Assignment 4

March 20, 2024

1 Exercises

1.1

Show that if the two cameras $P_1 = \begin{bmatrix} A_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} A_2 & t_2 \end{bmatrix}$ have the same camera center, then there is homography H that transforms the first image into the second image (can assume A_1 and A_2 invertible).

Let the camera center be C (homogenous coordinates). Same camera center so $C = null(P_1) = null(P_2)$. We then have

$$P_1 C = \begin{bmatrix} A_1 & t_1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = A_1 \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} + t_1 = 0 \rightarrow \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = -A_1^{-1} t_1$$

$$P_2 C = \begin{bmatrix} A_2 & t_2 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = A_2 \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} + t_2 = 0 \rightarrow \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = -A_2^{-1} t_2$$

This means we have $t_1 = A_1 A_2^{-1} t_2$ and therefore:

$$P_1 = \begin{bmatrix} A_1 & A_1 A_2^{-1} t_2 \end{bmatrix} = A_1 \begin{bmatrix} I & A_2^{-1} t_2 \end{bmatrix} = A_1 A_2^{-1} \begin{bmatrix} A_2 & t_2 \end{bmatrix} = A_1 A_2^{-1} P_2$$

If 3D-point $\mathbf{X}$ projects to x_1 in image 1 and x_2 in image 2:

$$\lambda_1 x_1 = P_1 \mathbf{X} = A_1 A_2^{-1} P_2 \mathbf{X} = A_1 A_2^{-1} \lambda_2 x_2$$

We see $H = A_1 A_2^{-1}$ transforms point in image 2 to image 1.

1.2

Suppose that we want to find a homography that transforms one 2D point set into another. How many degrees of freedom does a homography have? What is the minimal number of point correspondences that you need to determine the homgraphy? If the number of incorrect correspondences is 10 % how many iterations of RANSAC do you need to find an outlier free sample set with 98% probability?

Considering a set of n 2D-points y_i, i=1,2...,n that maps to the points x_i, i=1,2...,n. H is 3x3, meaning 9 elements but only 8 degrees of freedom because scale is arbitrary. So 8 degrees of freedom for homography. Regarding minimal number of point correspondences needed: every point leads to 3 equations and one new unknown λ_i meaning for the whole system 3n equations and 8+n deg. of freedom $\rightarrow 3n \geq 8 + n \rightarrow n \geq 4$. So a minimum of 4 points.

To find outlier free-set with probability p=0.98, we assume total number of points is large so portion of outliers remain roughly constant when removing a point. Choosing 4 points, the probability of not picking out a outlier free set is $p = 1 - 0.9^4$ for one iteration. So we get with k iterations the probability that we don't pick out a single outlier-free set:

$$p = (1 - 0.9^4)^k \leq 1 - 0.98 \rightarrow k \geq log(0.02)/log(1 - 0.9^4) \approx 3.67$$

So 4 iterations are enough.

Figure 1: Picture a, from comp. ex 1.

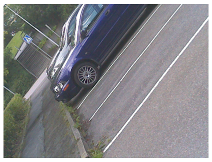

Figure 2: Picture b, from comp. ex 1.

1.3

Suppose that we want to estimate the epipolar geometry between two calibrated cameras. How many degrees of freedom does an Essential matrix have? What is the minimal number of point correspondences that you need to determine the essential matrix? If the number of incorrect correspondences is 10% how many iterations of RANSAC do you need to find an outlier free sample set with 98% probability?

The essential matrix has 6 degrees of freedom but since scale is arbitrary we have 5 degrees of freedom. The minimal number of correspondences is five. As before:

$$(1-0.9^5)^n \leq 0.02 \rightarrow n \geq log(0.02)/log((1-0.9^5)) = 4.3818$$

So n=5 iterations is enough.

1.4

2 Computer exercises

2.1

Figures 1, 2 and 3 show the pictures a and b, and the panorama stitched toghether from the two. 947 sift features were found in image a and for image b 865 sift features. The number of matches was 204. For this panorama, the number of good points was 143.

2.2

The number of inliers was 1465. In figure 4 is the 3D-reconstruction. Figures 5 and6 show the histograms of the reprojecting errors, and rms value computed to 0.3159.

2.3

The final rms value was calculated to 0.3141. Plot of error against number of iterations is seen in fig 6.

2.4

The final rms value was calculated to 0.2396. Plot of error against number of iterations is seen in fig 7.

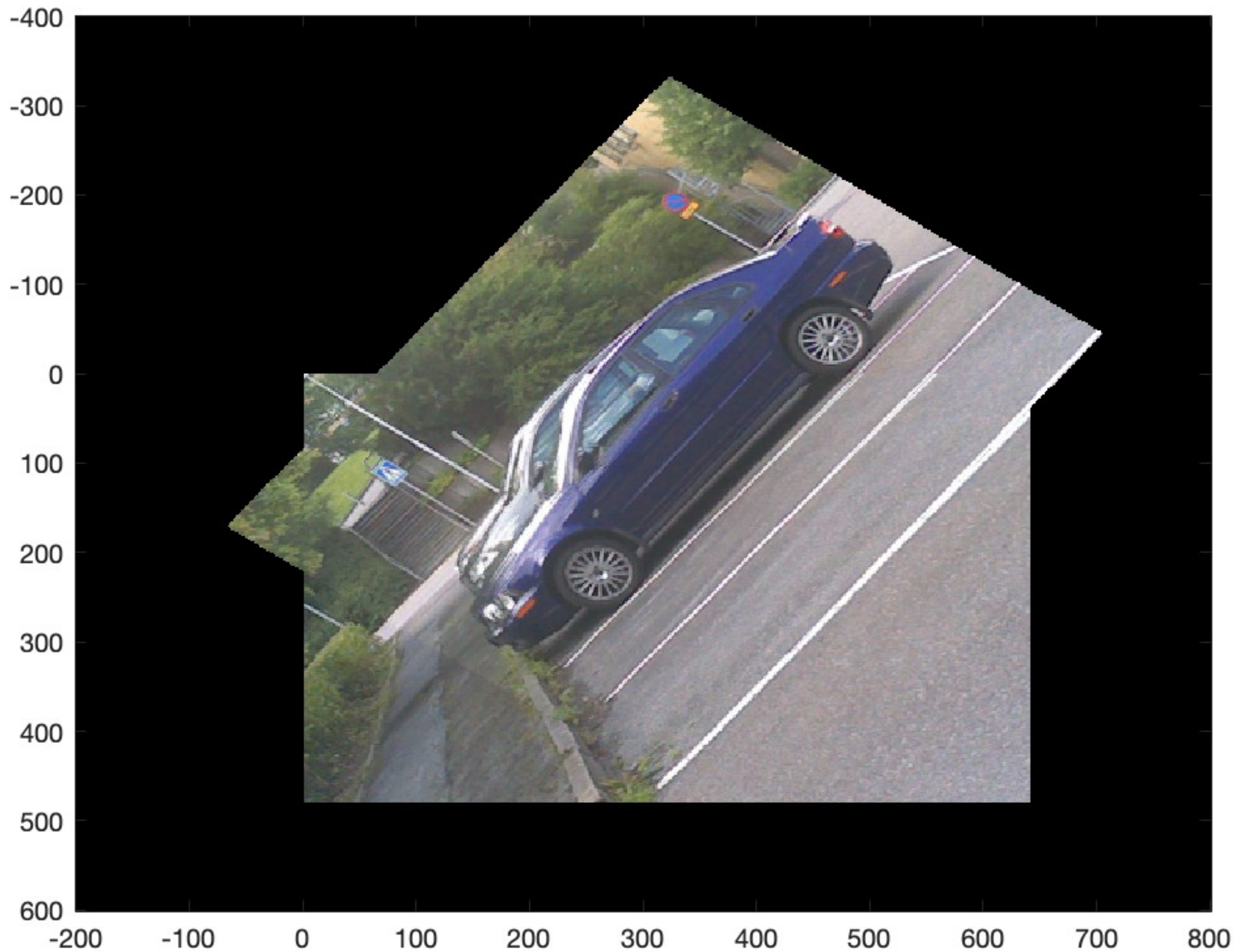

Figure 3: Panorama of car, from comp. ex 1.

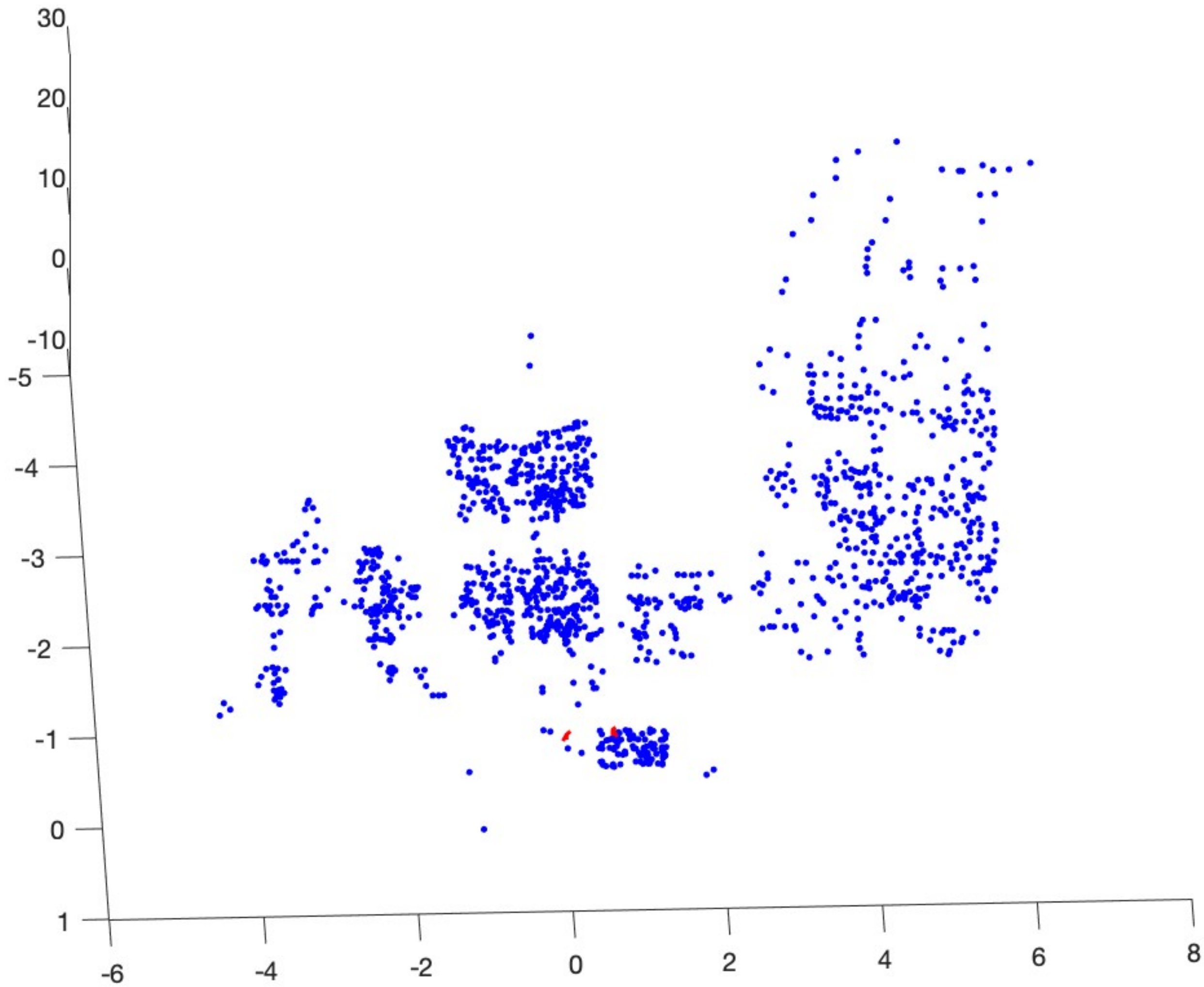

Figure 4: 3D reconstruction from comp. ex. 2

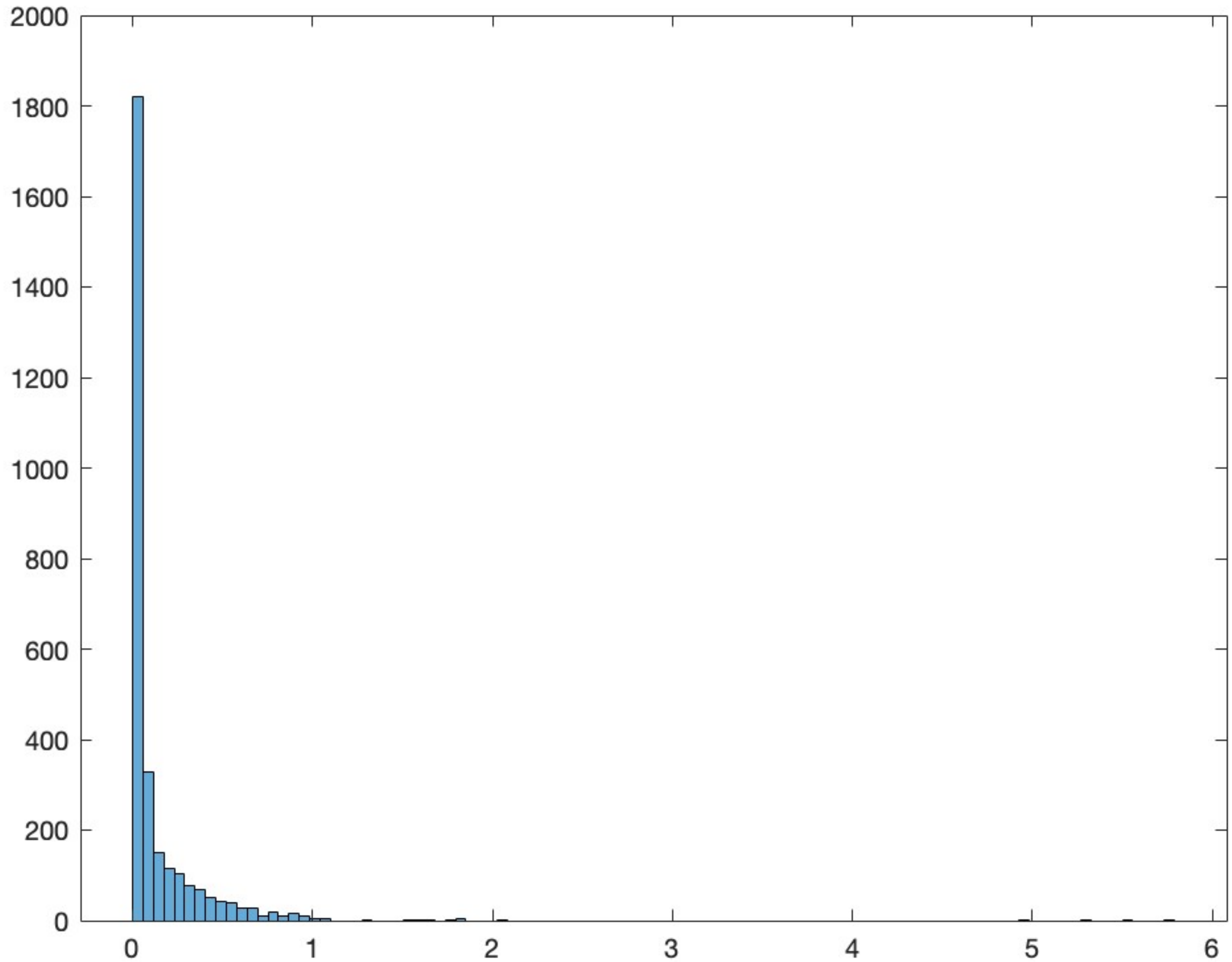

Figure 5: Histogram of reprojection errors from comp. ex. 2

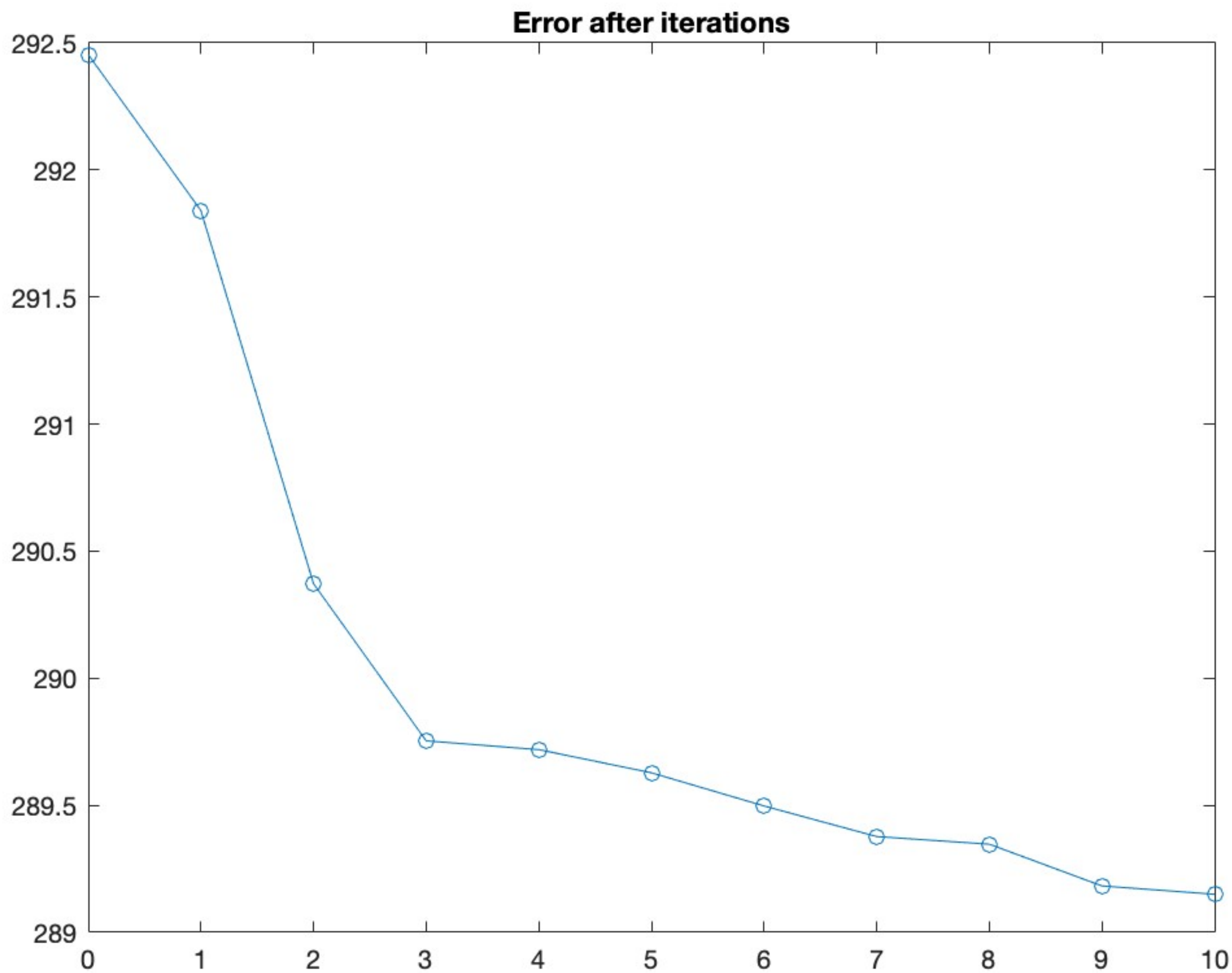

Figure 6: Computed error against number of iterations up to 10 iterations, from comp. ex. 3

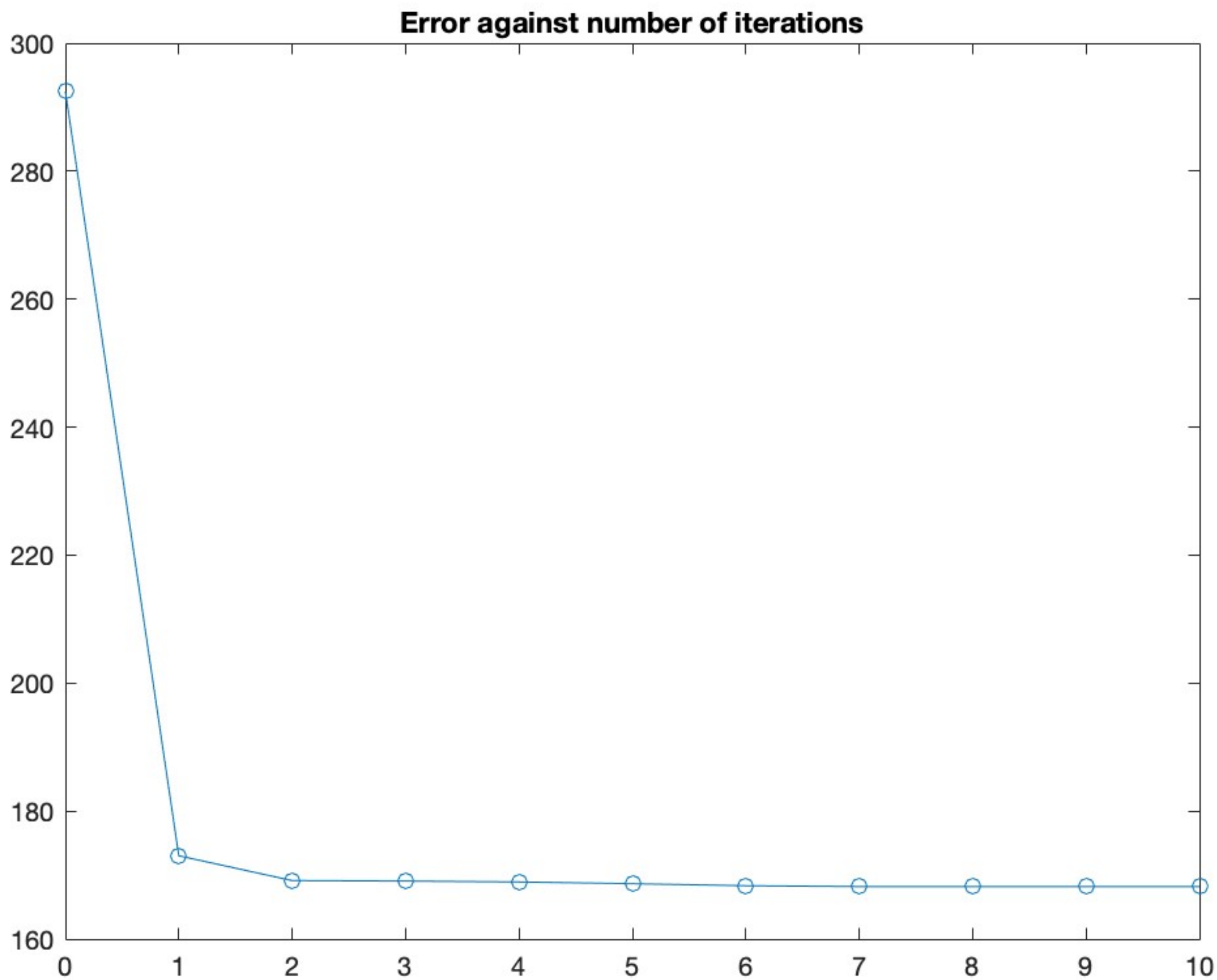

Figure 7: Plot of sum of residuals against iteration number, 10 iterations. From comp. ex. 4