TREX: CT IMAGE FEATURE EXTRACTION

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First Order Histogram Features

Image I of size $L_y \times L_x \times L_k$ with a total number of voxels, N.

P is the ordered image histogram, where $P(i)$ is the number of voxels of the i^{th} gray level.

HIST Feature List:

- Sum:

$$\text{Sum} = \sum_{i=1}^{N} I(i)$$

- Mean:

$$\text{Mean} = \overline{I} = \frac{1}{N}\sum_{i=1}^{N} I(i)$$

- Median:

$$\text{Median} = \widetilde{I} = median(I)$$

- Minimum:

$$\text{Minimum} = min(I)$$

- Maximum:

$$\text{Maximum} = max(I)$$

- Variance:

$$\text{Variance} = \frac{1}{N-1}\sum_{i=1}^{N}(I(i) - \overline{I})^2$$

- Skewness:

$$\text{Skewness} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(I(i) - \overline{I}\right)^3}{\sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}\left(I(i) - \overline{I}\right)^2\right]^3}}$$

- Kurtosis:

$$\text{Kurtosis} = \frac{\frac{1}{N}\sum_{i=1}^{N}(I(i) - \overline{I})^4}{\left[\frac{1}{N}\sum_{i=1}^{N}(I(i) - \overline{I})^2\right]^2}$$

- Range:

$$\text{Range} = max(I) - min(I)$$

- Mean absolute deviation:

$$\text{MeanAbsDeviation} = mean(\left|I - \overline{I}\right|)$$

- Median absolute deviation:

$$\text{MedianAbsDeviation} = median(\left|I - \widetilde{I}\right|)$$

- 1^{st} Percentile:
$$\text{Per01} = P_{1\%}(I)$$
- 10^{th} Percentile:
$$\text{Per10} = P_{10\%}(I)$$
- 25^{th} Percentile:
$$\text{Per25} = P_{25\%}(I)$$
- 75^{th} Percentile:
$$\text{Per75} = P_{75\%}(I)$$
- 90^{th} Percentile:
$$\text{Per90} = P_{90\%}(I)$$
- 95^{th} Percentile:
$$\text{Per95} = P_{95\%}(I)$$
- 99^{th} Percentile:
$$\text{Per99} = P_{99\%}(I)$$
- Interquartile range:
$$\text{IQR} = P_{75\%}(I) - P_{25\%}(I)$$
- Energy:
$$\text{Energy} = \sum_{i=1}^{N} I(i)^2$$
- Root mean square:
$$\text{RMS} = \sqrt{\frac{\sum_{i=1}^{N} I(i)^2}{N}}$$

Entropy and Uniformity are both calculated after linearly downsampling the image bit depth to 8 (i.e. 256 gray levels).

- Entropy:
$$\text{Entropy} = \sum_{i=1}^{256} P(i) \log_2 P(i)$$
- Uniformity:
$$\text{Uniformity} = \sum_{i=1}^{256} P(i)^2$$

Preprocessing and Filter List:

- None: Features extracted from the original, unfiltered image.
- 2D Gradient Sobel Filter: 2D correlation 3-by-3 sobel (gradient) filter applied slice by slice (z direction) to the 3D CT image set.
- 3D Gradient Sobel Filter: 3D correlation 3-by-3-by-3 sobel (gradient) filter applied to the 3D CT image set.
- Local Entropy Filter (NHood=3): Resulting filtered image contains the entropy of the 3-by-3 neighborhood around each pixel. 2D filter applied slice by slice (z direction) to the 3D CT image set.

- Local Range Filter (NHood=3): Resulting filtered image contains the range (max - min) of the 3-by-3 neighborhood around each pixel. 2D filter applied slice by slice (z direction) to the 3D CT image set.
- Local Standard Deviation Filter (NHood=3): Resulting filtered image contains the standard deviation of the 3-by-3 neighborhood around each pixel. 2D filter applied slice by slice (z direction) to the 3D CT image set.
- Laplacian of Gaussian Filter (Size = 4, $\sigma = 1.0$): 2D correlation LoG filter of size = 4 and $\sigma = 1.0$ created using:

$$h_g(n_1, n_2) = e^{\frac{-(n_1^2+n_2^2)}{2\sigma^2}}$$

$$h(n_1, n_2) = \frac{(n_1^2 + n_2^2 - 2\sigma^2)h_g(n_1, n_2)}{2\pi\sigma^6 \sum_{n_1} \sum_{n_2} h_g}$$

Applied slice by slice (z direction) to the 3D CT image set.

- Laplacian of Gaussian Filter (Size = 6, $\sigma = 1.5$): 2D correlation LoG filter of size = 6 and $\sigma = 1.5$ applied slice by slice (z direction) to the 3D CT image set.
- Laplacian of Gaussian Filter (Size = 8, $\sigma = 1.8$): 2D correlation LoG filter of size = 8 and $\sigma = 1.8$ applied slice by slice (z direction) to the 3D CT image set.
- Laplacian of Gaussian Filter (Size = 10, $\sigma = 2.0$): 2D correlation LoG filter of size = 10 and $\sigma = 2.0$ applied slice by slice (z direction) to the 3D CT image set.
- Laplacian of Gaussian Filter (Size = 12, $\sigma = 2.5$): 2D correlation LoG filter of size = 12 and $\sigma = 2.5$ applied slice by slice (z direction) to the 3D CT image set.
- Threshold at 0 HU: All voxels greater than 0 HU are removed from the image.
- Threshold at -250 HU: All voxels greater than -250 HU are removed from the image.
- Threshold at -500 HU: All voxels greater than -500 HU are removed from the image.
- Threshold at -750 HU: All voxels greater than -750 HU are removed from the image.

Gray Level Co-occurrence Features

The gray level co-occurrence matrix (GLCM) is used to define texture in an image by determining the distribution of co-occurring voxel values that occur along a given displacement [1]. The displacement vector is defined by both the direction and distance for comparison of voxel pairs. Thirteen unique directions in 3-dimensional space were considered. Two additional GLCMs were calculated by summing the GLCM from all axial (i.e. 2D) directions and from summing all 13 3D directions.

N_g is the maximum number of gray levels in the image.

P is a $N_g \times N_g$ gray level co-occurrence matrix for an image I of size $L_y \times L_x \times L_k$ along a given displacement $\Delta y, \Delta x, \Delta z$.

$$P_{\Delta y, \Delta y, \Delta z}(i, j) = \sum_{r=1}^{L_z} \sum_{p=1}^{L_y} \sum_{q=1}^{L_x} \begin{cases} 1, \text{if } I(p, q, r) = i \text{ and } I(p + \Delta y, q + \Delta x, r + \Delta z) = j \\ 0, \text{otherwise} \end{cases}$$

The i^{th} entry in the marginal row matrix is $P_x(i) = \sum_{j=1}^{N_g} P(i, j)$.

The i^{th} entry in the marginal column matrix is $P_y(i) = \sum_{i=1}^{N_g} P(i, j)$.

μ is the mean of P, μ_x is the mean of P_x, μ_y is the mean of P_y, σ_x is the standard deviation of P_x, and σ_y is the standard deviation of P_y.

$P_{x+y}(k) = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i, j), i + j = k, k = 2, 3, 4, ..., 2N_g$.

$P_{x-y}(k) = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i,j), |i+j| = k, k = 0, 1, 2, ..., N_g - 1.$

The entropy of P_x is $HX = -\sum_{i=1}^{N_g} P_x(i) \log_2 [P_x(i)]$.

The entropy of P_y is $HY = -\sum_{i=1}^{N_g} P_y(i) \log_2 [P_y(i)]$.

The entropy of P is $HXY = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i,j) \log_2 [P(i,j)]$.

$HXY1 = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P(i,j) \log_2 [P_x(i) P_y(j)]$.

$HXY2 = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_x(i) P_y(j) \log_2 [P_x(i) P_y(j)]$.

GLCM Feature List:

- Autocorrelation [2]:
$$\text{Autocorrelation} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} ijP(i,j)$$
- Cluster prominence [2]:
$$\text{ClusterProminence} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} [i + j - \mu_x(i) - \mu_y(j)]^4 \, P(i,j)$$
- Cluster shade [2]:
$$\text{ClusterShade} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} [i + j - \mu_x(i) - \mu_y(j)]^3 \, P(i,j)$$
- Cluster tendency [2]:
$$\text{ClusterTendency} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} [i + j - \mu_x(i) - \mu_y(j)]^2 \, P(i,j)$$
- Contrast [2]:
$$\text{Contrast} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} |i - j|^2 \, P(i,j)$$
- Correlation [1]:
$$\text{Correlation} = \frac{\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} ijP(i,j) - \mu_x \mu_y}{\sigma_x \sigma_y}$$
- Difference entropy [1]:
$$\text{DiffEntropy} = \sum_{i=1}^{N_g - 1} P_{x-y}(i) \log_2 [P_{x-y}(i)]$$
- Dissimilarity [2]:
$$\text{Dissimilarity} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} |i - j| \, P(i,j)$$
- Energy [1]:
$$\text{Energy} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} [P(i,j)]^2$$

- Entropy [1]:

$$\text{Entropy} = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P(i,j)\log_2\left[P(i,j)\right]$$

- Homogeneity 1 [2]:

$$\text{Homogeneity1} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} \frac{P(i,j)}{1+[i-j]}$$

- Homogeneity 2 [2]:

$$\text{Homogeneity2} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} \frac{P(i,j)}{1+[i-j]^2}$$

- Information measures of correlation 1 [1]:

$$\text{InfoMC1} = \frac{HXY - HXY1}{max([HX, HY])}$$

- Information measures of correlation 2 [1]:

$$\text{InfoMC2} = \sqrt{1 - e^{-2(HXY2 - HXY)}}$$

- Inverse difference moment normalized [2]:

$$\text{InDiffMomNorm} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} \frac{P(i,j)}{1+\frac{|i-j|^2}{N^2}}$$

- Inverse difference moment [1]:

$$\text{InDiffMom} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} \frac{1}{1+(i-j)^2} P(i,j)$$

- Inverse difference normalized [2]:

$$\text{InDiffNorm} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} \frac{P(i,j)}{1+\frac{|i-j|}{N}}$$

- Inverse variance [2]:

$$\text{InVariance} = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} \frac{P(i,j)}{|i-j|^2}, i \neq j$$

- Max probability [2]:

$$\text{MaxProb} = max(P(i,j))$$

- Sum average [1]:

$$\text{SumAverage} = \sum_{i=2}^{2N_g} iP_{x+y}(i)$$

- Sum entropy [1]:

$$\text{SumEntropy} = -\sum_{i=2}^{2N_g} P_{x+y}(i)\log_2\left[P_{x+y}(i)\right]$$

- Sum variance [2]:

$$\text{SumVariance} = \sum_{i=2}^{2N_g} (i - \text{Sum entropy})^2 P_{x+y}(i)$$

- Variance [2]:

$$\text{Variance} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i - \mu)^2 P(i,j)$$

Gray Level Run Length Features

The gray level run length matrix (GLRLM), similar to the GLCM, is used to define texture in an image by considering strings of consecutive voxels that have similar gray values along a given direction [3]. As with the GLCM, 13 individual 3D directions as well as the 2D and 3D summed GLRLMs were calculated.

N_g is the maximum number of gray levels in the image and N_r is the number of different run lengths in the image.

P is a $N_g \times N_r$ gray level run length matrix for an image I of size $L_y \times L_x \times L_k$ along a given displacement $\Delta x, \Delta y, \Delta z$.

N_p is the total numer of voxels in the image which can be extracted from P using $N_p = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} P(i,j)j$.

The sum of P serves as a normalizing factor and is defined as $C = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} P(i,j)$

GLRLM Feature List:

- Short Run Emphasis [3]:

$$\text{SRE} = \frac{1}{C} \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{P(i,j)}{j^2}$$

- Long Run Emphasis [3]:

$$\text{LRE} = \frac{1}{C} \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} P(i,j) j^2$$

- Gray Level Nonuniformity [3]:

$$\text{GLNU} = \frac{1}{C} \sum_{i=1}^{N_g} \left[\sum_{j=1}^{N_r} P(i,j) \right]^2$$

- Run Length Nonuniformity [3]:

$$\text{RLNU} = \frac{1}{C} \sum_{j=1}^{N_r} \left[\sum_{i=1}^{N_g} P(i,j) \right]^2$$

- Run Percentage/Fraction [3]:

$$\text{Fraction} = \frac{C}{N_p}$$

- Low gray-level run emphasis [4,5]:

$$\text{LGRE} = \frac{1}{C} \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{P(i,j)}{i^2}$$

- High gray-level run emphasis [4,5]:

$$\text{HGRE} = \frac{1}{C}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r} P(i,j)i^2$$

- Short run low gray-level emphasis [5,6]:

$$\text{SRLGE} = \frac{1}{C}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r} \frac{P(i,j)}{i^2 j^2}$$

- Short run high gray-level emphasis [5,6]:

$$\text{SRHGE} = \frac{1}{C}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r} \frac{P(i,j)i^2}{j^2}$$

- Long run low gray-level emphasis [5,6]:

$$\text{LRLGE} = \frac{1}{C}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r} \frac{P(i,j)j^2}{i^2}$$

- Long run high gray-level emphasis [5,6]:

$$\text{LRHGE} = \frac{1}{C}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r} P(i,j)i^2 j^2$$

Neighborhood Gray-Tone Difference Features

The neighborhood gray-tone difference matrix (NGTDM) defines texture in an image by calculating the average gray level difference between each voxel and its neighboring voxels [7]. Each neighborhood was defined by considering only immediately adjacent voxels (i.e. distance = 1) in both two and three dimensions.

N_g is the maximum number of gray levels in the image.

$N_{g,\text{actual}}$ is the actual number of gray levels in the image (i.e. counting only the different gray levels that occur in the image).

Generalized to 3 dimensions, the average gray level (or gray-tone) of a voxel contained within the region of interest over a neighborhood centered at but excluding (p,q,r) is:

$$\overline{A}_i = \overline{A}(p,q,r) = \frac{1}{w-1} \times \left[\sum_{l=-1}^{1}\sum_{m=-1}^{1}\sum_{n=-1}^{1} I(p+l, q+m, r+n)\right], (l,m,n) \neq 0$$

Where $w = (2d+1)^2 - c, c = sum\{(p+l, q+m, r+n) \notin ROI\}$ accounts for any voxels that are not included in the possibly irregularly shaped ROI.

N_i is the set of all voxels with a gray level i, such that the $N_g \times 1$ neighborhood gray tone difference matrix is:

$$S(i) = \begin{cases} \sum_i i - \overline{A}_i, \text{for } i \in N_i \text{ if } N_i \neq 0 \\ 0, \text{otherwise} \end{cases}$$

NGTDM Feature List:

- Coarseness [7]:

$$\text{Coarseness} = \left[\epsilon + \sum_{i=0}^{Ng} P(i)S(i)\right]^{-1}$$

- Contrast [7]:

$$\text{Contrast} = \left[\frac{1}{N_{g,\text{actual}}(N_{g,\text{actual}} - 1)} \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P(i)P(j)(i-j)^2\right] \left[\frac{1}{N} \sum_{i=1}^{N_g} S(i)\right]$$

- Busyness [7]:

$$\text{Busyness} = \frac{\sum_{i=1}^{Ng} P(i)S(i)}{\sum_{i=1}^{Ng} \sum_{j=1}^{Ng} [iP(i) - jP(j)]}, P(i) \neq 0, P(j) \neq 0$$

- Complexity [7]:

$$\text{Complexity} = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} \frac{|i-j| \left[P(i)S(i) + P(j)S(j)\right]}{N^2 \left[P(i) + P(j)\right]}, P(i) \neq 0, P(j) \neq 0$$

- Strength [7]:

$$\text{Strength} = \frac{\sum_{i=1}^{Ng} \sum_{j=1}^{Ng} \left[P(i) + P(j)\right](i-j)^2}{\epsilon + \sum_{i=1}^{Ng} S(i)}, P(i) \neq 0, P(j) \neq 0$$

Laws' Filtered Features

Laws' filters are a series of 5 one dimensional spatial filters that are convolved with the image to emphasize textural structure [8]. Fifteen pairs of the Laws' 1x5 filters were applied to each axial slice of the image.

The Laws filters utilized are:

$$\text{Level 5: L5} = \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$\text{Energy 5: E5} = \begin{bmatrix} -1 & -2 & 0 & 2 & 1 \end{bmatrix}$$

$$\text{Spot 5: S5} = \begin{bmatrix} -1 & 0 & 2 & 0 & -1 \end{bmatrix}$$

$$\text{Wave 5: W5} = \begin{bmatrix} -1 & 2 & 0 & -2 & 1 \end{bmatrix}$$

$$\text{Ripple 5: R5} = \begin{bmatrix} 1 & -4 & 6 & -4 & 1 \end{bmatrix}$$

Image I of size $L_y \times L_x \times L_k$ with a total number of voxels, N.

P is the ordered image histogram, where $P(i)$ is the number of voxels of the i^{th} gray level.

LAWS2D Feature List:

- Mean:

$$\text{Mean} = \overline{I} = \frac{1}{N} \sum_{i=1}^{N} I(i)$$

- Variance:

$$\text{Variance} = \frac{1}{N-1} \sum_{i=1}^{N} (I(i) - \overline{I})^2$$

- Skewness:

$$\text{Skewness} = \frac{\frac{1}{N}\sum_{i=1}^{N}(I(i)-\overline{I})^3}{\sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}(I(i)-\overline{I})^2\right]^3}}$$

- Kurtosis:

$$\text{Kurtosis} = \frac{\frac{1}{N}\sum_{i=1}^{N}(I(i)-\overline{I})^4}{\left[\frac{1}{N}\sum_{i=1}^{N}(I(i)-\overline{I})^2\right]^2}$$

- Energy:

$$\text{Energy} = \sum_{i=1}^{N} I(i)^2$$

- Root mean square:

$$\text{Root mean square} = \sqrt{\frac{\sum_{i=1}^{N} I(i)^2}{N}}$$

Entropy and Uniformity are both calculated after linearly downsampling the image bit depth to 8 (i.e. 256 gray levels).

- Entropy:

$$\text{Entropy} = \sum_{i=1}^{256} P(i)\log_2 P(i)$$

- Uniformity:

$$\text{Uniformity} = \sum_{i=1}^{256} P(i)^2$$

Lung-Specific CT Features

Multiple methods have been proposed to quantify COPD in patients using CT by representing the volume and cluster density of low attenuation areas in the lung.

Image I of size $L_y \times L_x \times L_k$ with a total number of voxels, N, with voxel volume v.

P is the ordered image histogram, where $P(i)$ is the number of voxels of the i^{th} gray level.

LUNG Feature List:

- Mode [9]:

$$\text{Mode} = \text{gray level with the highest frequency}$$

- Volume of low attenuation area voxels below -856 HU [10]:

$$LAA_{-856} = v \cdot \sum_{i=-1000}^{-856-1} P(i)$$

- Percent volume of low attenuation area voxels below -856 HU [10]:

$$\%LAA_{-856} = \frac{1}{N} \cdot \sum_{i=-1000}^{-856-1} P(i)$$

- Volume of low attenuation area voxels below -864 HU [9]:

$$LAA_{-864} = v \cdot \sum_{i=-1000}^{-864-1} P(i)$$

- Percent volume of low attenuation area voxels below -864 HU [9]:

$$\%LAA_{-864} = \frac{1}{N} \cdot \sum_{i=-1000}^{-864-1} P(i)$$

- Volume of low attenuation area voxels below -900 HU [11]:

$$LAA_{-900} = v \cdot \sum_{i=-1000}^{-900-1} P(i)$$

- Percent volume of low attenuation area voxels below -900 HU [11]:

$$\%LAA_{-900} = \frac{1}{N} \cdot \sum_{i=-1000}^{-900-1} P(i)$$

- Volume of low attenuation area voxels below -910 HU [9,12,13]:

$$LAA_{-910} = v \cdot \sum_{i=-1000}^{-910-1} P(i)$$

- Percent volume of low attenuation area voxels below -910 HU [9,12,13]:

$$\%LAA_{-910} = \frac{1}{N} \cdot \sum_{i=-1000}^{-910-1} P(i)$$

- Volume of low attenuation area voxels below -960 HU [11,14]:

$$LAA_{-960} = v \cdot \sum_{i=-1000}^{-960-1} P(i)$$

- Percent volume of low attenuation area voxels below -960 HU [11,14]:

$$\%LAA_{-960} = \frac{1}{N} \cdot \sum_{i=-1000}^{-960-1} P(i)$$

- Volume of non-emphysematous lung [11]:

$$\text{Reserve} = v \cdot \sum_{i=-850}^{-700} P(i)$$

- Percent volume of non-emphysematous lung [11]:

$$\%\text{Reserve} = \frac{1}{N} \cdot \sum_{i=-850}^{-700} P(i)$$

Using thresholds of -910 HU and -960 HU, emphysematous lesion clusters were defined using 26-neighbor connectivity. We let $V(i)$ denote the volume of the i^{th} cluster where N is the total number of clusters at a given HU threshold.

- Number of clusters below -910 HU [15]:

$$\text{Cluster Count } LAA_{-910} = N$$

- Mean volume of clusters below -910 HU [15]:

$$\text{Mean Cluster Volume } LAA_{-910} = \overline{V}_{-910} = \frac{1}{N}\sum_{i=1}^{N} V(i)$$

- Standard deviation of volume of clusters below -910 HU [15]:

$$\text{Std Cluster Volume } LAA_{-910} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(V(i) - \overline{V}_{-910})^2}$$

- Number of clusters below -960 HU [14]:

$$\text{Cluster Count } LAA_{-960} = N$$

- Mean volume of clusters below -960 HU [14]:

$$\text{Mean Cluster Volume } LAA_{-960} = \overline{V}_{-960} = \frac{1}{N}\sum_{i=1}^{N} V(i)$$

- Standard deviation of volume of clusters below -960 HU [14]:

$$\text{Std Cluster Volume } LAA_{-960} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(V(i) - \overline{V}_{-960})^2}$$

A power law relationship was used to describe the relationship between the cumulative number of cluster lesions (Y) and the lesion volume (V) such that:

$$Y = K \cdot V^{-D}$$

Least squares regression of $\log(Y)$ versus $\log(A)$ gives estimates of the fit parameters D and K.

- Power law slope for emphysematous lesion clusters below -910 HU [15]:

$$D_{-910}$$

- Power law intercept for emphysematous lesion clusters below -910 HU [15]:

$$K_{-910}$$

- Power law slope for emphysematous lesion clusters below -960 HU [14]:

$$D_{-960}$$

- Power law intercept for emphysematous lesion clusters below -960 HU [14]:

$$K_{-960}$$

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