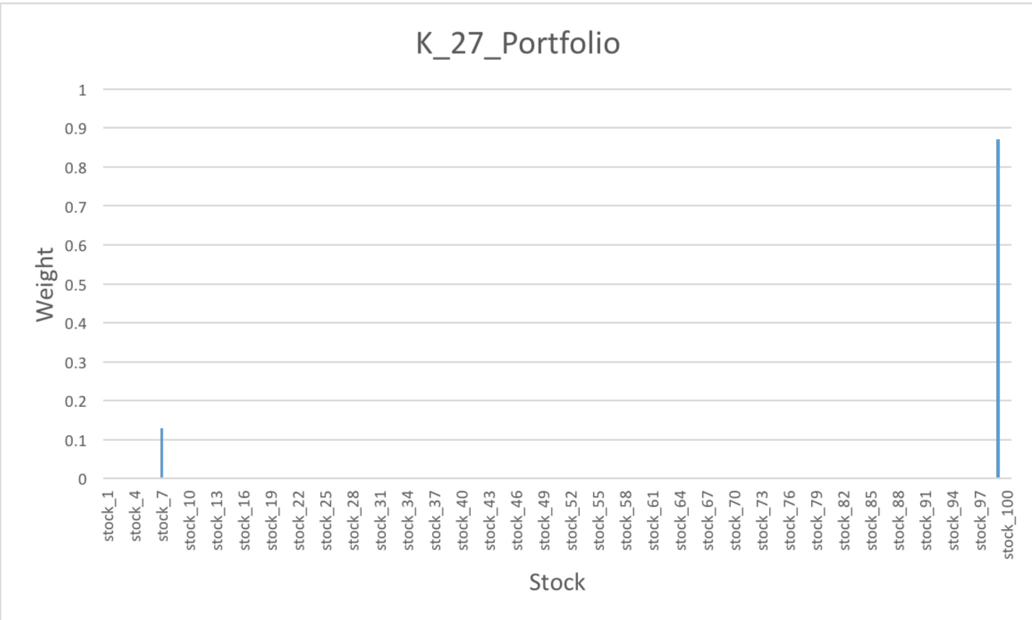

Figure 33: Return Lower Bound Plus 27 Step

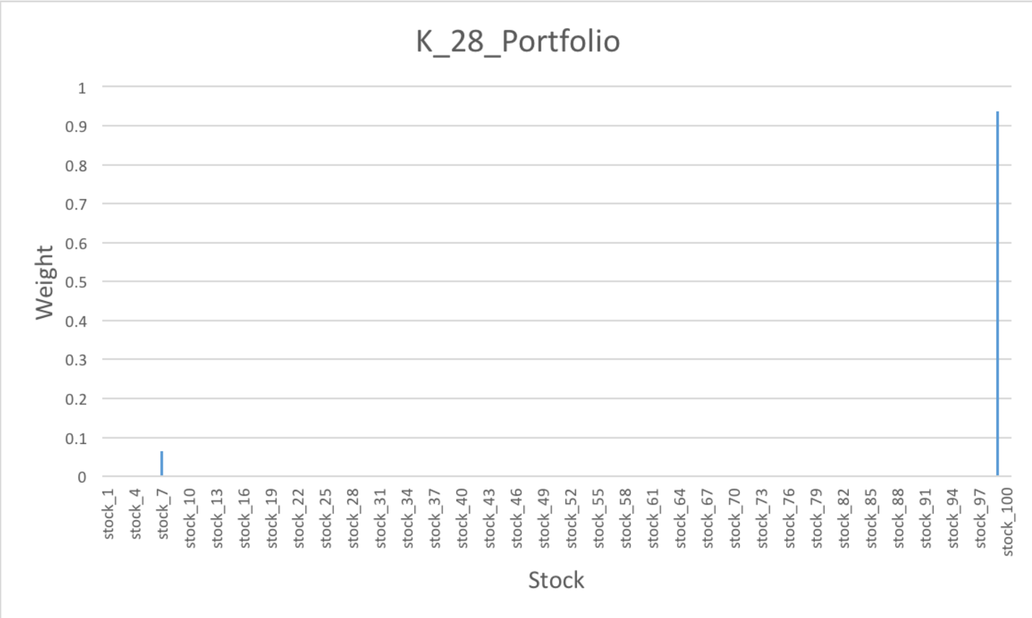

Figure 34: Return Lower Bound Plus 28 Step

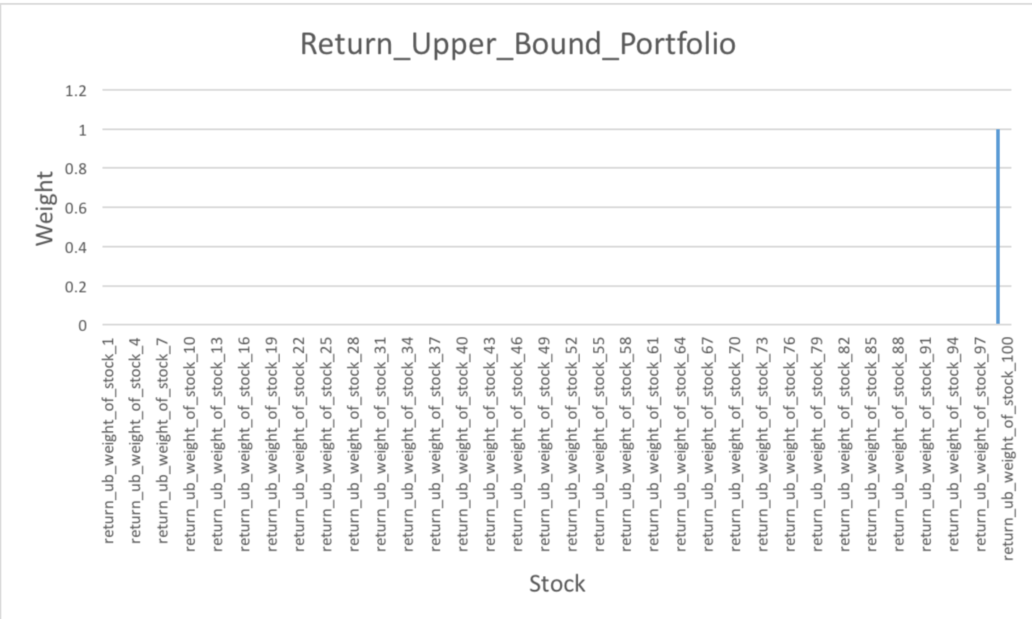

Figure 35: Return Upper Bound Portfolio

0.2 Task 2

The purpose of task 2 is to analyze the performance of five particular portfolios, which investors are most likely to be interested in. These five portfolios are:
1. The global minimum variance portfolio (Return Lower Bound)
2. The maximum return portfolio (Return Upper Bound)
3. 10-step from the lower bound
4. 15-step from the lower bound
5. 20-step from the lower bound

In the figure below, the five portfolios are highlighted with red circle. The corresponding returns and risks are given.

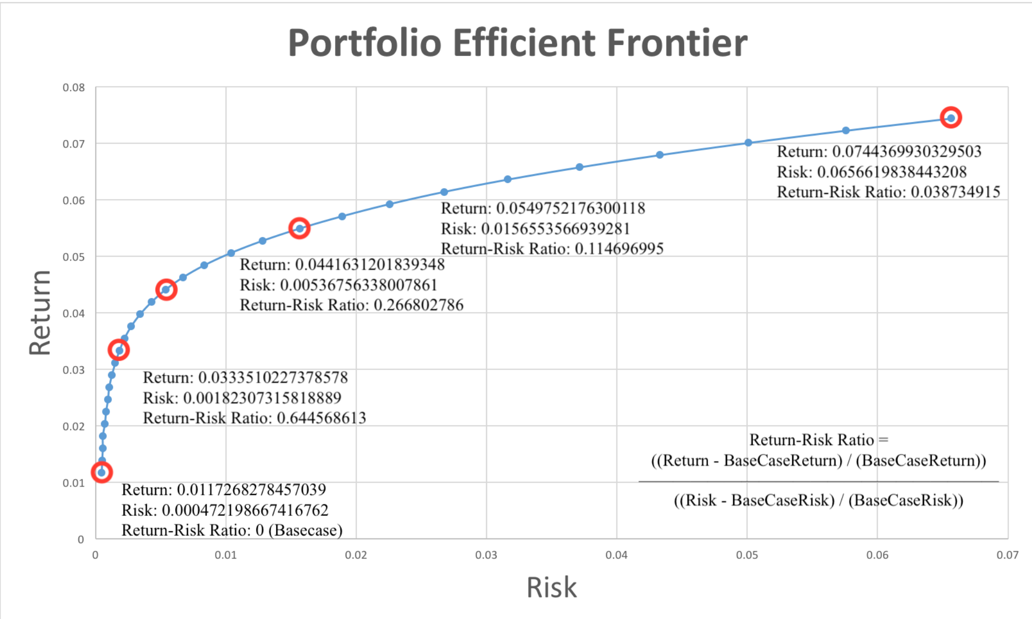

Figure 36: Portfolio Efficient Frontier with Highlights

Now, we are going to perform the Sharp Ratio measure on these five portfolios. The Sharp Ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. The Sharpe ratio is often used to compare the change in a portfolio's overall risk-return characteristics. Generally, the greater the value of the Sharpe ratio, the more attractive the risk-adjusted return. The computation of Sharp Ratio requires annual return of a portfolio, annual deviation of the portfolio, and the risk-free rate(which is the theoretical rate of return of an investment with zero risk).

However, the return and risk given here is the monthly return and monthly variance. We need to first convert them into annual return and annual risk to calculate the sharp ratio. In addition, the risks are represented with variance instead of deviation, so we need to first convert the risk from variance to deviation. Here are the functions to do the conversion:

From variance to deviation: $\sqrt{variance}$

From monthly return to annual return: $(1 + MonthlyReturn)^{12} - 1$

From monthly risk to annual risk: $(1 + MonthlyRisk)^{12} - 1$

After the calculation, the monthly standard deviations of the five portfolios are:

1. Lower Bound Portfolio: 0.021730133
2. Upper Bound Portfolio: 0.256245944
3. 10-K Portfolio: 0.042697461
4. 15-K Portfolio: 0.073263657
5. 20-K Portfolio: 0.125121368

The annual returns of the five portfolios are:

1. Lower Bound Portfolio: 0.150162478
2. Upper Bound Portfolio: 1.366853802
3. 10-K Portfolio: 0.482430985
4. 15-K Portfolio: 0.679655431
5. 20-K Portfolio: 0.900671634

The annual risks of the five portfolios are:

1. Lower Bound Portfolio: 0.294298419
2. Upper Bound Portfolio: 14.44884697
3. 10-K Portfolio: 0.65158067
4. 15-K Portfolio: 1.336022813
5. 20-K Portfolio: 3.115214456

The sharp ratio:
Since the risk-free rate is not given in the problem, we assume the risk-free rate is 5%. Assuming risk-free rate is 5%, the sharp ratio of the five portfolios are:

1. Lower Bound Portfolio: 0.340343242
2. Upper Bound Portfolio: 0.091139023
3. 10-K Portfolio: 0.663664539
4. 15-K Portfolio: 0.471290927
5. 20-K Portfolio: 0.273070007

Given the sharp ratios, we can rank these five portfolios based on their risk-return characteristics. The best one is the 10-K Portfolio, followed by 15-K Portfolio, Lower Bound Portfolio, 20-K Portfolio. The Upper Bound Portfolio has the least sharp ratio, which means that it is the worst among these five portfolios. The reason is that, although it has a very high return, it comes with too much risk. The return is not high enough to cover the risk portion.

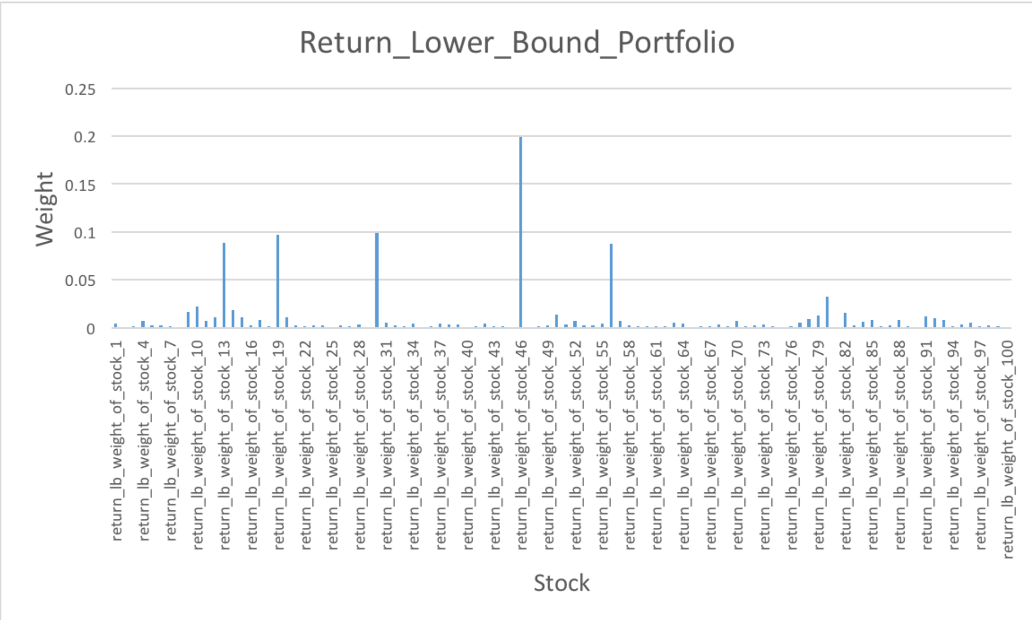

Figure 37: Return Lower Bound Portfolio

Analysis:

This portfolio is very well diversified. It invests the money in every stock to minimize the risk. Only 5 stock have more than 5% of the total investment. However, there are two disadvantages of this portfolio. First, there are many stocks to keep track on. Compare to other portfolios, it is more difficult to supervise stocks in this portfolio. Second, it is difficult to implement. Some of the stocks have the portion less than 0.001%, for most investors, 0.001% may not be able to buy one share of that stock. Therefore, to obtain this portfolio, it requires a huge amount of investment.

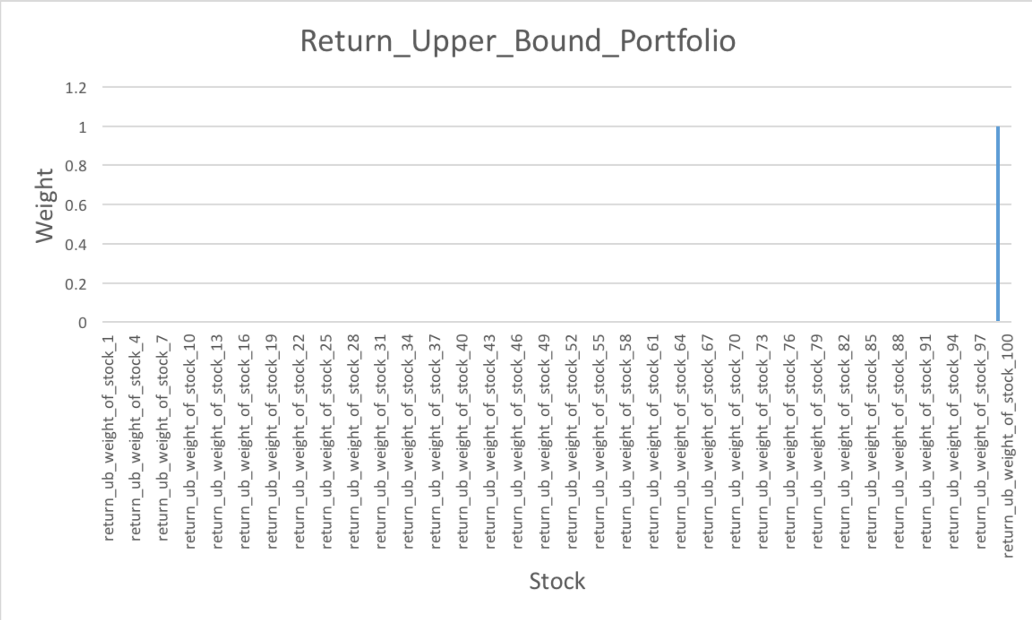

Figure 38: Return Upper Bound Portfolio

Analysis:

This portfolio is not diversified at all. It invested all the money in one stock, which results in a high risk.

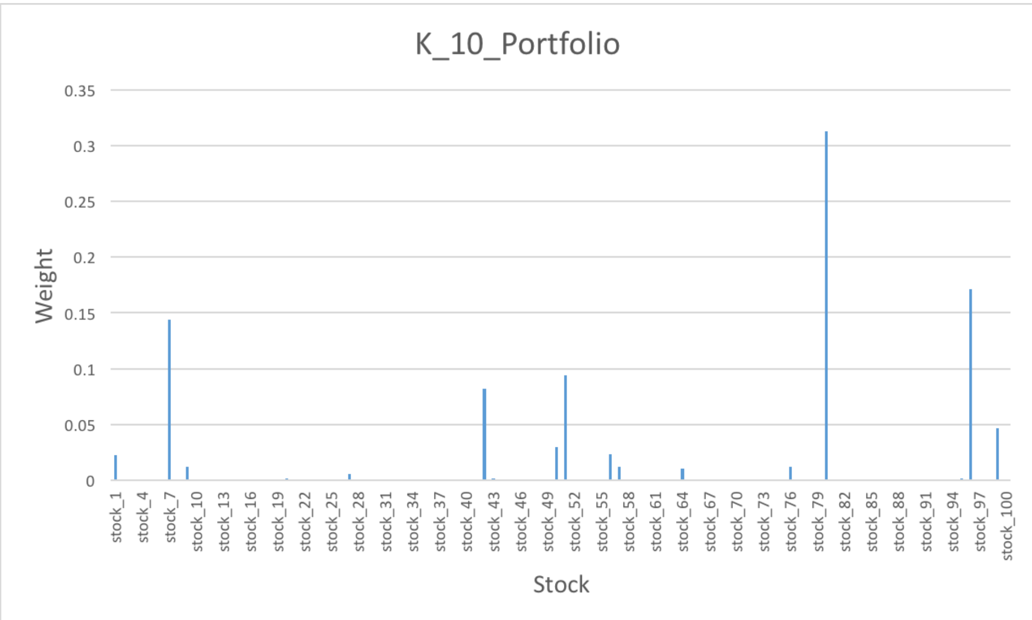

Figure 39: Return Lower Bound Plus 10 Step Portfolio

Analysis:

This portfolio is very also well diversified and it has a very high return, which makes it the best among these five portfolios.

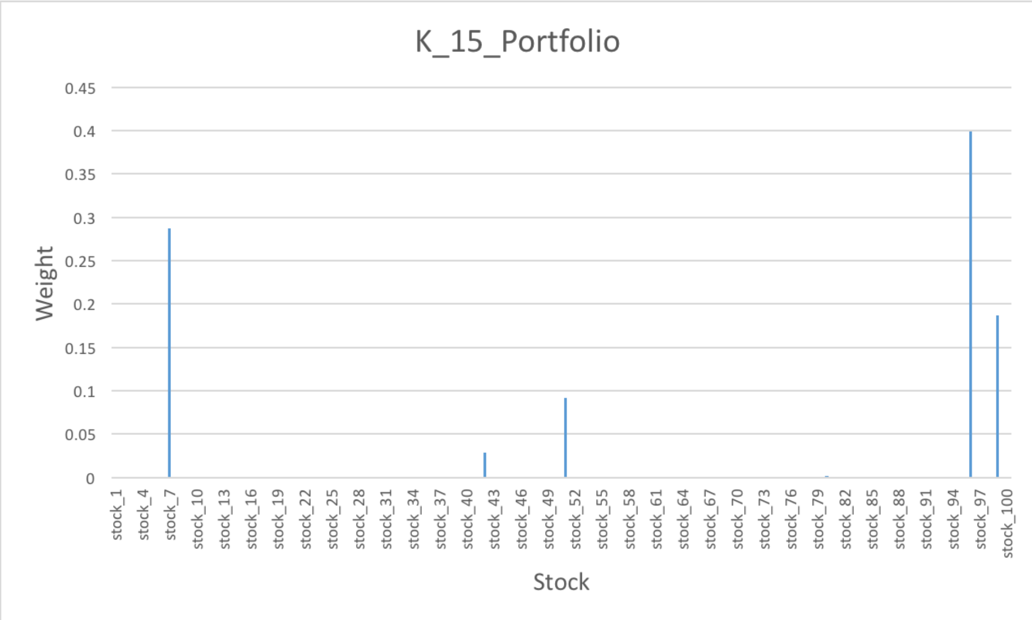

Figure 40: Return Lower Bound Plus 15 Step

Analysis:

This portfolio is somewhat diversified. Compare to the 10-K Portfolio. You can see a lot of stocks which has a portion around 3% are eliminated. First is because they do not have high enough return. Second is because they might have a high covariance with the high return stock.

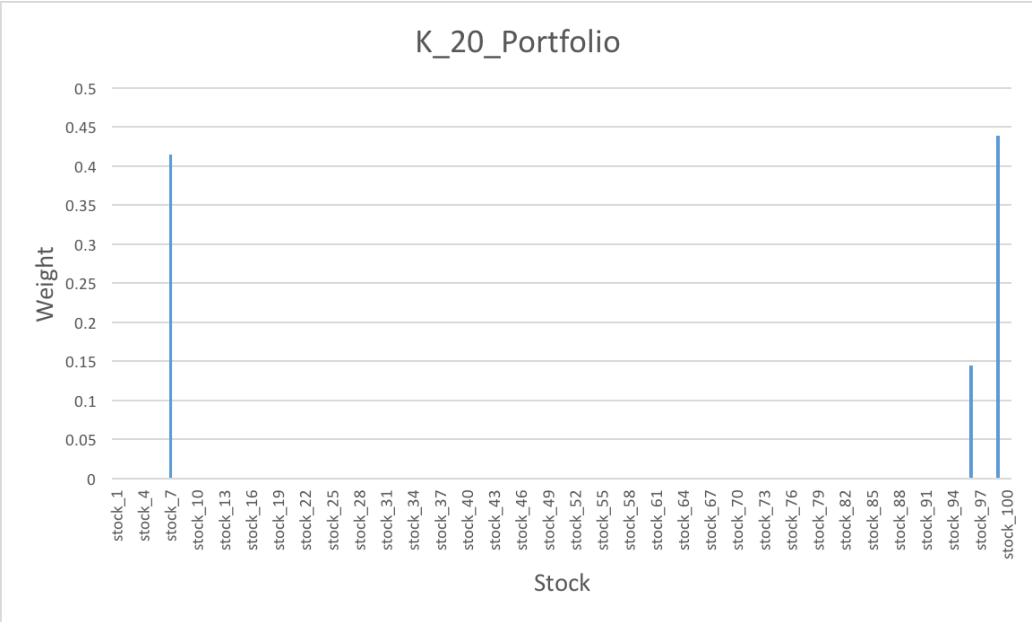

Figure 41: Return Lower Bound Plus 20 Step Portfolio

Analysis:

This portfolio is not well diversified. It is consist of only three stocks. Other stocks are eliminated because they would either lower the portfolio return too much or increase the total covariance of the portfolio.

Final Conclusion:

Based on the diversification, feasibility, and the Sharp Ratio. It is the best to invest money to obtain the 10-K portfolio.

0.3 References

Code reference:

Education, K., 2017. Markowitz Portfolio Optimization in MATLAB. [online] YouTube. Available at: <https://www.youtube.com/watch?v=N1KfFeW-HOg> [Accessed 30 Mar. 2018].

Analysis reference:

Segal, T., 2018. Measure Your Portfolio's Performance. [online] Investopedia. Available at: <https://www.investopedia.com/articles/08/performance-measure.asp> [Accessed 30 Mar. 2018].

Hayes, A., 2017. Sharpe Ratio. [online] Investopedia. Available at: <https://www.investopedia.com/terms/s/sharperatio.asp> [Accessed 30 Mar. 2018].