Hand-in in 2 Spectral Analysis

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I. INTRODUCTION

This hand-in explores advanced topics in time-frequency analysis, specifically focusing on quadratic class methods, novel envelope shape detectors, the number of multitapers in spectral estimation, and multitaper spectral estimation of stationary processes. It aims to contribute to the understanding of signal processing techniques by investigating the time-frequency distribution of signals, the effectiveness of envelope shape detection in transient signals, and the impact of multitaper choices on spectral estimates. The findings and methodologies discussed herein are grounded in theoretical frameworks and practical applications, providing insights into the advantages and limitations of these approaches. This work reflects a comprehensive effort to apply and evaluate signal processing methods in various contexts, underlining the importance of precise parameter selection and methodological innovation in enhancing signal analysis outcomes.

II. TASKS

A. A Small Study of Some Quadratic Class Methods

In Task 1, a complex-valued signal comprising Gaussian-modulated components is generated. Some parameters are chosen to construct the signal:

- The number of samples, N, is selected to 300
- A constant sampling frequency, Fs, is set at 1 Hz, simplifying the time-scale.
- The FFT sample size, $NFFT$, is fixed at 1024 (default value).

The signal itself is synthesized from four Gaussian components, each specified by:

- Linearly spaced center time points, $Tvect$ from 0 to $(300 - 50)$ (so that the last envelope is not pushed to the end of the time sample), and frequencies and $Fvect$ are chosen of the signal's duration and the frequency spectrum.
- Equal amplitudes for all components, $Avect = 1$, a consistent signal strength.
- The Gaussian shape parameters, $Cvect$, are selected to vary linearly from 5 to 12. This choice is implemented by defining $Cvect$ as `linspace(5, 12, 4)`, which generates four values linearly spaced within this range. The introduction of this range of shape parameters yields a variety of envelope widths for the Gaussian components, directly influencing the time-frequency spread of the signal's components.

The signal is also represented visually in figure 1 and is now subject to further analysis and visualization in this assignment.

Then analyzing this signal was analyzed using three different quadratic class time-frequency distribution (TFD) methods: Choi-Williams, Lag-independent, and Pseudo-Wigner. I will briefly go over each method's advantages, drawbacks, and their effects on the marginal properties. With separable kernels [1]

$$\phi(\nu, \tau) = G_1(\nu) g_2(\tau). \tag{1}$$

we see that Pseudo-Wigner and Lag-Independent TFD are two special cases where the doppler-independent kernel Pseudo-Wigner is when $G_1(\nu) = 1$ and the lag-independent kernel $g_2(\tau) = 1$.

Summary on Marginals

Marginals refer to the property that the integral of the time-frequency distribution over time should yield the signal's power spectral density, and the integral over frequency should yield the signal's time-dependent power. A TFD fulfills or does not fulfill these marginal conditions that ensure energy conservation in both time and frequency domains.

To preserve the time- and frequency marginals of the time-frequency distribution, the ambiguity kernel must fulfill eq 2 [1].

$$\phi(0, \tau) = \phi(\nu, 0) = 1 \tag{2}$$

- **Choi-Williams** should in theory always fulfill the marginals as the Choi-Williams kernel is symmetric in frequency ν and time τ (see eq 3) and it fulfills eq 2. As we see from separable kernels in eq 1 this is not guaranteed in the other two TFD methods as it is impossible for eq 2 to always be fulfilled if they are independent of time or frequency.
- **Pseudo-Wigner** distributions tend to preserve marginals well however, with effectiveness depending on parameter choices.
- The **Lag-Independent** distribution's ability to fulfill marginals depends on the chosen lags' representation of the signal's energy distribution in time and frequency.

In practice, no single TFD method is universally best for all signals. The choice depends on the specific characteristics of the signal and the analysis goals, balancing between resolution, cross-term reduction, and computational efficiency.

Choi-Williams Distribution

The Choi-Williams distribution uses an exponential kernel to reduce cross-term interference, enhancing clarity in time-frequency representation.

The ambiguity kernel of the Choi-Williams distribution is found in eq 3.

$$\phi_{ED}(\nu, \tau) = e^{-\frac{\nu^2 \tau^2}{\sigma}} \tag{3}$$

Advantages:

- Effective at reducing cross-terms compared to the basic Wigner-Ville distribution.
- Product kernel means that it has one-parameter setting: it has only one variable, σ, which is an advantage for optimizing the kernel for specific signal performances.
- Offers better clarity in time-frequency representation for signals with closely spaced components or rapidly varying frequencies.

Drawbacks:

- Parameter-dependent: The choice of sigma parameter is crucial to balance time-frequency resolution and cross-term suppression.
- Computationally intensive due to the kernel operation.

Marginals: Generally preserves the energy distribution over time or frequency. Effectiveness of the method varies based on the exponential kernel parameter chosen.

Lag-Independent Distribution

This method attempts to reduce the cross-terms without needing a kernel function, focusing on certain lags that contribute most to the auto-terms.

Advantages:

- Reduces cross-terms with less computational complexity than kernel-based methods.
- Simplicity in application.

Drawbacks:

- May not be as effective in reducing cross-terms for complex signals as methods with adaptive or signal-dependent kernels.
- The choice of lags is crucial and may not work equally well for all signal types.

Marginals: Tends to maintain the energy distribution across time or frequency well, but effectiveness can vary with signal characteristics and lag selection.

Pseudo-Wigner Distribution

A variant of the Wigner-Ville distribution that applies a windowing function to mitigate cross-term issues.

Advantages:

- Offers a balance between time-frequency resolution and cross-term reduction.
- More flexible due to windowing, allowing some control over the analysis.

Drawbacks:

- Still produces some cross-terms, though less severe than the Wigner-Ville distribution.
- The choice of windowing function and parameters significantly affects performance.

Marginals: Generally preserves the time and frequency marginals, influenced by the windowing function and parameters.

Summary comment on produced plots

While the same signal is being studied in all the plots the results can differ significantly comparing the different methods and one has to take into account how the different methods produce their results when analysing. In the time-frequency distributions on the generated signal three frequencies are identified in all cases. This is evident from the three peaks in height looking at the heat color map. Also not so surprisingly we see that the three peaks lie on a ”straight line” with proportional relationship between time and frequency, as it was defined in its the signal’s generation. The third peak is split into two as it lie on the end of the frequency spectrum.

We see that for the doppler-independent kernel that their signal in fig 6 is smoothed-out in frequency dimension frequency but sharper in the other dimension, while the peaks are assumed to be balanced in the two dimensions for Choi-Williams with circular shapes reasonable for a Gaussian envelope signal, and this seems to also be the case for lag-independent time-frequency distribution in fig 4. For this specific case with our signal we see that the level is generally quite high in fig 4.

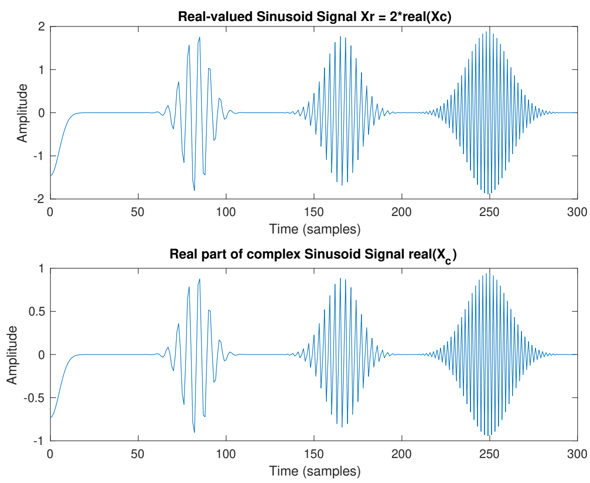

Fig. 1. Generated signals complex-valued signal showing both the real and the complex part.

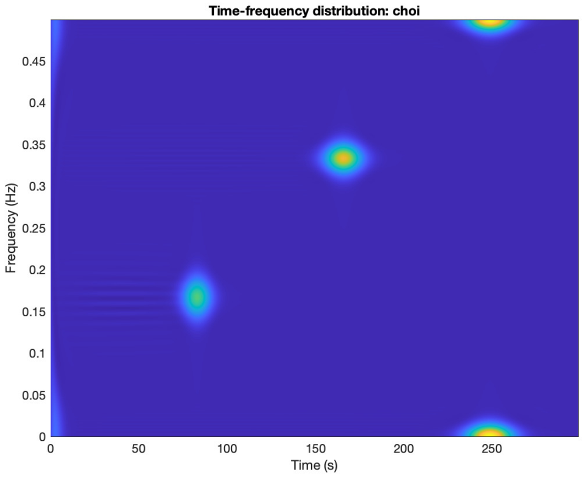

Fig. 2. Time-frequency distribution of the generated signal using the method Choi-Williams.

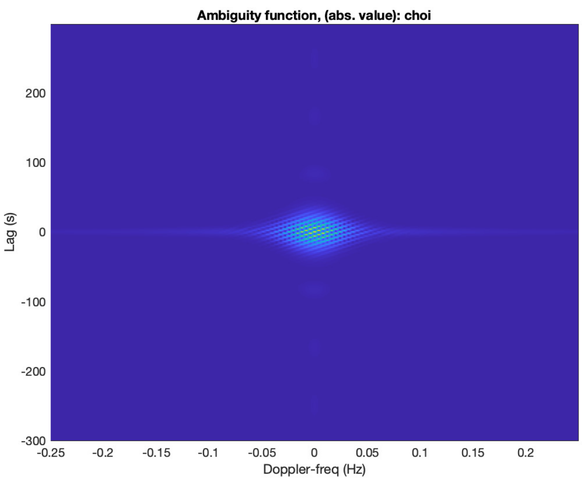

Fig. 3. The ambiguity function plot of the generated signal using the method Choi-Williams.

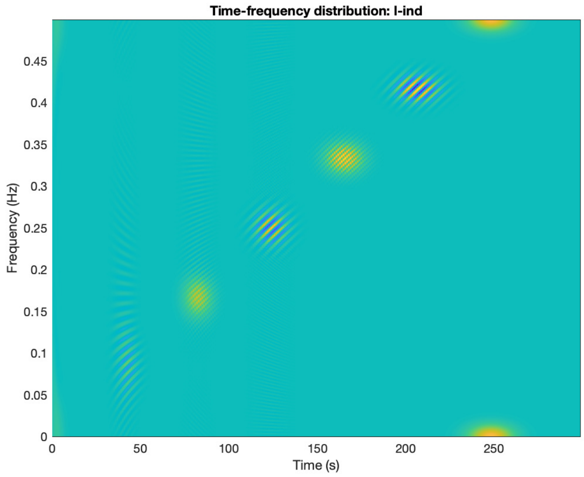

Fig. 4. Time-frequency distribution of the generated signal using the lag-independent method.

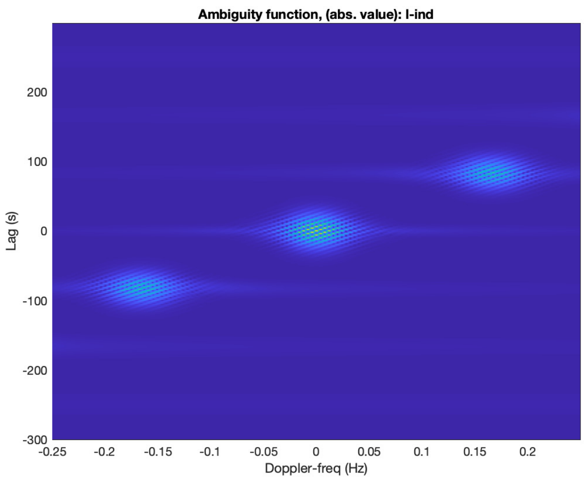

Fig. 5. The ambiguity function plot of the generated signal using the lag-independent method..

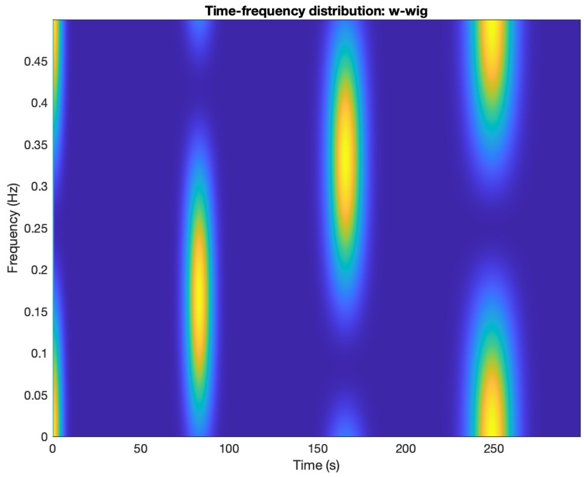

Fig. 6. Time-frequency distribution of the generated signal using the method pseudo-Wigner.

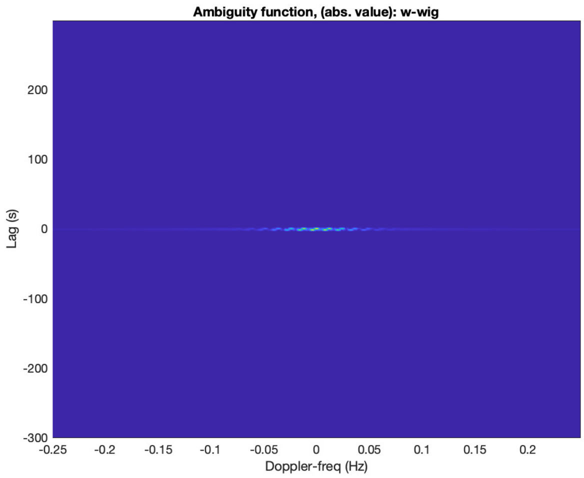

Fig. 7. The ambiguity function plot of the generated signal using the method pseudo-Wigner.

B. A Novel Envelope Shape Detector

I checked the views of the spectrogram and the SR-spectrogram by 3-D visualization using mesh. When comparing the energy-scale axes in fig 8 it is apparent that the signal is significantly stronger in the Scaled Reassigned Spectrogram (SRS) by many orders of magnitude. The peaks in SRS are essentially spikes when the energy mass is centered on the time-frequency centers (t_m, f_m). Early attempts for the rest of the exercise was made but nothing interesting to show here unfortunately.

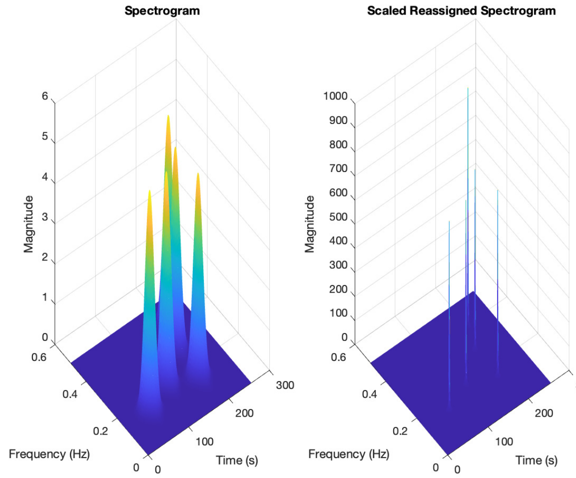

Fig. 8. The spectrogram and the SRS of the unknown sequence given in the task.

C. Choice of the Number of Multitapers

Early attempts for the rest of the exercise was made but nothing interesting to show here unfortunately.

D. Multitaper Spectral Estimation of Stationary Processes

The MATLAB code performs spectral analysis on simulated time series data. It simulates $N = 1000$ realizations of an AR(4) process with a specific pole value and Gaussian white noise, then computes their Thomson multitaper spectral estimates using MATLAB's pmtm function. It calculates and plots the mean and standard deviation of these spectral estimates in decibels (dB). Additionally, it normalizes the standard deviation by the mean and plots the results. The same procedure is applied to the Gaussian white noise realizations. Finally, it performs the Welch spectral estimation for the white noise and visualizes the mean and normalized standard deviation.

Number of tapers is set to $K = 5$ and Time-bandwidth product is set to $NW = 3$. NW parameter is the product of the time duration of the data window (in seconds) and the bandwidth (in Hz) over which the spectral estimate is smoothed. Increasing NW improves frequency resolution but reduces the number of independent estimates that contribute to the averaged estimate. K is the number of tapers used in the multitaper spectral estimation. It relates to the number of independent estimates of the spectrum that you will average together to get the final estimate. Increasing K can lead to a lower variance of the spectral estimate but also results in a wider main lobe and, consequently, poorer frequency resolution.

The mean spectral estimate is consistently estimated over the true spectral density a potential indicator of bias in the estimator, see figures 10 and 11. However, since the AR(4) process is not very complicated I would guess it is rather spectral leakage due to limited amount of data available or because of sidelobes in the multiple window functions (tapers). We also see that the standard deviation of the estimate is lower for higher frequencies, an inherent trait. When the standard deviation is normalized against the mean an interesting peak is discovered for low frequency. This is partly due because of a lower spectral resolution for lower frequencies but I also believe it is because of ”edge” effects of the AR(4) that is initialized at one point and that is almost random walk with such high parameters, see figure 9.

In the figures 12 and 13 we are able to compare the mean multitaper spectral estimate against the welch method. It is apparent that the welch method reduces the variance, (while the mean is retained in both of them). Why is this the case? The Multitaper method also reduces variance by using multiple tapers (data windows), but it might not achieve the same reduction in variance as Welch’s method because it does not involve segmenting the data and overlapping segments. The multitaper method applies several orthogonal tapers (windows) to the entire data set to obtain multiple PSD estimates, which are then averaged. However, this is seemingly not as effective as Welch’s method in reducing variance, particularly since I am not sure if the multitaper’s time-bandwidth product and the number of tapers are optimally chosen for the data at hand.

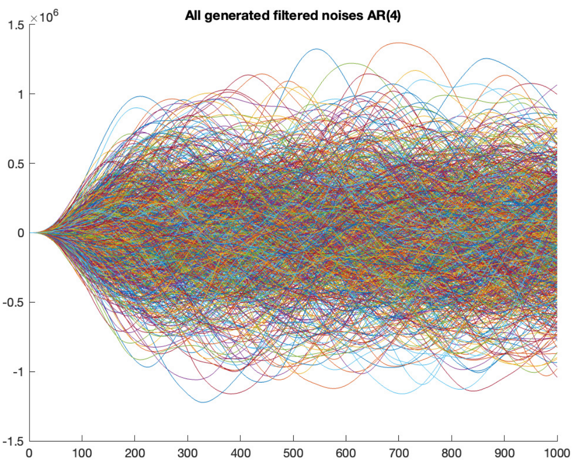

Fig. 9. A thousand AR(4) noises generated of size 1000.

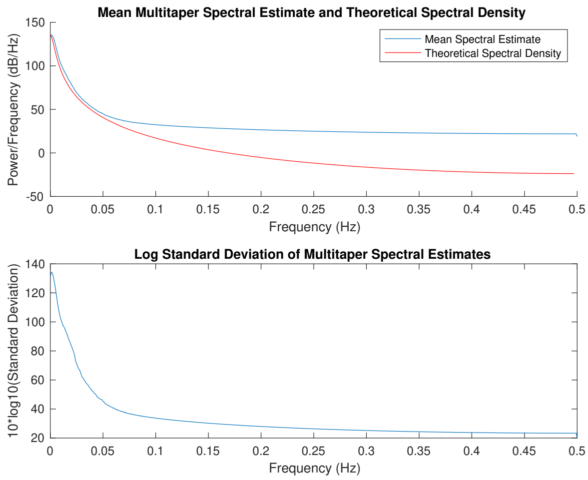

Fig. 10. Mean multitaper spectral estimate and log standrad deviation of multitaper spectral estimate on a AR(4) noise signal.

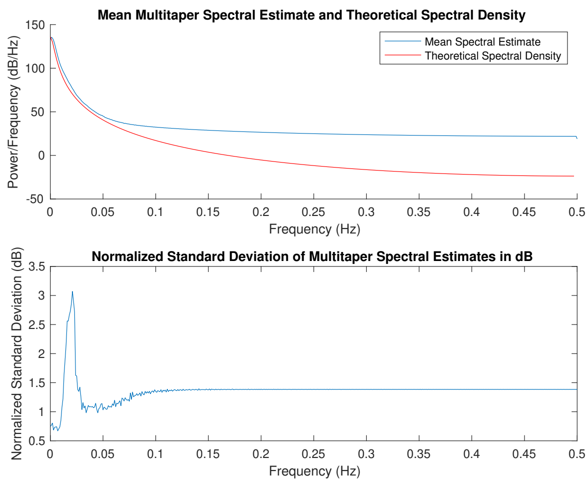

Fig. 11. Mean multitaper spectral estimate and normalized standard deviation of multitaper spectral estimate on a AR(4) noise signal.

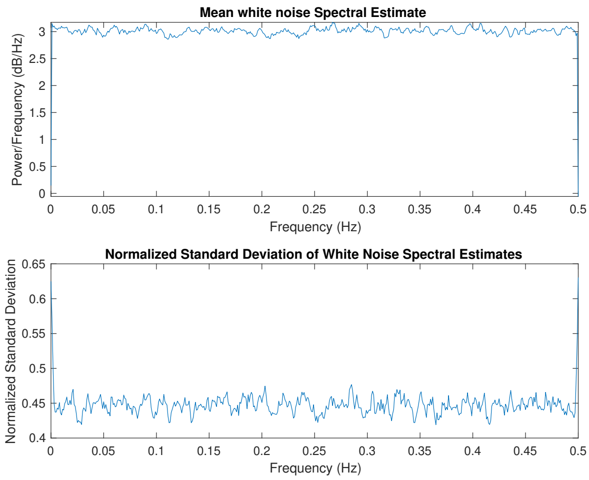

Fig. 12. Mean multitaper spectral estimate and normalized standard deviation of white noise spectral estimate on a Gaussian white noise signal.

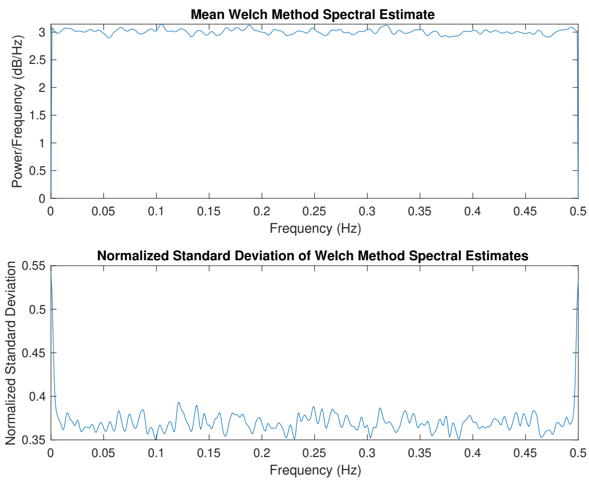

Fig. 13. Mean welch method spectral estimate and normalized standard deviation of welch method spectral estimate on Gaussian white noise signal.

III. References

[1] Maria Sandsten. *Time-Frequency Analysis of Time-Varying Signals and Non-Stationary Processes*. Lund University - Centre of Mathematical Statistics, 2024.