Hand-in in 1 Spectral Analysis

Eliot Montesino Petrén, el6183mo-s, 990618-9130, LU Faculty of Engineering F19 , eliot.mp99@gmail.com

I. Introduction

This assignment explores the evaluation and comparison of spectral estimation techniques under different noise conditions. The focus is on the implementation of AR(3) noise and its impact on various estimators, including the periodogram, RELAX, MUSIC, PSC, ASC, APES, IAA, SPICE, q-SPICE and LASSO. The task involves assessing estimator robustness against non-white noise assumptions and their precision in amplitude and frequency estimation. This approach offers a practical insight into the real-world application and limitations of spectral analysis methods, enhancing understanding of their functionalities in complex scenarios.

This assignment is a hand-in in the course of stationary and non-stationary spectral analysis (FMSN35) at Lund University Faculty of Engineering.

II. Tasks

A. AR(3) Noise Simulation and Estimator Robustness Analysis

After having played around with different signals for a while: white noise on signal, different AR(3) noises on signal, damped AR(3) noise on signal, different frequencies of the sinusoidal signals, frequencies of the freq-grid and so on, the task was to make the disturbing white noise into a AR(3) process. This was done in the code with the Matlab command "filter()":

$$\text{ar3\_noise} = filter(1, a, w); \quad (1)$$

$$a = [1, -(0.9), -(-0.6), -(0.4)]; \quad (2)$$

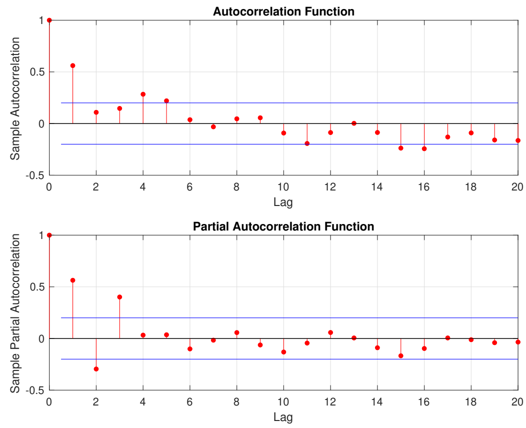

Fig. 1. Auto correlation-and partial auto correlation function of the AR(3) noise model in eq 2.

Where w is randomly generated white noise, a defines the AR(3) process parameters that were chosen to be highly correlated noise (can also be seen in fig 1) and they were chosen to be stable and this was tested, please see fig 2.

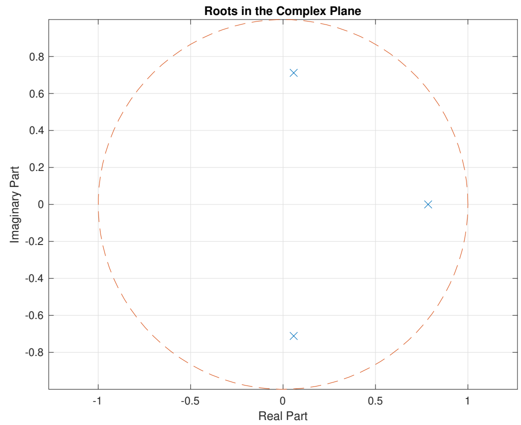

Fig. 2. Roots of the AR(3) noise parameters. All roots are withing the unit circle and stable.

Keeping all else equal produces the results in fig 3 and 4

The results from fig 3 and 4 show that the noise does affect some of the estimators (the models assumes white noise) but many do still preserve good estimates, indicating robustness. APES and MUSIC are effected by the introduced AR(3) noise, especially MUSIC. Thus the spectral density figure might be interesting to show here for APES, but the fig 5 does not show much when observed with the naked eye and the differences for APES introducing AR(3) noise is not clearly noticeable in the view of a figure.

```
True frequencies: 0.090000, 0.120000
  Periodogram:    0.088000, 0.120000
  MUSIC:          0.092238, 0.120375
  RELAX:          0.090000, 0.120000
  PSC:            0.090000, 0.120000
  ASC:            0.090000, 0.120000
  APES:           0.091000, 0.120000
  IAA:            0.090000, 0.120000
  SPICE:          0.090000, 0.120000
  q-SPICE, q=2:   0.090000, 0.120000
  LASSO:          0.090000, 0.120000
Periodogram resolution: 1/N = 0.010000 (roughly)
Grid resolution limit:  1/P = 0.001000
```

Fig. 3. No changes in the code "ex1" produced these estimates. Noise is white and the true freq are seperated further than the resolution and above the Rayleigh resolution limit.

So far the results are not that interesting and does not tell us much about the robustness of the methods, therefore the signal to noise ration (SNR) will be investigated further by

```
The AR(3) process is stable.
True frequencies: 0.090000, 0.120000
  Periodogram:    0.089000, 0.120000
  MUSIC:          0.066069, 0.117157
  RELAX:          0.090000, 0.120000
  PSC:            0.090000, 0.120000
  ASC:            0.090000, 0.120000
  APES:           0.091000, 0.119000
  IAA:            0.090000, 0.120000
  SPICE:          0.090000, 0.120000
  q-SPICE, q=2:   0.090000, 0.120000
  LASSO:          0.090000, 0.120000
Periodogram resolution: 1/N = 0.010000 (roughly)
Grid resolution limit:  1/P = 0.001000
```

Fig. 4. Estimates when making the previously white noise into an AR(3) process according to eq 2 and keeping the default SNR (noise amplitude $w = 0.1 * ...$).

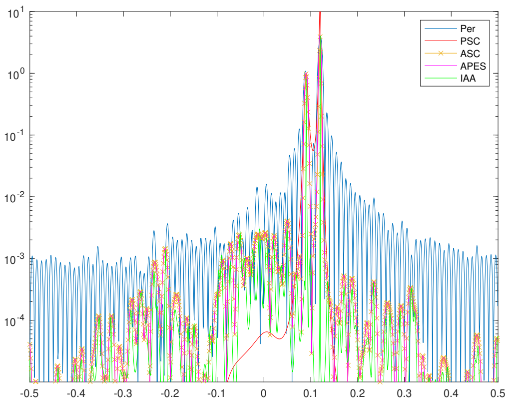

Fig. 5. Comparative pseudo-spectra analysis on a signal with AR(3) noise using periodogram, PSC, ASC, APES, and IAA methods. The plot shows the magnitude of the spectral density estimates across a normalized frequency range with notable peaks at 0.9 and 0.12. SNR ≈ 24 tested with amp $= 0.1$

increasing the amplitude of the noise. In the table I below different SNR values are displayed that were tested (the first one is from the default provided code). Note that the SNR values are an average observed by the naked eye, since the SNR value is stochastic in nature.

TABLE I
TESTED SIGNAL TO NOISE RATIOS AND CORRESPONDING INSERTED AMPLITUDE OF THE ORIGINAL WHITE NOISE $w =' amp' * ....$

0.1	24
0.3	25
0.6	8.5

B. Amplitude Estimation Comparison in Signal Analysis

C. Frequency Estimation Accuracy in Simulated Signals

D. Frequency Analysis of Real-World Data Sets

III. CONCLUSION

IV. REFERENCES